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Climate Policy and Catastrophic Change: Be Prepared and Avert Risk

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CLIMATE POLICY AND CATASTROPHIC CHANGE:
Be Prepared and Avert Risk*

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Abstract

The optimal reaction to a pending climate catastrophe is to accumulate capital to be better prepared for the disaster and levy a carbon tax to reduce the risk of the hazard by curbing global warming. The optimal carbon tax consists of the present value of marginal damages, the non-marginal expected change in welfare caused by a marginal higher risk of catastrophe, and the expected loss in after-catastrophe welfare. The last two terms offset precautionary capital accumulation. A linear hazard function calibrated to an expected time of 15 years for a 32% drop in global GDP if temperature stays at 6 degrees Celsius implies with a discount rate of 1.4% a precautionary return of 1.6% and a carbon tax of 136 US $/tC. More intertemporal substitution lowers precautionary capital accumulation and lessens the need for a high carbon tax, but implies less intergenerational inequality aversion which pushes up the carbon tax.

Keywords: non-marginal climate policy, tipping points, risk avoidance, economic growth, social cost of carbon, precaution, adaptation capital

JEL codes: D81, H20, O40, Q31, Q38.

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1. Introduction

A crucial feature of climate policy is how to prepare for catastrophic change and how to reduce the risk of such disasters. Policy thus has to deal with abrupt, often irreversible catastrophic events such as destroying a large chunk of productive capacity or unleashing positive feedback loops at higher temperatures. Examples are the reduced cooling associated with sudden melting of ice sheets (the 'albedo' effect), death of rain forests and reduced carbon capture, and methane release from the tundra in the Arctic.[1] Most conventional analysis of economic policy, however, uses smooth climate damage functions to relate production losses to temperature and accumulation of greenhouse gases in the atmosphere. It abstracts from regime switches in deriving the social cost of carbon and thus the first-best price of carbon as the present value of all future marginal global warming damages (e.g., Tol, 2002; Nordhaus, 2008; Stern, 2007; Golosov et al., 2013; Gerlagh and Liski, 2012). These integrated assessment studies do not deal with non-marginal regime switches induced by climate change.

Pindyck (2013) is very critical of conventional integrated assessment models and argues that the most important driver of the social cost of carbon is the possibility of a catastrophic outcome with a substantial drop in welfare. He also argues that the case for stringent climate policy should not be based on arbitrary downward adjustments of the discount rate but on trying to reduce the chance of catastrophic events occurring. Lemoine and Traeger (2012) and Cai et al. (2012) take up this challenge by analyzing climate tipping in extended versions of the well-known DICE model and showing that the threat of a tipping point induces significant and immediate increases in the social cost of carbon and therefore the optimal carbon tax. Our contribution is to analyze a Ramsey growth model with climate tipping in the form of a hazard of an abrupt and irreversible substantial loss in total factor productivity. We highlight that the regime switches induced by climate catastrophes warrant precautionary capital accumulation (possibly induced by a capital subsidy) and, to the extent that the hazard of a climate catastrophe depends on the stock of atmospheric carbon, a carbon tax to lower this hazard. We thus find that a combination of precautionary saving and a carbon tax is needed to internalize one *non-marginal* regime switch. We have the following four objectives.

First, we wish to take up the challenge posed by Pindyck (2013) and derive a formal framework for the optimal social cost of carbon if there is a hazard of a future climate

[1] Other positive feedback loops such as the build-up of water vapour in the upper atmosphere and the reduced ability of oceans to absorb carbon occur more gradually as temperature rises.

catastrophe. We thus derive the non-marginal costs of a disastrous regime switch resulting from a climate disaster at some random date in the future. If this hazard is exogenous, there is no need to adjust the conventional social cost of carbon. However, if the hazard of the climate catastrophe increases with the stock of atmospheric carbon, the social cost of carbon must be increased with the present value of the marginal increase in expected climate catastrophe damages that results from a higher hazard associated with more carbon in the atmosphere.

