Center for
Energy and
Environmental
Economic
Studies

Stefano Bosi

David Desmarchelier

Lionel Ragot

Pollution effects on labor supply and growth

Working paper CE3S-05/13

St. Petersburg
2013

УДК 330.35
ББК 65.012.2
B79

European University at St. Petersburg
Department of Economics

Université Catholique de Louvain
Center for Operations Research and Econometrics

Center for Energy and Environmental Economic Studies

Bosi S., Desmarchelier D., Ragot L.
B79 Pollution effects on labor supply and growth / Stefano Bosi, David Desmarchelier, Lionel Ragot: CEEES paper CE3S-05/13; Center for Energy and Environmental Economic Studies. — St. Petersburg: EUSP, 2013. — 19 p.

Some recent empirical contributions have pointed out a significant negative impact of pollution on labor supply. These impacts have been largely ignored in the theoretical literature, which, instead, focused on the case of pollution effects on consumption demand. In this paper, we study the short and long-run effects of pollution in a Ramsey model where pollution and labor supply are nonseparable arguments in households' preferences. We determine sufficient conditions for existence and uniqueness of a longterm equilibrium and we show how large (negative) effects of pollution on labor supply may promotes macroeconomic volatility (deterministic cycles near the steady state) through a flip bifurcation.

Stefano Bosi, EPEE, University of Evry. Corresponding author.

David Desmarchelier, EQUIPPE, University of Lille 1

Lionel Ragot, ECONOMIX, University of Paris 10 and CEPII

Издание осуществлено за счет средств проекта создания специализации по природным ресурсам и экономике энергетики «ЭксонМобил».

Pollution effects on labor supply and growth*

Stefano BOSI[†]
EPEE, University of Evry

David DESMARCHELIER
EQUIPPE, University of Lille 1

Lionel RAGOT
ECONOMIX, University of Paris 10 and CEPII

November 2, 2013

Abstract

Some recent empirical contributions have pointed out a significant negative impact of pollution on labor supply. These impacts have been largely ignored in the theoretical literature, which, instead, focused on the case of pollution effects on consumption demand. In this paper, we study the short and long-run effects of pollution in a Ramsey model where pollution and labor supply are nonseparable arguments in households' preferences. We determine sufficient conditions for existence and uniqueness of a long-term equilibrium and we show how large (negative) effects of pollution on labor supply may promotes macroeconomic volatility (deterministic cycles near the steady state) through a flip bifurcation.

Keywords: pollution, endogenous labor supply, Ramsey model.

JEL Classification: E32, O44.

*This work was funded by the French *Agence Nationale de la Recherche* under grant ANR-09-BLAN-0350-02. We are also grateful to participants to the CLEANER workshop held in Annecy on January 2013 for valuable comments.

[†]Corresponding author: stefano.bosi@cnrs-dir.fr.

1 Introduction

Recent empirical studies have documented significant impacts of pollution on labor supply, in its extensive margin, hours worked (Graff Zivin and Neidell (2010), Carson, Koundouri and Nauges (2011), Hanna and Oliva (2011)), and also in its intensive margin, labor productivity (Schlenker and Walker (2011), Graff Zivin and Neidell (2012)). There is nowadays a consensus on the negative impacts of pollution on labor productivity because of the negative effect on human capital and, in particular, on health. Evidences on the impacts of pollution on the hours worked also converge to negative effects. The more recent attempt to measure these effects and figure them out is Hanna and Oliva (2011). Using a recent data set for Mexico City, they find that a one percent increase in air pollution (mainly, SO2 concentration) results in a 0.61 percent decrease in the worked hours. They consider a partial equilibrium model to capture the driving forces between pollution and worked hours. Our paper aim at representing these forces in a (dynamic) general equilibrium model to address the issue of their macroeconomic incidence in the short and long run.

The literature on the consequences of pollution in the long run seems to neglect the possible influence of pollution on the consumption-leisure arbitrage and, thereby, on labor supply. In general, the interplay between pollution and labor supply remains ambiguous from a theoretical point of view. On the one hand, pollution may worsen working conditions (for instance, the negative impact of global warming rests on a positive correlation between heat and work painfulness) and give an incentive to substitute leisure to working time. On the other hand, households like to enjoy leisure in a healthy and pleasant environment (for example, air pollution may dissuade people from going outdoor and encourage them to work more).

Theoretical literature has pointed out the same ambiguity in the role of pollution on consumption.[1] From an exogenous growth perspective, Keeler, Spence and Zeckhauser (1972) pioneered the class of Ramsey models with pollution accumulation. Pollution lowers the level of welfare as negative externality. Focusing on nonseparable preferences, they assume that consumption and environmental quality are normal goods in order to ensure the uniqueness of the steady state. Van der Ploeg and Withagen (1991) assume additively separable preferences or a negative cross derivative (a marginal utility of consumption decreasing in the pollution level). These conditions are sufficient for uniqueness and saddle-point stability of the competitive steady state. Tahvonen and Kuuluvainen (1993) removed any restriction on the sign of the cross derivative in a Ramsey model. The most complete characterization of the interplay between consumption and pollution in a Ramsey model was given by Ryder and Heal (1973).[2] Satiation is possible under assumptions on the first-order derivatives of the utility function and may promote the multiplicity of steady states. Assumptions on second-order derivatives and intertemporally dependent pref-

[1] All the papers cited in this paragraph represent pollution as a stock instead of a flow.

[2] In Ryder and Heal (1973) pollution comes from consumption and it is interpreted as a habit effect. Heal (1982) considers explicitly the same variable as a pollution stock.

erences may promote the occurrence of cycles through a Hopf bifurcation in the case of adjacent complementarity.

The effects of pollution on growth through the consumption channel were also studied by Michel and Rotillon (1995) in an endogenous growth model (AK). They find that a *distaste effect* of pollution on consumption (negative cross derivative) or separable preferences are incompatible with optimal endogenous growth. Conversely, sustained long-term growth is optimal when the utility function exhibits a *compensation effect* (positive cross derivative). Endogenous growth occurs in the competitive equilibrium regardless of the effects of pollution on consumption.

Surprisingly and paradoxically, many theoretical works have considered the pollution effects on consumption behavior and the growth path, while, to the best of our knowledge, there are no empirical studies on that. Conversely, a rising number of empirical works show pollution effects on labor supply, whereas these effects are largely ignored in theoretical papers.

