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Scarcity vs. Pollution in Public Policy toward Fossil Fuels

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DRAFT

Scarcity vs. Pollution in Public Policy toward Fossil Fuels

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Abstract

Most policy exercises that model the optimal control of greenhouse gas emissions have focused almost exclusively on the pollution problem in isolation from the fossil fuels scarcity problem. We argue that this approach misses important interactions between the two issues and, contrary to what is claimed, will lead to sub-optimal policies, at least within the framework of the models employed. To demonstrate, we employ an intertemporally optimizing model of economy and climate, with carbon resource scarcity and a backstop technology. Using plausible parameter values, we conclude that the initial resource shadow price is approximately twice the value of the pollution shadow price. Therefore, the optimal carbon tax is approximately three times what would be recommended if we focused solely on the pollution problem. This result is robust to changes in the values of key parameters, including the social discount rate and the backstop price.

(JEL CODES: Q3, Q4. Keywords: pollution, scarcity, carbon tax, climate policy)

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I. Introduction.

During the past two decades, there has been extensive economic research on pollution associated with fossil fuels, such as the emission of greenhouse gases. However, relatively little attention paid to the scarcity of the underlying resource base.

Nonetheless, there is reason to believe that the scarcity of fossil fuels is an important issue. In particular, research shows that producers in the oil and gas industry use a high rate of discount (in excess of 25 % per annum) to evaluate new projects (Farzin 1986, 2001; Pesaran 1990). In contrast, most public agencies now use a social discount rate for evaluating projects which is much lower – between 3 and 5 % per annum in many cases. In these circumstances, the market price charged by producers will significantly undervalue the social user cost of extracting the resource.

It is straightforward to show that the gross price of the polluting non-renewable resource in the first-best world includes the resource and pollution shadow prices. In contrast, in deriving the decentralized market equilibrium one can demonstrate that social prices of pollution and resource stocks are ignored. Therefore, the optimal tax program that would make the decentralized solution identical to the socially optimum must include the shadow values of the both stocks. If the central planner charges a carbon tax only and the resource shadow price is negligible, the optimal path of the decentralized economy will be close or identical to the socially optimal one. However, if the fuel stock shadow price is not negligible, but ignored, the economy will not exhibit the optimal path, even with the carbon tax in place.

Nonetheless, most policy exercises to model the optimal control of greenhouse gas emissions have focused almost exclusively on the pollution problem in isolation from the scarcity problem (e.g. Nordhaus and Boyer 2000; Popp 2004; Gerlagh 2008). We argue that this approach misses important interactions between the two issues and, contrary to what is claimed, will lead to sub-optimal policies, at least within the framework of the models employed.

To demonstrate, we employ an intertemporally optimizing model of economy and climate, with carbon resource scarcity and a backstop technology. The economy-climate model is DICE (Nordhaus and Boyer 2000).[2] Carbon resource scarcity is based on the estimates of increasing supply price provided by Rogner (1997). For the backstop, we employ a simple switching regime from carbon fuels to an unlimited, non-polluting source at a high threshold price.

Using plausible parameter values, we conclude that the initial resource shadow price is approximately twice the value of the pollution shadow price. Therefore, the optimal carbon tax is approximately three times what would be recommended if we focused solely on the pollution problem. This result is robust to changes in the values of key parameters, including the social discount rate and the backstop price.

II. Models

1. Centrally planned economy

Consider the problem of the central planner, who makes consumption-production decisions for the whole economy. The gross output, $Y(t)$, is produced by means of capital, $K(t)$,energy services, $E(t)$, and labour, $L(t)$. The production function $Q(K,E,L)$ is assumed to be strictly concave, twice differentiable and exhibits constant returns to scale: $Q_i > 0, Q_{ii} < 0$ where $i = K,E$.[3] We also assume that the inputs are essential: $Q(0,E,L)=0$ and $Q(K,0,L)=0$ and Inada conditions are satisfied. The world is endowed with some initial capital stock, K_0. We assume that labour is fully employed and supplied inelastically. Energy services are provided by fossil fuels, $F(t) : E(t)=F(t)$. The use of fossil fuels generates a pollution stock of greenhouse gases, $S(t)$, which accumulate in the atmosphere according to the following law of motion:

$$\dot{S} = F - \delta_S S, \qquad (1)$$

where δ_S is the decay rate of the pollution stock.[4]