Second, we also want to take up the challenge of how to best prepare for a climate disaster occurring at some unknown future date. We show that this requires precautionary capital accumulation.[2] At the time of the disaster, total factor productivity and therefore consumption and output drop, putting the economy at a lower post-catastrophe path. In order to mitigate this effect and to smooth consumption over time, more capital has to be accumulated. However, this boosts fossil fuel consumption and global mean temperature and might thus lead to some unappetizing positive feedback loops making catastrophe more imminent. The optimal social cost of carbon thus not only has to lower the probability of a climate disaster but also to balance the adverse effect of precautionary saving and capital accumulation. More generally we show that the risk of a climate catastrophe also requires more adaptation investment to protect against the damages of the impending climate disaster.

Third, we specify and calibrate an integrated assessment model of the global economy and use this to derive some optimal policy simulations. When a sudden drop in total factor productivity occurs at the time of the disaster, the capital stock and the stock of atmospheric CO2 do not change but consumption jumps down by a discrete amount putting the economy on a lower post-catastrophe path. The possibility of a climate catastrophe thus induces the economy to act in a precautionary way to stave off disaster by accumulating more capital, thereby mitigating the post-catastrophe drops in output and consumption. In the period before disaster demand for energy is boosted due to the extra demand for capital, which implies that carbon emissions are higher in the period leading up to a catastrophe. Of course, this sets in motion a poisonous feedback as the resulting increase in atmospheric carbon before a catastrophe increases the hazard of the catastrophe itself. The carbon tax is in place to reduce the risk of a climate catastrophe. With a linear hazard function calibrated to an expected time of 15 years for a 32% drop in global GDP to occur when global mean temperature stays at 6 degrees Celsius, we find that the first-best optimal global policy requires the equivalent of a

[2] We use the term 'being prepared' informally, since it differs from the *precautionary* saving resulting in stochastic environments with positive third derivatives of the utility function (Kimball, 1990).

capital subsidy of 1.6% to best prepare the global economy for a climate catastrophic and a global carbon tax of 136 US $/tC to reduce the risk of such a catastrophe. With a quartic hazard calibrated to the same hazard at 6 degrees Celsius the carbon tax rises to 381 US $/tC.

Fourth, we highlight the role played by the elasticity of intertemporal substitution whose inverse captures both relative risk aversion and intergenerational inequality aversion. A higher value for this elasticity implies less risk aversion and thus society engages in less precautionary capital accumulation. As a result, fossil fuel demand and carbon emissions will be less and thus there is less need for a high carbon tax. On the other hand, a higher value for this elasticity implies less intergenerational inequality aversion so that current generations are more prepared to sacrifice consumption to improve the climate for future generations. This tends to push up the carbon tax. Our simulations suggest that a higher elasticity of intertemporal substitution leads to a lower carbon tax.

Our analysis models the possibility of a tipping point where the economy shifts from a regime with a high productive capacity to a future regime with a lower one. These regimes are different domains of attraction with different steady states for the capital stock, the carbon stock, output and consumption. This type of modelling is quite common in natural sciences (e.g. Scheffer, 1997), where natural systems develop redundancies to be more resilient to possible bad regime shifts (e.g. Elmqvist et al., 2003). These redundancies look inefficient but only if the possibility of the regime shift is ignored. Similarly, additional capital accumulation resulting from a capital subsidy may look inefficient with a standard Ramsey-Pigouvian lens abstracting from catastrophic change but is optimal when taking account of an imminent climate disaster at some future point of time.

As already mentioned, our study is related to two earlier studies on climate tipping and optimal climate policy.[3] Lemoine and Traeger (2012) add learning to the analysis by modelling the tipping point as an uncertain temperature threshold: when the temperature increases into the support of the probability distribution and nothing happens, the probability distribution is adjusted. Cai et al. (2012) model the tipping point with a hazard rate that shifts the damage function and calculate the effect on the optimal carbon tax. They also show that

[3] There are also analytical papers on climate catastrophes (e.g., Tsur and Zemel, 1996; Karp and Tsur, 2011) and studies on collapses of the resource stock and changes in system dynamics (e.g., Naevdal, 2006; Polasky et al., 2011). Related is the use of a Markov two-regime switching model to capture an exogenous risk of a large drop in GDP growth in which case the prospect of a regime switch curbs the efficient discount rate (for long maturities) and implies a higher carbon tax (Gollier, 2012).

the intertemporal elasticity of substitution and the degree of risk aversion increase the optimal carbon tax even further. These studies focus on numerical methods in advanced integrated assessment models and deal with multiple tipping points.