Our work contributes to shed light on the interplay between pollution and labor supply in the particular case of a productive economy with capital and pollution accumulation. An ideal framework to carry out this task is a Ramsey model with nonseparable preferences between pollution and labor supply (separability rules out any direct effect of pollution on labor disutility and supply).

First, we build a growth model à la Ramsey without specifying any functional form for technology and preferences, and we show that, if consumption and leisure are normal goods, a distaste effect (a marginal utility of consumption decreasing in the pollution level) jointly with a leisure effect (a positive effect of pollution on labor disutility) are sufficient conditions for the existence and uniqueness of a competitive steady state.

Second, in order to characterize the dynamic properties of the competitive equilibrium, we specify production and utility as isoelastic functions. Moreover, the novelty of this separable model rests on the interchange of nonseparability assumptions: between pollution and leisure instead of between pollution and consumption.

In this oversimplified context, we show that, under sufficiently large (negative) effects of pollution on labor supply, the economy may experience deterministic fluctuations around the steady state through flip and period-doubling bifurcations. Thus, the pollution effect on labor supply, highlighted by a recent empirical literature, seems to promote macroeconomic volatility and destabilize the economic dynamics.

The rest of the paper is articulated in three sections: (1) presentation of a general setting, (2) application to the separable case (separability between consumption and labor), (3) conclusion.

2 The model

In the following, we consider a discrete-time Ramsey economy with pollution and capital accumulation. A representative household faces a consumption-leisure arbitrage by supplying a labor force to a sector of perfectly competitive firms. These firms produce a single commodity working either as capital or a consumption good. Because of the constant returns to scale, firms can be represented by a single aggregate firm. Pollution is a by-product of industrial activities and affects the individual welfare by distorting the consumption-leisure arbitrage.

2.1 Firms

At each date $t = 0, 1, \ldots$, a representative firm produces a single output Y_t. Technology is represented by a constant returns to scale production function: $Y_t = F(K_t, L_t)$, where K_t and L_t are the demands for capital and labor respectively.

Assumption 1 *The production function $F : \mathbb{R}_+^2 \to \mathbb{R}_+$ is C^1, homogeneous of degree one, strictly increasing and concave. Standard Inada conditions hold.*

The firm chooses the amount of capital and labor to maximize the profit taking as given the real interest rate r_t and the real wage w_t. The program is correctly defined under Assumption 1: $\max_{K_t, L_t} [F(K_t, L_t) - r_t K_t - w_t L_t]$, and the first-order conditions write:

$$\begin{aligned} r_t &= f'(k_t) \equiv r(k_t) \\ w_t &= f(k_t) - k_t f'(k_t) \equiv w(k_t) \end{aligned}$$

where $k_t \equiv K_t/L_t$ denotes the capital intensity. We introduce the capital share in total income α and the elasticity of capital-labor substitution σ:

$$\begin{aligned} \alpha(k_t) &\equiv \frac{k_t f'(k_t)}{f(k_t)} \\ \sigma(k_t) &= \alpha(k_t) \frac{w(k_t)}{k_t w'(k_t)} \end{aligned} \quad (1)$$

In addition, the elasticities of factor prices write:

$$\frac{k_t r'(k_t)}{r(k_t)} = -\frac{1-\alpha(k_t)}{\sigma(k_t)} \quad (2)$$

$$\frac{k_t w'(k_t)}{w(k_t)} = \frac{\alpha(k_t)}{\sigma(k_t)} \quad (3)$$

2.2 Preferences

At each date $t = 0, 1, \ldots$, the household earns a capital income $r_t h_t$ and a labor income $w_t l_t$ where h_t and l_t denote the individual wealth and labor supply respectively. Income is consumed and saved/invested according to the budget constraint:

$$c_t + h_{t+1} - (1-\delta) h_t \leq r_t h_t + w_t l_t \quad (4)$$

The gross investment includes the capital depreciation at the rate δ.

For simplicity, the population of consumers-workers is constant over time: $N = 1$. Such normalization implies $L_t = Nl_t = l_t$, $K_t = Nh_t = h_t$ and $h_t = K_t/N = k_t l_t$.

The representative agent takes a utility from the consumption c_t and a disutility from the labor supply l_t and the amount of pollution P_t, that is an aggregate externality. The utility function $u_t = u(c_t, l_t, P_t)$ satisfies the following assumption.

Assumption 2 *The utility function $u : \mathbb{R}^3_+ \to \mathbb{R}$ is C^2, strictly increasing in c_t and strictly decreasing in l_t and P_t, and concave with respect to (c_t, l_t).*

If consumption and leisure are both normal goods, we have $u_{cc} - u_{lc}u_c/u_l < 0$ and $u_{ll} - u_{cl}u_l/u_c < 0$. These inequalities hold for instance if $u_{lc} \leq 0$ that is a sufficient condition.

According to Michel and Rotillon (1995), pollution has a *distaste effect* on consumption if $u_{cP} < 0$: an increase in pollution reduces the marginal utility of consumption and thereby households' propensity to consume. These authors call the opposite effect ($u_{cP} > 0$) the *compensation effect.* An increase in pollution raises the propensity to consume.

This terminology can be extended to the effects of pollution on labor supply. Focusing on leisure, we will call *leisure effect* the positive effect of pollution on labor disutility ($u_{lP} < 0$) which decreases labor supply and increases in turn leisure demand. Conversely, we will call *disenchantment effect* the negative effect of pollution on labor disutility ($u_{lP} > 0$). In this case, leisure time decreases with pollution. As seen in the introduction, disenchantment for leisure comes from a more polluted and unpleasant environment.

Any household maximizes an intertemporal utility function $\sum_{t=0}^{\infty} \beta^t u(c_t, l_t, P_t)$ under the budget constraint (4) where $\beta \in (0, 1)$ is a constant discount factor. This program is correctly defined under Assumption 2. The first-order conditions result in a static consumption-leisure arbitrage

$$u_l(c_t, l_t, P_t) = -u_c(c_t, l_t, P_t)\, w_t$$

and a dynamic Euler equation

$$\frac{u_c(c_t, l_t, P_t)}{u_c(c_{t+1}, l_{t+1}, P_{t+1})} = \beta(1 - \delta + r_{t+1})$$

jointly with the budget constraint (4) now binding.