The pollution stock has a negative effect on production, represented by the

[2] *DICE* – *D*ynamic *I*ntegrated model of *C*limate and *E*conomy

[3] To denote the partial derivative of function F with respect to x the following notation is used: F_x

[4] The notation $\dot{F}$ denotes the time derivative.

damage function $D(S)$ which possesses the following properties: $D(0)=1,\quad D'<0,\quad D''<0$. The output adjusted to climate damages is $D(S)Q(K,E,L)$. In each period some amount of fuels is extracted, building the stock of cumulative extraction, Cm_t:

$$\dot{Cm}=F \tag{2}$$

The total fuels extraction costs at time *t* is the product of the current extraction $F(t)$ and $F(t)$ and the unit extraction cost $Z\ \ Cm(t)$ .The unit extraction cost is assumed to be the strictly convex function of cumulative extraction: $Z'>0,\ Z''>0$. The stock of fuels in carbon units is assumed to be unlimited, meaning that we rely on the economic scarcity.

At each period *t* the planner distributes the output between the capital investment, K, consumption, C, and the purchase of fossil fuel:

$$\dot{K}=D(S)Q(K,E,L)-C-Z\ \ Cm\ \ F-\delta_k K \tag{3}$$

where δ_K is the decay rate of capital stock. The planner maximizes an aggregate welfare function $V=\int_0^\infty LU(\frac{C}{L})e^{-\rho t}dt$ where L denotes the population of the economy (assumed equal to labour supply), $\frac{C}{L}$ is the per-capita consumption, $U\ \cdot$ is the strictly concave utility function, that exhibits Inada conditions, $U\ \ 0\ =0, U_{\frac{C}{L}}>0$, and ρ is the pure rate of time preference. Also: $K(0), S(0), L(0)>0$ and $\lim_{t\to\infty} e^{-\rho t}\lambda(t)K(t)=0$, $\lim_{t\to\infty} e^{-\rho t}\mu(t)S(t)=0$ and $\lim_{t\to\infty} e^{-\rho t}\theta(t)Cm(t)=0$, where co-state variables λ,μ and θ represent the shadow values of capital, pollution and resource stocks respectively. The current-value Hamiltonian of the problem is written as[5]:

$$H=LU(\frac{C}{L})+\lambda\left[D(S)Q(K,F,L)-C-Z\ \ Cm\ \ F-\delta_k K\right]+\mu[F-\delta_S S]+\theta F$$

The control variables are C and F and the state variables are K, S and Cm. If an optimum exists, then the necessary conditions for optimality are as follows:

[5] Time notation is dropped where appropriate.

$$\frac{\partial H}{\partial C}=U'(\frac{C}{L})-\lambda=0 \qquad (4)$$

$$\frac{\partial H}{\partial F}=\lambda \ \ D(S)Q_F(K,F,L)-Z(Cm) \ \ +\mu+\theta=0 \qquad (5)$$

$$-\frac{\partial H}{\partial K}=\dot{\lambda}-\rho\lambda=-\lambda[D(S)Q_K(K,F,L)-\delta_K] \qquad (6)$$

$$-\frac{\partial H}{\partial S}=\dot{\mu}-\rho\mu=\mu\delta_s-\lambda D'(S)\,Q(K,F,L) \qquad (7)$$

$$-\frac{\partial H}{\partial Cm}=\dot{\theta}-\rho\theta=\lambda Z'(Cm)F \qquad (8)$$

(1)-(3) and the transversality conditions described above.

In our subsequent analysis, we will need expressions for the pollution and resource shadow prices. Integrating (6) and (7) yields:

$$\lambda(t)=\lambda(v)e^{-\int_t^v \ \delta_K+\rho-D(S)Q_K(K,F,L) \ dq}, \qquad v\geq t \qquad (9)$$

$$\mu(t)=\int_t^\infty \exp^{-(\delta_s+\rho)(v-t)}\lambda(v)D'(S(v))\,Q(K(v),F(v),L(v))\,dv \qquad (10)$$

Substituting (9) into (10) for $\lambda(v)$ we obtain the pollution shadow price:

$$\mu(t)=\lambda(t)\int_t^\infty \exp^{-\int_t^v \ \delta_s+D(S)Q_K(K,F,L)-\delta_K \ dq} D'(S(v))\,Q(K(v),F(v),L(v))\,dv \qquad (11)$$

The pollution shadow price according to (11) is the sum of the discounted values of the marginal damages, which happen due to the increase in the carbon stock by one unit at time *t*. This shadow price of pollution can also be considered as the discounted marginal benefits if the additional unit of emission is not emitted at *t*.