Section 2 sets up a Ramsey growth model with climate disaster leading to a discrete downward adjustment of total factor productivity as an impending regime switch, where the hazard of a disaster increases with the stock of atmospheric carbon. Section 3 characterizes the social optimum after the catastrophe. Section 4 derives the social optimum before the catastrophe for a constant hazard rate and shows that not a carbon tax, but precautionary saving is needed. No carbon tax is required because we abstract from conventional marginal damages and the hazard rate is not affected by pollution. Section 5 considers a hazard rate that increases with the stock of atmospheric carbon which yields in addition an optimal carbon tax. Before the disaster the economy aims for the so-called disaster-distorted steady state. Our approach yields a catastrophe underpinning of a non-marginal damage function which is the product of the hazard rate and the gap between pre-disaster and post-disaster welfare. Section 6 discusses the calibration of our model, especially the hazard function. Section 7 offers policy simulations of the before-catastrophe outcomes and analyzes the sensitivity to the hazard rate, the shape of the hazard function and the coefficient of relative intergenerational inequality aversion or relative risk aversion. Section 8 generalizes our model to allow for marginal as well as non-marginal damages of global warming in production. This gives an expression for the optimal carbon tax which differs from the conventional Pigouvian carbon tax in that there is a 'make hay while the sun shines' effect, a 'raise the stakes' effect and an 'avert risk' effect. The 'be prepared' effect is realized via precautionary saving. Section 9 introduces adaptation capital to reduce the adverse impact of climate disaster (e.g., dykes) alongside capital used in the production process. The risk of climate disaster typically reduces capital used in production but boosts adaptation capital. Section 10 concludes.

2. A Ramsey Growth Model with Climate Tipping Points

Consider a continuous-time Ramsey growth model with a constant population and an abundant supply of fossil fuel and a carbon-free imperfect substitute for fossil fuel. The abundance of fossil fuel is a strong assumption, but with the advent of huge new reserves of shale gas and other forms of unconventional gas and oil it is becoming less unrealistic. Fossil fuel E is an input into the production process and has constant marginal cost $d > 0$. The carbon-free substitute (renewable) R is also an input into the production process and has constant marginal cost $c > 0$. The capital stock is denoted by K. We assume that capital, fossil fuel and the renewable are cooperative factors of production. Utility is denoted by U, consumption by C, the production function by F, total factor productivity before the catastrophe by $A > 0$, total factor productivity after the catastrophe by $0 < A - \Delta < A$, the depreciation rate of capital by $\delta > 0$, and the pure rate of time preference by $\rho > 0$. For simplicity, we abstract from population growth and technical progress.

To highlight our messages about the policy implications of climate tipping as clearly as possible, we abstract from the usual smooth global warming damages in the production function or the welfare function. Instead, we will give a micro foundation based on tipping points of such damage functions (see section 6). We thus focus on the *non-marginal* regime switch costs of climate change arising from a sudden drop in total factor productivity from A to A - Δ, as a consequence of a potential climate disaster at some uncertain date T in the future. This is, in our view, a way to capture the core of the issue of regime switches resulting from climate change in the Ramsey growth model.[4]