2.3 Pollution

The aggregate stock of pollution P_t is a pure externality. Technology is dirty and pollution persists. More explicitly, we assume that the stock of pollution tomorrow will depend on pollution and production today according to a simple linear process:

$$P_{t+1} = aP_t + bY_t \tag{5}$$

where $1 - a \in (0, 1]$ captures the natural rate of pollution absorption and $b > 0$ the environmental impact of production. Under Assumption 1, the process of pollution accumulation (5) writes:

$$P_{t+1} = aP_t + bL_t f(k_t) = aP_t + bl_t f(k_t)$$

2.4 Equilibrium

Good and labor markets clear. Noticing that $h_t = k_t l_t$, we find

$$\begin{aligned} c_t + k_{t+1} l_{t+1} &= [1 - \delta + r(k_t)] k_t l_t + w(k_t) l_t && (6) \\ \frac{u_c(c_t, l_t, P_t)}{u_c(c_{t+1}, l_{t+1}, P_{t+1})} &= \beta [1 - \delta + r(k_{t+1})] && (7) \\ P_{t+1} &= aP_t + bl_t f(k_t) && (8) \\ u_l(c_t, l_t, P_t) &= -u_c(c_t, l_t, P_t) w(k_t) && (9) \end{aligned}$$

Applying the Implicit Function Theorem to the static arbitrage (9), we are able to compute the derivatives of the labor supply function $l_t = l(c_t, k_t, P_t)$. Indeed, differentiating $u_l(c_t, l_t, P_t) + u_c(c_t, l_t, P_t) w(k_t) = 0$ and keeping in mind that $w_t = -u_l/u_c$, we get

$$\frac{dl}{dc_t} = -\frac{\frac{u_{cl}}{u_l} - \frac{u_{cc}}{u_c}}{\frac{u_{ll}}{u_l} - \frac{u_{cl}}{u_c}}, \quad \frac{dl}{dP_t} = -\frac{\frac{u_{lP}}{u_l} - \frac{u_{cP}}{u_c}}{\frac{u_{ll}}{u_l} - \frac{u_{cl}}{u_c}}, \quad \frac{dl}{dk_t} = \frac{\frac{w'(k_t)}{w_t}}{\frac{u_{ll}}{u_l} - \frac{u_{cl}}{u_c}}$$

These derivatives allow us to compute the second-order elasticities of the utility function:

$$E \equiv \begin{bmatrix} \varepsilon_{cc} & \varepsilon_{cl} & \varepsilon_{cP} \\ \varepsilon_{lc} & \varepsilon_{ll} & \varepsilon_{lP} \\ \varepsilon_{Pc} & \varepsilon_{Pl} & \varepsilon_{PP} \end{bmatrix} \equiv \begin{bmatrix} \frac{c_t u_{cc}}{u_c} & \frac{c_t u_{cl}}{u_l} & \frac{c_t u_{cP}}{u_P} \\ \frac{l_t u_{lc}}{u_c} & \frac{l_t u_{ll}}{u_l} & \frac{l_t u_{lP}}{u_P} \\ \frac{P_t u_{Pc}}{u_c} & \frac{P_t u_{Pl}}{u_l} & \frac{P_t u_{PP}}{uP} \end{bmatrix}$$

Using (1), we find the elasticities of labor supply:

$$\begin{aligned} \lambda_c &\equiv \frac{c_t}{l_t} \frac{dl}{dc_t} = \frac{\varepsilon_{cc} - \varepsilon_{cl}}{\varepsilon_{ll} - \varepsilon_{lc}} && (10) \\ \lambda_P &\equiv \frac{P_t}{l_t} \frac{dl}{dP_t} = \frac{\varepsilon_{Pc} - \varepsilon_{Pl}}{\varepsilon_{ll} - \varepsilon_{lc}} \\ \lambda_k &\equiv \frac{k_t}{l_t} \frac{dl}{dk_t} = \frac{\alpha}{\sigma} \frac{1}{\varepsilon_{ll} - \varepsilon_{lc}} && (11) \end{aligned}$$

All our economic analysis will rest on these crucial elasticities.

Proposition 1 *If consumption and leisure are normal goods, $\lambda_c < 0$ and $\lambda_k > 0$.*

Proof. *Normality entails $u_{cc}/u_c - u_{lc}/u_l < 0$ and $u_{ll}/u_l - u_{cl}/u_c > 0$, that is $\varepsilon_{cc} - \varepsilon_{cl} < 0$ and $\varepsilon_{ll} - \varepsilon_{lc} > 0$. Apply (10) and (11).* ■

Replacing the labor supply $l(c_t, k_t, P_t)$ in (6), (7) and (8), we obtain a three-dimensional dynamic system.

Proposition 2 *An intertemporal equilibrium with perfect foresight is a nonnegative sequence* $(k_t, c_t, P_t)_{t=0}^{\infty}$ *satisfying the dynamic system*

$$c_t + k_{t+1} l(c_{t+1}, k_{t+1}, P_{t+1}) = ([1-\delta + r(k_t)] k_t + w(k_t)) l(c_t, k_t, P_t) \quad (12)$$

$$\frac{u_c(c_t, l(c_t, k_t, P_t), P_t)}{u_c(c_{t+1}, l(c_{t+1}, k_{t+1}, P_{t+1}), P_{t+1})} = \beta [1-\delta + r(k_{t+1})] \quad (13)$$

$$P_{t+1} = aP_t + bf(k_t) l(c_t, k_t, P_t) \quad (14)$$

We observe that this system is three-dimensional with two predetermined variables (k_t, P_t) and one non-predetermined (c_t).

2.5 Steady state

Variables are constant over time: $(k_t, c_t, P_t) = (k, c, P)$ for every t. At the steady state, the dynamic system (12)-(14) writes:

$$r(k) = \frac{1}{\beta} - 1 + \delta \quad (15)$$

$$c = \left[\frac{1-\beta}{\beta} k + w(k)\right] l(c, k, P) \quad (16)$$

$$P = \frac{b}{1-a} f(k) l(c, k, P) \quad (17)$$

We obtain the stationary capital k from the first equation. Replacing k in (16) and (17) and solving system (16)-(17), we obtain also (c, P).