Integrating (8) and substituting (9) we obtain the resource stock shadow price:

$$\theta(t)=-\lambda(t)\int_t^\infty e^{\int_t^v - \ D(S)Q_K(K,F,L)-\delta_K \ dq} Z' \ \ Cm(v) \ \ F(v)dv \qquad (12)$$

The equation (12) gives the present value of the increase in the costs of extraction over all future periods due to the extraction of one unit of fuels at time *t*. Extracting one unit of fuels at *t* increases the extraction costs over the remaining periods of the planning horizon of the model.

In what follows we will also need the equilibrium condition of resource extraction. Rearranging (5) we obtain:

$$D(S)Q_F(K,F,L)=Z(Cm)-\frac{\mu}{\lambda}-\frac{\theta}{\lambda}$$

or $$p_F=Z(Cm)-\frac{\mu}{\lambda}-\frac{\theta}{\lambda} \qquad (5')$$

where p_F is optimal fuel price. It can be seen from (4), (11) and (12) that $\lambda>0,\ \mu,\theta<0$. Hence equation $(5')$ suggests that the gross price of the resource (i.e. the value of the marginal product of the additional unit of fuels) equals the value of extraction costs plus pollution and resource shadow prices. Also note that equation $(5')$ gives the implicit demand function for energy. Since $\lambda>0,\ \mu,\theta<0$, equation $(5')$ implies:

$$D(S)Q_F(K,F,L)>Z(Cm) \quad . \qquad (13)$$

2. Decentralized equilibrium. Optimal taxation program.

Consider an economy that consists of a representative consumer, a producer and the government. The representative consumer maximizes the *intertemporal* utility function by choosing consumption and savings paths. The producer extracts a polluting non-renewable resource (fossil fuels) that is used in the production of the composite consumption good. The government redistributes taxes and profits.

The representative producer, in contrast to the consumer, maximizes profit period by period. In other words, the producer maximizes the *static* profit. This way of modeling captures the myopic behavior of oil producers, which is confirmed by empirical literature. Farzin (1986) found that U.S. oil producers have a 33% discount rate and a four year time horizon.[6] The results of the empirical study of U.K. oil producers by Pesaran (1990) also imply static profit maximization, since he found the evidence of the zero discount factor and rejected the hypothesis of the present-value maximization. Hence, fuels producers are expected to ignore the effect of current extraction on the future extraction costs, taking the unit cost of extraction as given each period. In other words, the user cost (scarcity rent) disappears. Also, given static profit maximization, the representative producer ignores the effect of emissions on future production and consumer's utility. Hence all pollution cost is external. Thus the dynamics of cumulative extraction and pollution stock are exogenous.

The producer is endowed with some stock of fuels that is not exogenously fixed. The consumer maximizes the utility function $V=\int_0^{\infty} Lu(\frac{C}{L})e^{-\rho t}dt$, subject to the budget constraint: $\dot{K}=\pi+rK+wL+\Gamma-C-\delta_K K$, where π is the profit of the representative firm, which is distributed to the consumer, r is the rental rate of capital, w is the equilibrium wage rate, Γ is the lump sum transfer of fuel tax to consumer. The tax is redistributed by the government to the consumer. Labour is fully employed and supplied inelastically. The current-value Hamiltonian is given by:

$H=Lu(\frac{C}{L})+\xi \;\; \pi+rK+wL+\Gamma-C-\delta_K K$. The necessary conditions for the interior optimum are the following:

$$\frac{\partial H}{\partial C}=u'(\frac{C}{L})-\xi=0\,; \qquad (14)$$

$$-\frac{\partial H}{\partial K}=\dot{\xi}-\rho\xi=-\xi(r-\delta_K) \qquad (15)$$

$\lim_{t\to\infty} e^{-\rho t}\lambda_t K_t=0$ and capital law of motion, described above.

The laws of motion of pollution and cumulative extraction stocks are: $\dot{S}=F-\delta_S S$ and $C\dot{m}=F$ as before.