Given k, solution of (15), let

$$\mu(c) \equiv l\left(c, k, \frac{b}{1-a} \frac{f(k)}{f(k) - \delta k} c\right)$$

Proposition 3 *Let Assumptions 1 and 2 hold. If* $\lim_{c \to 0^+} \mu(c) > 0$ *and* $\mu'(c) < 0$ *for every* $c > 0$*, then there exists a unique steady state.*

Proof. Under Assumption 1, k is uniquely determined by (15). Replacing k in (16) and (17) we obtain a two-dimensional system in (c, P). We observe from (16) that

$$\frac{c}{l(c, k, P)} = \frac{1-\beta}{\beta} k + w(k) = f(k) - \delta k > 0 \quad (18)$$

Dividing equations (16) and (17) side by side and using (18), we get

$$P = \frac{b}{1-a} \frac{f(k)}{f(k) - \delta k} c \quad (19)$$

Replacing (19) in (16), we find

$$g(c) \equiv c - [f(k) - \delta k] \mu(c) = 0 \quad (20)$$

a single equation in c. Under Assumption 2.2, we find $\lim_{c\to 0^+} g(c) < 0$ and $\lim_{c\to +\infty} g(c) = +\infty$. Under Assumption 1, g is a continuous function. Thus, a solution of equation (20) exists.

In addition, this solution is unique because, under Assumption 2.2, $g'(c) > 0$ for any $c > 0$. ■

We will see that, in the case of separable isoelastic preferences, the assumptions of Proposition 3 hold and a unique steady state exists.

Corollary 4 *Let Assumptions 1 and 2 hold. If consumption and leisure are normal goods, a distaste effect ($u_{cP} < 0$) jointly with a leisure effect ($u_{lP} < 0$) holds and* $\lim_{c\to 0^+} \mu(c) > 0$*, then there exists a unique steady state.*

Proof. We observe that

$$\begin{aligned}\mu'(c) &= l_c + l_P \frac{b}{1-a} \frac{f(k)}{f(k) - \delta k} = l_c + l_P \frac{P}{c} \\ &= -\frac{\frac{u_{cl}}{u_l} - \frac{u_{cc}}{u_c}}{\frac{u_{ll}}{u_l} - \frac{u_{cl}}{u_c}} - \frac{\frac{u_{lP}}{u_l} - \frac{u_{cP}}{u_c}}{\frac{u_{ll}}{u_l} - \frac{u_{cl}}{u_c}} \frac{P}{c} = \frac{\frac{u_{cc}}{u_c} - \frac{u_{cl}}{u_l} + \left(\frac{u_{cP}}{u_c} - \frac{u_{lP}}{u_l}\right)\frac{P}{c}}{\frac{u_{ll}}{u_l} - \frac{u_{cl}}{u_c}}\end{aligned}$$

Since consumption and leisure are both normal goods, we have

$$\frac{u_{cc}}{u_c} - \frac{u_{lc}}{u_l} < 0 \text{ and } \frac{u_{ll}}{u_l} - \frac{u_{cl}}{u_c} > 0$$

Thus, $\mu'(c) < 0$ iff

$$\frac{u_{cc}}{u_c} - \frac{u_{cl}}{u_l} + \left(\frac{u_{cP}}{u_c} - \frac{u_{lP}}{u_l}\right)\frac{P}{c} < 0 \tag{21}$$

If a distaste effect ($u_{cP} < 0$) jointly with a leisure effect ($u_{lP} < 0$) holds, we get (21), that is $\mu'(c) < 0$ for any $c > 0$. Eventually, Proposition 3 applies. ■

2.6 Long run

Focus on the comparative statics. In this general section, we have not specified technology and preferences. The only parameters we consider are a, b, β and δ. We compute their impact on c, k, P. In the isoelastic case (see below), we will consider also the impact of technology and preferences on the endogenous variables.

Differentiating (15) and using (2), we obtain the usual elasticities of Modified Golden Rule:

$$\frac{\beta}{k}\frac{\partial k}{\partial \beta} = \frac{\sigma(k)}{1-\alpha(k)}\frac{1}{\beta r(k)} > 0 \tag{22}$$

$$\frac{\delta}{k}\frac{\partial k}{\partial \delta} = -\frac{\sigma(k)}{1-\alpha(k)}\frac{\delta}{r(k)} < 0 \tag{23}$$

where $r(k)$ is given by (15).

Differentiating system (16)-(17) and using (22) and (23), we obtain:

$$\begin{aligned}
\rho\left(1-\lambda_c\right)\frac{dc}{c}-\rho\lambda_P\frac{dP}{P} &= \frac{\sigma_2}{\beta}\frac{d\beta}{\beta}-\delta\left(1+\sigma_2\right)\frac{d\delta}{\delta} \\
-\lambda_c\frac{dc}{c}+\left(1-\lambda_P\right)\frac{dP}{P} &= -\frac{dz}{z}+\frac{db}{b}+\sigma_1\frac{d\beta}{\beta}-\beta\delta\sigma_1\frac{d\delta}{\delta}
\end{aligned}$$

where $z \equiv 1-a$ and

$$\begin{aligned}
\rho &\equiv \frac{c}{kl}=\frac{1-\beta}{\beta}+\frac{1-\alpha}{\alpha}r \\
\sigma_1 &\equiv \frac{\sigma}{1-\alpha}\frac{\alpha+\lambda_k}{\beta r} \\
\sigma_2 &\equiv \lambda_k\frac{\sigma}{\alpha}+\left(1+\lambda_k\right)\frac{\sigma}{1-\alpha}\frac{1-\beta}{\beta r}
\end{aligned}$$

that is

$$\begin{bmatrix} \frac{dc}{c} \\ \frac{dP}{P} \end{bmatrix} = \frac{M}{1-\lambda_c-\lambda_P}\begin{bmatrix} \frac{dz}{z} \\ \frac{db}{b} \\ \frac{d\beta}{\beta} \\ \frac{d\delta}{\delta} \end{bmatrix}$$

with

$$M \equiv \begin{bmatrix} -\lambda_P & \lambda_P & \lambda_P\sigma_1+\left(1-\lambda_P\right)\frac{\sigma_2}{\beta\rho} & -\lambda_P\beta\delta\sigma_1-\left(1-\lambda_P\right)\frac{\delta(1+\sigma_2)}{\rho} \\ \lambda_c-1 & 1-\lambda_c & \lambda_c\frac{\sigma_2}{\beta\rho}+\left(1-\lambda_c\right)\sigma_1 & -\lambda_c\frac{\delta(1+\sigma_2)}{\rho}-\left(1-\lambda_c\right)\beta\delta\sigma_1 \end{bmatrix} \tag{24}$$

We find the following elasticity of comparative statics:

$$\begin{bmatrix} \frac{z}{c}\frac{\partial c}{\partial z} & \frac{b}{c}\frac{\partial c}{\partial b} & \frac{\beta}{c}\frac{\partial c}{\partial \beta} & \frac{\delta}{c}\frac{\partial c}{\partial \delta} \\ \frac{z}{P}\frac{\partial P}{\partial z} & \frac{b}{P}\frac{\partial P}{\partial b} & \frac{\beta}{P}\frac{\partial P}{\partial \beta} & \frac{\delta}{P}\frac{\partial P}{\partial \delta} \end{bmatrix} = \frac{M}{1-\lambda_c-\lambda_P} \tag{25}$$

Proposition 5 *In the case of negative effects of consumption and pollution on labor supply, the following inequalites hold:*

$$\frac{\partial c}{\partial z}>0,\ \frac{\partial c}{\partial b}<0,\ \frac{\partial P}{\partial z}<0,\ \frac{\partial P}{\partial b}>0 \tag{26}$$

Proof. Consider (24) with (25). Since $\lambda_c, \lambda_P < 0$, we obtain $1-\lambda_c > 0$ and $1-\lambda_c-\lambda_P > 0$. ■

Unsurprisingly, the consumption of steady state is increasing with the natural rate of pollution absorption and decreasing with the pollution emission rate. These parameters of pollution accumulation have opposite effects on the pollution stock in the long run.

Corollary 6 *Under the sufficient conditions for existence and uniqueness of the steady state (normality of consumption and leisure, distaste and leisure effects: see Corollary 4), inequalities (26) hold.*

Proof. Normality implies $\varepsilon_{ll} - \varepsilon_{lc} > 0$ and $\lambda_c < 0$ (Proposition 1). We observe that $u_{cP} < 0$ (distaste effect) and $u_{lP} < 0$ (leisure effect) entail $\varepsilon_{Pc} < 0$ and $\varepsilon_{Pl} > 0$ respectively, that is $\lambda_P = (\varepsilon_{Pc} - \varepsilon_{Pl}) / (\varepsilon_{ll} - \varepsilon_{lc}) < 0$. Apply Proposition 5. ■

2.7 Short run

Focus on local dynamics. We linearize the dynamic system (12)-(14) around the steady state:

$$\begin{bmatrix} 1+\lambda_k & \lambda_c & \lambda_P \\ \varepsilon_{lc}\lambda_k - \frac{1-\alpha}{\sigma}\beta r & \varepsilon_{cc}+\varepsilon_{lc}\lambda_c & \varepsilon_{lc}\lambda_P + \varepsilon_{Pc} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{dk_{t+1}}{k} \\ \frac{dc_{t+1}}{c} \\ \frac{dP_{t+1}}{P} \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1}{\beta} + (1+\rho)\lambda_k & (1+\rho)\lambda_c - \rho & (1+\rho)\lambda_P \\ \varepsilon_{lc}\lambda_k & \varepsilon_{cc}+\varepsilon_{lc}\lambda_c & \varepsilon_{lc}\lambda_P+\varepsilon_{Pc} \\ (1-a)(\alpha+\lambda_k) & (1-a)\lambda_c & a+(1-a)\lambda_P \end{bmatrix} \begin{bmatrix} \frac{dk_t}{k} \\ \frac{dc_t}{c} \\ \frac{dP_t}{P} \end{bmatrix}$$

In order to study the local stability of system (12)-(14), that is the shape of the characteristic polynomial, we need to assume preferences to be additively separable in consumption and labor.

3 The separable model

In the case of separable utility:[3] $u(c_t, l_t, P_t) = \tilde{u}(c_t) - \omega v(l_t, P_t)$, the elasticity matrix E becomes

$$E \equiv \begin{bmatrix} \varepsilon_{cc} & 0 & 0 \\ 0 & \varepsilon_{ll} & \varepsilon_{lP} \\ 0 & \varepsilon_{Pl} & \varepsilon_{PP} \end{bmatrix} \qquad (27)$$

and the elasticities of labor supply $l_t = l(c_t, k_t, P_t)$ write:

$$\begin{aligned} \lambda_c &= \frac{\varepsilon_{cc}}{\varepsilon_{ll}} \\ \lambda_P &= -\frac{\varepsilon_{Pl}}{\varepsilon_{ll}} \\ \lambda_k &= \frac{\alpha}{\sigma}\frac{1}{\varepsilon_{ll}} \end{aligned} \qquad (28)$$

When pollution does not affect the labor supply $\varepsilon_{Pl} = 0$. Thus, the very difference with respect to the standard consumption-labor arbitrage is the elasticity ε_{Pl}. In the following, we will show that σ and ε_{Pl} play a role in the occurrence of endogenous business cycles.

From a theoretical point of view, the effect of pollution on labor supply is ambiguous: $\lambda_P \lesseqgtr 0$. However, according to the recent empirical studies and, in

[3] Separability between consumption and leisure and between consumption and pollution, but nonseparability between leisure and pollution.

particular, to Hanna and Oliva (2011) work on Mexico City, a rise in pollution seems to have a negative effect on labor supply: $\lambda_P < 0$. In the following, we will consider this case.

3.1 Isoelastic form

In the isoelastic case, the elasticities are constant and notation simplifies: $\varepsilon \equiv -\varepsilon_{cc}$ and $\varphi \equiv \varepsilon_{ll}$. We consider explicit isoelastic separable preferences:

$$\tilde{u}(c_t) \equiv \frac{c_t^{1-\varepsilon}}{1-\varepsilon} \text{ and } v(l_t, P_t) \equiv \frac{(l_t P_t^{\gamma})^{1+\varphi}}{1+\varphi} \tag{29}$$

where $1/\varepsilon \geq 0$ is the consumption elasticity of intertemporal substitution and $1/\varphi \geq 0$ is the Frisch elasticity of intertemporal substitution.

In addition, this form allows us to express the key elasticity ε_{Pl} in terms of the structural parameters: $\varepsilon_{Pl} = \gamma(1+\varphi)$. Since

$$\lambda_P = -\frac{\varepsilon_{Pl}}{\varepsilon_{ll}} = -\gamma\frac{1+\varphi}{\varphi} \tag{30}$$

and, according to Assumption 2, the utility function decreases with P_t, we obtain $\gamma > 0$ and, thus, $\lambda_P < 0$ or, equivalently, $dl/dP_t < 0$ (the labor supply decreases with pollution in turn).