The static profit maximization problem of producer is:

$$\max_{\{F,K,L\}} \pi=D(S)Q(K,F,L)-rK-wL-Z \;\; Cm \;\; F-\tau F$$

Government charges fuel tax $\tau:\Gamma=\tau F$. First-order conditions of the problem, assuming interior optimum, are the following:

$$\frac{\partial \pi}{\partial K}=D(S)Q_F(K,F,L)-Z \;\; Cm \;\; -\tau=0 \qquad (16)$$

[6] This observation was used in Farzin (2001) in the context of modeling the exploration activity of oil producers.

$$\frac{\partial\pi}{\partial K}=D(S)Q_K(K,F,L)-r=0 \qquad (17)$$

$$\frac{\partial\pi}{\partial L}=D(S)Q_L(K,F,L)-w=0 \qquad (18)$$

Combining first-order conditions of consumer and producer problems, we can describe the decentralized equilibrium:

$$u'(\frac{C}{L})=\xi\,; \qquad (19)$$

$$D(S)Q_F(K,F,L)=Z\ \ Cm\ +\tau\,; \qquad (20)$$

$$\dot{\xi}-\rho\xi=-\xi\ \ D(S)Q_K(K,F,L)-\delta_K \qquad (21)$$

Comparing (4) and (19) reveals that $\lambda=\xi$. Equation (21) is identical to (6). It follows from (20) and (5): $\tau=-\left(\frac{\mu+\theta}{\lambda}\right)$, which suggests that decentralized equilibrium is going to be identical to the social optimum if the fuel tax charged by the government consists of two components. The first component, $\frac{\mu}{\lambda}$, represents social cost of pollution (pollution shadow price) in terms of consumption numeraire. This component is denoted as carbon tax. The second component, $\frac{\theta}{\lambda}$, can be denoted as the depletion fee and equals the shadow value of the resource stock that is ignored by the fuels producers.

To summarize, in order for economy to follow the optimal path, both stock externalities (pollution and resource) have to be addressed. If the government charges carbon tax only and the resource shadow price is negligible, the optimal path of the decentralized economy will be close or identical to the socially optimal one. However, if the fuels depletion fee is not negligible, but ignored, the economy will not exhibit the optimal path, even with the carbon tax in place. This issue has not been discussed in the literature, with the exception of the work by Farzin (1996), who assumed the constant depletion fee.

3. Backstop technology

The long-term considerations of climate-economy related issues should include the existence of the non-pollution backstop technology, otherwise the results of the model simulations could be misleading (e.g. Nordhaus 2008). We now modify the planner's model described above by adding the non-polluting backstop technology, which does not have a capacity constraint. We model backstop technology, B, as the perfect substitute to fossil fuels as, for example, in Nordhaus and Boyer (2000). Thus energy services are now written as follows: $E(t)=F(t)+B(t)$. The capital law of motion has to be modified as well:

$$\dot{K}=D(S)Q(K,E(F,B),L)-C-Z\ \ Cm\ \ F-\delta_k K-bpB \tag{3`}$$

where bp is the constant unit price (marginal cost) of backstop technology.

The current-value Hamiltonian of the problem is defined as:

$$H=LU(\frac{C}{L})+\lambda\left[D(S)Q(K,E(F_t,B_t),L)-C-Z\ \ Cm\ \ F-\delta_k K-bpB\right]+\mu[F-\delta_S S]+\theta F$$

We are particularly interested in the first-order conditions with respect to F and B:

$$\frac{\partial H}{\partial F}=\lambda\ \ D(S)Q_E E_F-Z(Cm)\ \ +\mu+\theta\leq 0\ ,F\geq 0,\ \frac{\partial H}{\partial F}F=0 \tag{23}$$

$$\frac{\partial H}{\partial B}=\lambda\ \ D(S)Q_E E_B-bp\ \ \leq 0\ ,B\geq 0,\ \frac{\partial H}{\partial B}B=0 \tag{24}$$

Thus we define three cases:

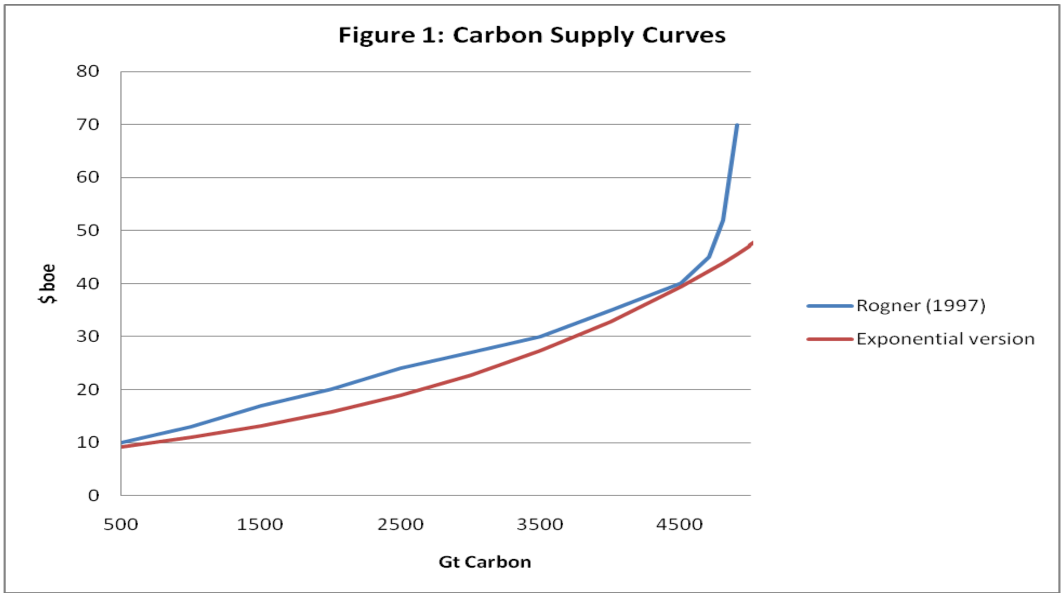
$$\begin{aligned}
&\text{i)}\ \ Z(Cm)-\frac{\mu}{\lambda}-\frac{\theta}{\lambda}<bp,\ B=0,\ F>0,\ \frac{\partial H}{\partial F}=0\\
&\text{ii)}\ \ Z(Cm)-\frac{\mu}{\lambda}-\frac{\theta}{\lambda}=bp,\ F,B\geq 0,\ \frac{\partial H}{\partial F}=\frac{\partial H}{\partial B}=0\\
&\text{iii)}\ \ Z(Cm)-\frac{\mu}{\lambda}-\frac{\theta}{\lambda}>bp,\ F=0,B>0,\ \frac{\partial H}{\partial B}=0
\end{aligned} \tag{25}$$

From (25) we can see that backstop technology becomes operational at the point where $bp=Z(Cm)-\frac{\mu}{\lambda}-\frac{\theta}{\lambda}$ (note that $Q_E E_F = Q_E E_B$). In other words, as long as the price of backstop is higher than the efficient fuel price, defined in (5`), the central planner relies on polluting fuels only. At the point in time where the price of backstop equals the efficient fuel price, which includes resource and pollution shadow values, both sources of energy can be used; however in the following periods, the planner optimally chooses to use the backstop only.

III. Results

Popp (2004) modified the original DICE-99 model constructed by Nordhaus and Boyer (2000) by adapting the energy supply sector of Nordhaus' regionalized model RICE-99 (Nordhaus and Boyer 2000) to the single-region structure of DICE. We further extended Popp's (2004) version of DICE model by including non-polluting backstop technology which is modeled as a perfect substitute to the conventional fossil fuels, as in Nordhaus and Boyer (2000). The simulation model maintains the same structure as the theoretical model of the central planner with the backstop technology presented above. The climate module is, however, more complex.[7] The gross output is produced by means of labour, capital and energy services. Fossil fuels and backstop technology are modeled as perfect substitutes in energy production. We also assume technological change; however we restrict ourselves to the case of energy producing exogenous technological change. In particular, the energy production function becomes: $E=\frac{F}{\Phi}+B$, where Φ_t is the energy efficiency parameter that exogenously declines over time.