From (30), labor disutility writes also

$$v(l_t, P_t) \equiv \frac{l_t^{1+\varphi} P_t^{-\varphi\lambda_P}}{1+\varphi}$$

In the case (29), with an isoelastic intensive production function $f(k_t) = Ak_t^{\alpha}$, the labor supply function explicitly becomes

$$l_t = l(c_t, k_t, P_t) = m(c_t, k_t) P_t^{\lambda_P} \text{ with } m(c_t, k_t) \equiv \left(\frac{1-\alpha}{\omega}\frac{Ak_t^{\alpha}}{c_t^{\varepsilon}}\right)^{\frac{1}{\varphi}}$$

3.2 Long run (continued)

Preferences rationalized by functions (29) ensure the uniqueness of the steady state. Indeed, consumption and leisure are normal goods and a distaste effect jointly with a leisure effect hold. More explicitly, we find

$$\mu(c) = \left(\frac{1-\alpha}{\omega}Ak^{\alpha}\right)^{\frac{1}{\varphi}} \left(\frac{b}{1-a}\frac{Ak^{\alpha}}{Ak^{\alpha}-\delta k}\right)^{\lambda_P} c^{\lambda_P - \frac{\varepsilon}{\varphi}}$$

with $\lambda_P < 0$. Thus, $\lim_{c \to 0^+} \mu(c) = +\infty > 0$ and $\mu'(c) < 0$, and the assumptions of Proposition 3 hold.

Let

$$\lambda_P^* \equiv -\frac{1}{\varphi\sigma}\left(1 + \frac{r-\delta}{\delta}\frac{1+\varphi\sigma}{1-\alpha}\right) < 0$$

Proposition 7 *The long-run effects of the fundamental parameters on the pollution stock are given by*

$$\frac{\partial P}{\partial a} > 0,\ \frac{\partial P}{\partial b} > 0,\ \frac{\partial P}{\partial \beta} > 0$$

and

$$\frac{\partial P}{\partial \delta} < 0 \textit{ iff } \sigma > \frac{\frac{r-\alpha r}{r-\alpha\delta}\varepsilon - 1}{\frac{\delta-\alpha\delta}{r-\alpha\delta}\varepsilon + \varphi}$$

The long-run effects of these parameters on the consumption level depend on the pollution elasticity of labor supply λ_P*,*

$$\begin{aligned}
\frac{\partial c}{\partial a} &< 0 \\
\frac{\partial c}{\partial b} &< 0 \\
\frac{\partial c}{\partial \beta} &> 0 \textit{ iff } \lambda_P > \lambda_P^* \\
\frac{\partial c}{\partial \delta} &< 0 \textit{ iff } \sigma < \frac{r}{\delta} \textit{ or } \left(\sigma > \frac{r}{\delta} \textit{ and } \lambda_P > \frac{r-\sigma\delta\lambda_P^*}{r-\sigma\delta}\,(<0)\right)
\end{aligned}$$

Proof. Reconsider (30) and the impact matrix (25). Notice that $z \equiv 1-a$ and

$$(\lambda_c, \lambda_k) = \left(-\frac{\varepsilon}{\varphi}, \frac{1}{\varphi}\frac{\alpha}{\sigma}\right)$$

The denominator $1-\lambda_c-\lambda_P$ is positive. (24) becomes

$$M \equiv \begin{bmatrix} -\lambda_P & \lambda_P & \frac{\alpha\delta\sigma}{\beta r}\frac{\lambda_P-\lambda_P^*}{r-\alpha\delta} & \frac{\alpha\delta}{r}\frac{r-\sigma\delta}{r-\alpha\delta}\left(\lambda_P - \frac{r-\sigma\delta\lambda_P^*}{r-\sigma\delta}\right) \\ -\frac{\varepsilon+\varphi}{\varphi} & \frac{\varepsilon+\varphi}{\varphi} & \frac{1}{\varphi\beta r}\frac{\alpha}{1-\alpha}\left(1+\varphi\sigma+\varepsilon\sigma\frac{\delta-\alpha\delta}{r-\alpha\delta}\right) & \frac{\delta}{\varphi}\left[\frac{\varepsilon}{\rho} - \frac{1}{r}\frac{\alpha}{1-\alpha}\left(1+\varphi\sigma+\varepsilon\sigma\frac{\delta-\alpha\delta}{r-\alpha\delta}\right)\right] \end{bmatrix}$$

We observe that $r > \alpha\delta$ and

$$\begin{aligned}
\frac{1}{\varphi\beta r}\frac{\alpha}{1-\alpha}\left(1+\varphi\sigma+\varepsilon\sigma\frac{\delta-\alpha\delta}{r-\alpha\delta}\right) &> 0 \\
\frac{\delta}{\varphi}\left[\frac{\varepsilon}{\rho} - \frac{1}{r}\frac{\alpha}{1-\alpha}\left(1+\varphi\sigma+\varepsilon\sigma\frac{\delta-\alpha\delta}{r-\alpha\delta}\right)\right] &< 0 \text{ iff } \sigma > \frac{\frac{r-\alpha r}{r-\alpha\delta}\varepsilon - 1}{\frac{\delta-\alpha\delta}{r-\alpha\delta}\varepsilon + \varphi}
\end{aligned}$$

The proposition follows immediately. ■

Corollary 8 $\partial P/\partial\delta < 0$ *if*

$$\varepsilon < \frac{r-\alpha\delta}{r-\alpha r}\,(>1)$$

Corollary 9 *If* $\lambda_P = 0$ *(that is* $\gamma = 0$*), we have*

$$\frac{\partial c}{\partial a} = \frac{\partial c}{\partial b} = 0,\ \frac{\partial c}{\partial \beta} > 0,\ \frac{\partial c}{\partial \delta} < 0$$

We recover in this case the usual conclusions of the Ramsey model.

Let us provide an interpretation of Proposition 7.

Focus on equations (17), (22) and (28). The higher the β, the larger the capital stock k and, in turn, the higher the labor supply l and the stock of pollution P. The effect of β on c in the long run depends on λ_P. Since $\lambda_P < 0$, the households substitutes leisure to working time. However, if this effect does not compensate the positive effect of k on labor supply, a higher β entails a higher consumption in the long run.