The price of fuels is modeled as a function of cumulative fuel consumption, and therefore is determined endogenously and reflects scarcity. Rogner (1997) provides the assessment of hydrocarbon resources and derives the quantity-cost schedule that is used in the present model. Nordhaus and Boyer (2000) present the functional relationship that is aimed to mimic the Rogner's curve. However, according to Rogner's calculations when cumulative extraction of carbon reaches 1,500 GtC, the price doubles while in the case of Nordhaus and Boyer it changes by less than 3%. Grubler and Gritsevskyi (1997) assume an exponential relationship to fit Rogner's quantity-supply curve, while Greene et al. (2003) present logarithmic functional form to mimic the behavior of the carbon supply curve suggested by Rogner. In the present paper we assume an exponential relationship between cumulative extraction of carbon and marginal costs (Figure 1). Note that the fuels production costs map includes the forecasted technological advances in fuels extraction industry.



The carbon emissions, at each period *t* contribute to the stock of pollution, (M_t) according to the law of motion:

$$M_{t+1} - M_t = \alpha 10 F_t - \delta_M (M_t - 590) \qquad (26)$$

where α is the marginal atmospheric retention ratio, δ_M is the natural decay rate and 590 billion tons of carbon is the pre-industrial level of carbon stock. The carbon stock via a system of geophysical relationships leads to the increased mean temperature levels that negatively affect output production[8]. The climate feedback in the gross output equation is provided by the climate damage function: $D(TE_t)=1+d(TE_t)^2$, where d is the climate change damage parameter and TE_t is the increase in the atmospheric temperature level relative to the base period. The output net of climate damages is therefore

[7] For detailed description of DICE model, please see Nordhaus and Boyer (2000), Popp (2004). The model used here does not have three-box model of carbon flows as in DICE 1999 due to its unrealistic behaviour in the steady state. In fact, we use the climate module described in Nordhaus (1994).
[8] For details of climate module see Nordhaus (1994), Nordhaus and Boyer (2000)

$Y_t = \frac{Q_t}{D(TE_t)}$ .The starting values for the variables and the parameters were calibrated to their 1990 level. The initial period in the model is 1995 and the model operates in ten-year steps.

In the theoretical model discussed above, the time horizon was infinite. In contrast, the computational model requires a finite time horizon. Thus we deal with the problem of numerical approximation of the infinite time horizon: we are looking for a time horizon such that the values of the variables along the optimal path are on the turnpike. Shiell (2001) discusses the methods of approximating the infinite horizons. The less problematic is the one that requires the computation of the steady-state values of the state variables, which are then used in the dynamic objective function: terminal stock valuation method. This method ensures that the values of the control variables are on the turnpike and therefore provide the true scarcity-pollution relationship. Otherwise, the results would be arbitrary.[9] Thus we first compute the steady-state values of the shadow prices and then use them in the dynamic model as part of the objective function. In particular, we compute the modified golden rule, as discounting is involved. [10]

We also need the value for the price of the backstop technology. Popp (2006) reviews existing estimates of the clean backstop price in terms of carbon ton equivalent. In particular, he suggests $400 per ton of carbon equivalent (t.c.e) as low estimate and $1,200 as high estimate. These prices are in line with Nordhaus (2008) who sets the price of $1,200 for the backstop technology, which represents the marginal cost of abating the last unit of emissions. In our model, where backstop technology is modeled as a perfect substitute for fossil fuels energy, the clean technology becomes operational when fuels in all industries can be completely substituted by backstop technology. As Nordhaus (2008) observes, lower backstop price could represent the technology that substitutes polluting fuels in a particular sector only: at $500 per t.c.e the nuclear power can substitute the coal in electricity production. In the simulations, we use three estimates for the backstop technology: low ($400 per t.c.e), medium ($800 t.c.e) and high ($1,200 t.c.e.).

[9] Note that we are not able to characterize the steady-state of the model without backstop technology, as in Farzin (1996), due to non-constant marginal depletion cost and marginal damages.
[10] Appendix provides the details of steady-state computation.

The ratio, R, of scarcity and pollution shadow values the first period of model, (i.e. from the perspective of today's society) provides the relative importance of one issue over the other:[11]

$$R = \frac{\theta(t=1)}{\mu(t=1)}$$

Table 1: Scarcity-pollution ratio (R)

Backstop price ($/ t.c.e)	**No technological change ($\Phi_t = 1$)**				**Technological change ($\Phi_t < 1$)**	
	$\rho = 3\%$		$\rho = 1\%$		$\rho = 3\%$	
	R	switching period	R	switching period	R	switching period
$400	1.7236	T=11	0.6143	T=8	1.8794	T=25
$800	1.8953	T=28	1.1496	T=25	1.8824	T=43
$1,200	1.8965	T=38	1.2163	T=36	1.8825	T>60

A number of conclusions can be drawn using Table 1. Firstly, using conventional discounting scarcity is relatively more important than pollution. Secondly, the ratio increases with the rate of time preference. Thirdly, the transition period increases with the discount rate. Fourthly, technological change does not have a significant effect on the ratio, however, it does affect the switching time to backstop.

Although the rate of the time preference does not appear in equations (11) and (12) its effect comes indirectly through the marginal product of capital. Using equations (4) and (6) the following well-known expression can be derived:

$D(S)Q_k(K(t),F(t),L(t)) - \delta_K = \rho + \sigma\frac{\dot{c}}{c(t)}$ where σ is the elasticity of marginal utility with respect to consumption. Thus the change in the rate of time preference ρ affects the marginal product of capital $D(S)Q_K(K_t, F_t, L_t)$, which in turn affects the discount rate of the resource and pollution stocks shadow values.

[11] To be exact, the values for the ratio will be reported for the second period of the model, as shadow values for the first period are equal to zero. Second period of the model corresponds to year 2005.

We observe that the ratio increases with the discount rate, the pollution shadow value is more sensitive to the changes in the discount rate than scarcity rent. The apparent reason is that pollution damages last till the final period of the model, (i.e. pollution stock is long lasting), while scarcity rent becomes zero at the switching time.

The higher discount rate implies a higher emissions/extraction rate, which in turn means higher unit extraction costs per period. At the same time, pollution and resource shadow values are discounted more heavily and hence are lower in the present-value terms. The increase in the unit cost of extraction is more than offset by the lower shadow values; thus the social cost of fuels $(5')$ is lower, which leads to a longer transitional path to the backstop technology as seen in the Table 1 .

Table 1 also demonstrates the effect of technological change on the ratio. With technological change in place along the path described in (25 case i), the equilibrium condition on the fuels market is described by

$$D(S)Q_E(K,E,L)E_F = D(S)Q_E(K,E,L)\frac{1}{\Phi} = Z(Cm) - \frac{\mu}{\lambda} - \frac{\theta}{\lambda}$$

or

$$D(S)Q_E(K,E,L) = \left[Z(Cm) - \frac{\mu}{\lambda} - \frac{\theta}{\lambda}\right]\Phi \qquad (27)$$

Note that the term on the RHS of (27) is the social cost of energy. Since $\Phi < 1$, the technological change reduces the marginal cost of fossil-based energy *ceteris paribus*. With $\Phi < 1$, i.e. with technological change, the left-hand side of (27) should decrease to restore the equilibrium, which implies higher energy and therefore fuels use, given the state of technology Φ .The increase in fuel use increases the pollution cost (11).

The reduction in the equilibrium fuels price given by (27) , implies a longer transition period to backstop technology relative to the case $\Phi = 1$ and higher fuels levels, which in turn leads to higher cumulative extraction levels: $Cm_{\Phi=1} < Cm_{\Phi<1}, \ \forall t$, which leads to a higher scarcity rents. While it is clear that transition period to backstop technology becomes longer with energy related technological change, it is unclear if the degree to which the pollution cost rises due to technological change is fully offset by the increase in scarcity rent, (i.e. the ratio of the shadow values remains constant, decreases or increases comparatively to the case with no technological change). The simulations results reveal that the ratio is neutral with respect to technological change, as no significant effect on the ratio is observed.

IV. Conclusion.

The optimal price of the resource consists of three elements: the unit extraction costs and resource and pollution social shadow prices. We have shown that the price of the resource in the decentralized market system is lower than the optimal price by the value of fuel tax that includes the shadow values of resource and pollution stocks. In order to make the decentralized solution socially efficient, private agents must pay the tax that consists of both resource and pollution stocks shadow prices.

The objective of the present paper was to consider the relative importance of shadow values and to evaluate it quantitatively. As the indicator we chose the ratio of the initial values of the resource and pollution shadow prices. As long as the ratio is greater than unity then scarcity issues prevail over the pollution considerations.

Clearly, the ignorance of the scarcity issue in the carbon policy design will lead to sub-optimal outcomes, since our results demonstrate that the initial resource shadow price is approximately twice the value of the pollution shadow price. Therefore, the optimal carbon tax is approximately three times what would be recommended if we focused solely on the pollution problem.