Focus now on equations (17), (23) and (28). The higher the depreciation rate δ, the lower the capital stock k and, in turn, the lower the labor supply l and and the pollution stock P. According to equation (23), the impact of δ on k and, in turn, on l, depends on the elasticity of capital-labor substitution σ. A stronger σ induces a larger negative effect of δ on k. Since the negative effect of δ on P depends crucially on its impact on k, it follows that the negative impact of δ on P is also magnified under a large elasticity σ. Notice that $\lambda_P < 0$. We know also that a higher δ implies a lower P. Let the pollution elasticity of labor supply be not too negative. In this case, under a sufficiently large σ, the effect of k on l dominates the effect of P on l. Thus, a higher δ leads the household to substitute leisure to working time, that is to consume less in the long run.

3.3 Short run (continued)

Specification (29) gives the following Jacobian:

$$J = \begin{bmatrix} 1+\lambda_k & \lambda_c & \lambda_P \\ \varepsilon_{lc}\lambda_k - \frac{1-\alpha}{\sigma}\beta r & \varepsilon_{cc}+\varepsilon_{lc}\lambda_c & \varepsilon_{lc}\lambda_P+\varepsilon_{Pc} \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{\beta}+(1+\rho)\lambda_k & (1+\rho)\lambda_c-\rho & (1+\rho)\lambda_P \\ \varepsilon_{lc}\lambda_k & \varepsilon_{cc}+\varepsilon_{lc}\lambda_c & \varepsilon_{lc}\lambda_P+\varepsilon_{Pc} \\ (1-a)(\alpha+\lambda_k) & (1-a)\lambda_c & a+(1-a)\lambda_P \end{bmatrix}$$

with an elasticity matrix (27):

$$E = \begin{bmatrix} -\varepsilon & 0 & 0 \\ 0 & \varphi & \varepsilon_{lP} \\ 0 & \gamma(1+\varphi) & \varepsilon_{PP} \end{bmatrix}$$

Therefore,

$$J = \begin{bmatrix} 1+\lambda_k & \lambda_c & \lambda_P \\ -\frac{1-\alpha}{\sigma}\beta r & -\varepsilon & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{\beta}+(1+\rho)\lambda_k & (1+\rho)\lambda_c-\rho & (1+\rho)\lambda_P \\ 0 & -\varepsilon & 0 \\ (1-a)(\alpha+\lambda_k) & (1-a)\lambda_c & a+(1-a)\lambda_P \end{bmatrix} \tag{31}$$

with:

$$\rho = \frac{1-\beta}{\beta} + \frac{1-\alpha}{\alpha} r$$

and

$$\lambda_c = -\frac{\varepsilon}{\varphi},\ \lambda_P = -\gamma \frac{1+\varphi}{\varphi},\ \lambda_k = \frac{\alpha}{\sigma}\frac{1}{\varphi}$$

To know the location of the eigenvalues of the Jacobian matrix w.r.t. the unit circle, we study the sign of the characteristic polynomial $P(x)$ in $x = -1, 0, 1$. Tedious computations give the following values:

$$P(0) = (1-a) \frac{(1-\alpha)(1-\delta)\varepsilon\sigma}{(1-\alpha)\beta r \lambda_c - \varepsilon\sigma(1+\lambda_k)} (\lambda_P - \lambda_1) > 0 \text{ iff } \lambda_P < \lambda_1 \tag{32}$$

$$P(1) = \frac{\beta r \rho (1-\alpha)(1-a)(1-\lambda_c-\lambda_P)}{(1-\alpha)\beta r \lambda_c - \varepsilon\sigma(1+\lambda_k)} < 0 \tag{33}$$

$$P(-1) = (1-a) \frac{(1-\alpha)[\beta r \rho + 2\varepsilon\sigma(2-\delta)]}{(1-\alpha)\beta r \lambda_c - \varepsilon\sigma(1+\lambda_k)} (\lambda_P - \lambda_2) > 0 \text{ iff } \lambda_P < \lambda_2 \tag{34}$$

where

$$\lambda_1 \equiv -\frac{a}{1-a}\frac{1+(1+\rho)\beta\lambda_k}{\beta(1-\delta)(1-\alpha)} < 0$$

$$\lambda_2 \equiv -\frac{1+a}{1-a}\frac{(1-\alpha)\beta r[\rho-(2+\rho)\lambda_c] + 2\varepsilon\sigma[1+1/\beta+(2+\rho)\lambda_k]}{(1-\alpha)[\beta r\rho + 2\varepsilon\sigma(2-\delta)]} < 0$$

Assumption 3 *$a < \beta$.*

We notice that, under Assumption 3,

$$\lambda_P < 0 < \frac{\beta - a}{1-a} < \lambda_3 \equiv \frac{\beta-a}{1-a}\frac{1+\varphi\sigma-\beta(1-\alpha)(1-\delta)}{\varphi\sigma\beta(1-\alpha)(1-\delta)}$$

Let D and T be the determinant and the trace of J respectively.

Lemma 10 *Under Assumption 3, $D < 1$.*

Proof. $D < 1$ is equivalent to

$$D = -P(0) = (1-a)\frac{(1-\alpha)(1-\delta)\varepsilon\sigma}{\varepsilon\sigma(1+\lambda_k)-(1-\alpha)\beta r\lambda_c}(\lambda_P - \lambda_1) < 1$$

that is to $\lambda_P < \lambda_3$. ■

Focus now on the issue of equilibrium uniqueness.

Two variables are predetermined (k_t and P_t), one is not predetermined (c_t). $P(1) < 0$ implies that one eigenvalue is real and greater than one. Thus, equilibrium determinacy (locally). The question now is whether there are zero, one or two eigenvalues inside the unit circle.

There are two possible cases:
(1) $\lambda_1 < \lambda_2$,
(2) $\lambda_2 < \lambda_1$.
We observe that $\lambda_1 < \lambda_2$ iff

$$\frac{1+a}{a} < \frac{[1+(1+\rho)\beta\lambda_k][\beta r\rho + 2\varepsilon\sigma(2-\delta)]}{\beta(1-\delta)((1-\alpha)\beta r[\rho-(2+\rho)\lambda_c] + 2\varepsilon\sigma[1+1/\beta+(2+\rho)\lambda_k])}$$

Notice that the RHS does not depend on a.

Focus on the second case (for instance, if a is sufficiently small or $a = 0$). In this case, $\lambda_2 < \lambda_1$.