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APPENDIX

Constraints of the model, with backstop technology as energy services provider:

$$Y(t)=\frac{A(t)K(t)^{\gamma}L(t)^{1-\gamma-\beta}ENERGY(t)^{\beta}}{(1+0.0021TE(t)^{2})} \quad (A1)$$

$$K(t+1)-K(t)=10[I(t)-\frac{(1-(1-\delta_K))^{10}}{10}K(t)] \quad (A2)$$

$$C(t)=Y(t)-I(t)-bpB(t) \quad (A3)$$

$$ENERGY(t)=B(t) \quad (A4)$$

$$FORC(t)=4.1*\log(M(t)/590)/\log 2+FORCOTH(t) \quad (A5)$$

$$M(t+1)-M(t)=\delta_M(590-M(t)) \quad (A6)$$

$$TE(t+1)-TE(t)=C1[FORC(t)-(\lambda+C3)TE(t)+C3TL(t)] \quad (A7)$$

$$TL(t+1)-TL(t)=C4[TE(t)-TL(t)] \quad (A8)$$

The objective function of RA's problem is $\int_0^T e^{-10\rho t}[L(t)\ln\frac{C(t)}{L(t)}]dt$ .The control variables are C, B.

The state variables are: K, M, TE, TL and the corresponding co-state variables are: p^K, p^M, p^{TE}, p^{TL}. The first-order conditions of the problem are the following:

The first order conditions of the problem that are satisfied in all periods t=1,..T:

$$\frac{L(t)}{C(t)}=p^K(t) \quad (A9)$$

$$\dot{p}^K-10\rho p(t)^K=-10p(t)^K\left[\beta Y(t)/K(t)-\frac{(1-(1-\delta_K))^{10}}{10}\right] \quad (A11)$$

$$\dot{p}^M\text{-}10\rho p^M=p(t)^M\delta_M\text{-}p(t)^{TE}(4.1*C1)/(\log(2)*M(t)) \quad (A12)$$

$$\dot{p}^{TE}-10\rho p(t)^{TE}=p(t)^K\frac{0.0042TE(t)Y(t)}{(1+0.0021TE(t)^2)}+p(t)^{TE}(\lambda+C3)C1-p(t)^{TL}C4 \quad (A13)$$

$$\dot{p}^{TL}\text{-}10\rho p(t)^{TL}=-p(t)^{TE}C3C1+p(t)^{TL}C4 \quad (A14)$$

$$\gamma Y(t)/B(t)=bp \quad (A15)$$

The steady state is characterized by $K(t) = K(t+1), M(t) = M(t+1)$ etc. and $\dot{p}^K = \dot{p}^{MAT} = \dot{p}^{TE} = \dot{p}^{TL} = 0$ therefore the constraints of the model and the first-order conditions are the following:

$$I = \frac{(1-(1-\delta_K))^{10}}{10} K$$

$$C = Y - I - bpB$$

$$Y = \frac{AK^{\gamma} L^{1-\gamma-\beta} B^{\beta}}{(1+0.0021TE^2)}$$

$$FORC = FORCOTH$$

$$FORC = (\lambda + C3)TE - C3TL$$

$$TE = TL$$

$$M = 590$$

$$p^M(\delta_M + 10\rho) = p^{TE}(4.1*C1)/(\log(2)*M)$$

$$p^{TE}((\lambda + C3)C1 + 10\rho) = p^{TL}C4 - 10p^K \frac{Y(0.0042TE)}{(1+0.0021TE^2)}$$

$$p^{TL}(C4 + 10\rho) = p^{TE}C3C1$$

We assume that while there are no emissions caused by the use of fossil fuels, other GHG emissions (from agriculture, construction etc.) may still take place, which is reflected in parameter FORCOTH. Therefore, the mean temperature does not return to its pre-industrial level, in contrast to the carbon stock.

We also need the steady state values for all exogenous parameters such as labour(L), technological change(A), exogenous forcing of other GHG (FORCOTH) We calculated the following steady state values for these parameters:

$\lim_{t\to\infty} A(t) = =0.146$, $\lim_{t\to\infty} L(t) = 11424.911$, $\lim_{t\to\infty} FORCOTH(t) = 1.15$

We obtained the solution in GAMS software as a pseudo-optimization utilizing a dummy objective function unrelated to the equations.