Proposition 11 *(equilibrium uniqueness) Let a be null or sufficiently small and Assumption 3 hold. There are three cases.*

(1) $\lambda_P < \lambda_2 < \lambda_1 < 0$. The eigenvalues x_i are such that $x_1 < -1 < 0 < x_2 < 1 < x_3$: local overdeterminacy.

(2) $\lambda_2 < \lambda_P < \lambda_1 < 0$. The eigenvalues x_i are such that $-1 < x_1 < 0 < x_2 < 1 < x_3$: local determinacy.

(3) $\lambda_2 < \lambda_1 < \lambda_P < 0$. Under Assumption 3, there are two eigenvalues inside the unit circle and one outside: $|x_1|, |x_2| < 1 < x_3$. Thus, local determinacy.

When $\lambda_P = \lambda_2$ the system generically undergoes a flip bifurcation.

Proof. Consider the three eigenvalues: x_1, x_2 and x_3. We know that $x_3 > 1$ because $P(1) < 0$.

Points (1) and (2) are immediate: simply notice that $\lambda_2 < \lambda_1$ and consider the signs of expressions (32) and (34) in the cases $\lambda_P < \lambda_2 < \lambda_1 < 0$ and $\lambda_2 < \lambda_P < \lambda_1 < 0$ respectively.

Focus now on point (3). In this case, we get: $P(-1) < 0$, $P(0) < 0$ and $P(1) < 0$.

Under Assumption 3, Lemma 10 applies and $D = x_1x_2x_3 < 1$. Since $P(1) < 0$, we have $x_3 > 1$ and, so $x_1x_2 < 1$. There are two cases: these eigenvalues are (3.1) real or (3.2) nonreal.

In the subcase (3.1), $D < 1$ implies that at least one of the two eigenvalues x_1 and $x_2 > x_1$ is inside the unit circle. Let, without loss of generality, x_2 be in the unit circle that is $-1 < x_2 < 1$. If $0 < x_2 < 1$, since $P(1) < 0$, there exists $\bar{x} \in (0, x_2)$ such that $P(\bar{x}) > 0$. Since $P(0) < 0$, we have also $0 < x_1 < \bar{x} < 1$. Thus, $0 < x_1 < x_2 < 1$. Similarly, one can show that $-1 < x_2 < 0$ implies $-1 < x_1 < x_2 < 0$ because $P(-1) < 0$.

In the subcase (3.2), x_1 and x_2 are nonreal and conjugated. Thus, $|x_1||x_2| = |x_1x_2| < 1$ and, since they have the same modulus, $|x_1| = |x_2| < 1$. ■

Corollary 12 *Under Assumption 3, there is no room for Hopf bifurcations.*

Proof. We know that $P(1) < 0$, that is $x_3 > 1$. Under Assumption 3, Lemma 10 applies and, therefore, $D = x_1x_2x_3 < 1$. Thus, $x_1x_2 < 1$. The Hopf bifurcation generically arises when x_1 and x_2 are nonreal and $x_1x_2 = 1$. Then Assumption 3 is incompatible with the occurrence of a Hopf bifurcation. ■

The occurrence of deterministic fluctuations deserves an interpretation. Focus on the case of a sufficiently small a, that is $\lambda_2 < \lambda_1 < 0$, and a sufficiently negative impact of pollution on labor supply (λ_P close to λ_2).

In this case, an increase in pollution lowers enough the labor supply. The penury of labor input decreases considerably the production and pollution in turn. Thus, a rise in pollution is followed by a drop in pollution at the end: a cycle of period two arises.

We observe that the first part of the mechanism is essentially static through the consumption-labor arbitrage: indeed the negative impact on pollution on labor supply is captured by the elasticity λ_P coming from this arbitrage. Conversely, the second part is dynamic through the pollution accumulation process: the lower production today results in a lower pollution tomorrow.

Notice that a is a measure of pollution persistence. The occurrence of cycles is magnified when a is close to zero because this inertia fails, pollution becomes more volatile and the comparative effect of production on the pollution process becomes maximal.

4 Conclusion

We have considered an economy à la Ramsey where production pollutes and the negative externality distorts the household's consumption-leisure choice. In this framework, we have proved that a sufficiently large (negative) effects of pollution on labor supply may promotes macroeconomic volatility (deterministic cycles near the steady state) through a flip bifurcation. It seems that, in the empirical case considered by recent literature, pollution works as a destabilizing force also through its impact on labor supply. In this sense, our work provides a theoretical argument in favor of environmental friendly fiscal policies.

References

[1] Bovenberg L. and R. Mooij (1994). Environmental levies and distortionary taxation. *American Economic Review* **84**, 1085-1089.

[2] Carson R.T., P. Koundouri and C. Nauges (2011). Arsenic mitigation in Bangladesh: A household labor market approach. *American Journal of Agricultural Economics* **93**, 407-414.

[3] Graff Zivin J. and M. Neidell (2010). Temperature and the allocation of time: Implications for climate change. *NBER Working Paper* **15717.**

[4] Graff Zivin J. and M. Neidell (2012). The impact of pollution on worker productivity. *American Economic Review* **102**, 3652-3673.

[5] Hanna R. and P. Oliva (2011). The effect of pollution on labor supply: Evidence from a natural experiment in Mexico City. *NBER Working Paper* **17302**.

[6] Keeler E., M. Spence and R. Zeckhauser (1971). The optimal control of pollution. *Journal of Economic Theory* **4**, 19-34.

[7] Michel P. and G. Rotillon (1996). Disutility of pollution and endogenous growth. *Environmental and Resource Economics* **6**, 279-300.

[8] Ryder H.E. and G.M. Heal (1973). Optimal growth with intertemporally dependent preferences. *Review of Economic Studies* **40**, 1-33.

[9] Schlenker W. and W. Walker (2011). Airports, air pollution, and contemporaneous health. *NBER Working Paper* **17684**.

[10] Seegmuller T. and A. Verchère (2007). A note on indeterminacy in overlapping generations economies with environment and endogenous labor supply. *Macroeconomic Dynamics* **11**, 423-429.

[11] Tahvonen,O.and J. Kuuluvainen (1993). Economic growth, pollution, and renewable resources. *Journal of Environmental Economics and Management* **24**, 101-118.

[12] Van Der Ploeg F. and C. Withagen (1991). Pollution control and the Ramsey problem. *Environmental and Resource Economics* **1**, 215-236.

[13] Zhang J. (1999). Environmental sustainability, nonlinear dynamics and chaos. *Economic Theory* **14**, 489-500.