We model uncertainty of a catastrophe with the hazard rate $h(t) = \lim_{\Delta t \to 0} \Pr\left[T \in (t, t+\Delta t) \,|\, T \notin (0,t)\right] / \Delta t,$ so that $h(t)\Delta t$ approximates the probability that a disaster occurs between t and $t+\Delta t$ given that no disaster has taken place up to time t. A constant hazard rate h implies a probability density function $f(t) = he^{-ht}$ for the date T of climate change, which yields the cumulative density function $\Pr[T < t] = 1 - e^{-ht}$ (i.e., the probability of the catastrophe occurring before time t) with mean $1/h$, so that the probability of "survival" is given by e^{-ht}. If the hazard rate h is not constant, ht above has to be replaced by $\int_0^t h(s)ds.$ We will first consider the implications of a constant hazard rate of a climate catastrophe to show a specific effect in the Ramsey growth model. Then we will consider a stock-dependent hazard rate $h(t) = H(P(t))$ to capture the effect that a higher stock of carbon in the atmosphere increases the probability of climate change (i.e., $H'(P) > 0$). Hence, if the stock of atmospheric carbon increases over time, the expected duration before the catastrophe occurs, $1/H(P)$, decreases over time. A failing climate policy thus makes catastrophe more imminent.

[4] Alternatively, we could model a climate catastrophe as a sudden destruction of the capital stock or sudden release leading to a higher stock of atmospheric carbon but we will not pursue this option here.

A fraction ψ (about half) of carbon emissions remains in the atmosphere for a very long time; the remaining fraction returns fairly quickly to the surface of the earth. The stock of atmospheric carbon denoted by P has a rate of natural decay of γ (typically about 1/300). We measure fossil fuel use in GtC, so the carbon-emission ratio is unity. We abstract from carbon capture and sequestration and from learning by doing in the renewable sector and other forms of technical progress.

The problem for the global social planner is thus to choose aggregate consumption, fossil fuel use and renewable use to maximize the expected value of social welfare,

$$(1) \qquad \max_{C,E,R} \mathrm{E}_0 \left[\int_0^\infty e^{-\rho t} U(C(t)) dt \right],$$

subject to the capital accumulation equation,

$$(2) \qquad \dot{K}(t) = \tilde{A} F(K(t), E(t), R(t)) - dE(t) - cR(t) - C(t) - \delta K(t), \quad \forall t \geq 0, \quad K(0) = K_0,$$

the equation describing the stock of carbon in the atmosphere,

$$(3) \qquad \dot{P}(t) = \psi E(t) - \gamma P(t), \quad \forall t \geq 0, \quad P(0) = P_0,$$

the description of the catastrophe and total factor productivity,

$$(4) \qquad \tilde{A}(t) = A, \quad 0 \leq t < T, \qquad \tilde{A}(t) = A - \Delta, \quad \forall t \geq T, \qquad 0 < \Delta < A,$$

and the probability of the catastrophe occurring in the interval before time t,

$$(5) \qquad \Pr[T < t] = 1 - \exp\left(-\int_0^t H(P(s)) ds \right), \quad \forall t \geq 0.$$

Passing of time t increases the probability that a climate disaster has occurred at some time T before that, especially if the hazard rates are high and rising.

3. After the climate catastrophe

We use the principle of dynamic programming, so we first solve for the after-catastrophe regime (denoted by superscript A) and then solve for the before-catastrophe regime (denoted by superscript B). Section 4 does this for the case of an exogenous hazard rate h and section 5 for the general case of a stock-dependent hazard rate.

Since we abstract from multiple tipping points, carbon accumulation does not matter anymore after the climate catastrophe has occurred. We then have the standard Ramsey growth model where fossil fuel use E^A and renewable use R^A after the catastrophe are chosen optimally according to the usual marginal productivity conditions

(6) $$(A-\Delta)F_E(K,E^A,R^A)=d, \quad (A-\Delta)F_R(K,E^A,R^A)=c,$$

which yields the after-calamity net output function (*net* of costs of energy inputs and capital depreciation)

(6′) $$Y^A(K,d,c,A-\Delta)\equiv \underset{E,R}{Max}\left[(A-\Delta)F(K,E,R)-dE-cR\right]-\delta K$$

where $Y_K^A=(A-\Delta)F_K-\delta>0, Y_d^A=-E<0$ and $Y_c^A=-R<0.$ The Hamilton-Jacobi-Bellman (HJB) equation in the value function V^A after the climate disaster is

(7) $$\rho V^A(K,\Delta)=\underset{C}{\text{Max}}\left\{U(C)+V_K^A(K,\Delta)\left[Y^A(K,d,c,A-\Delta)-C\right]\right\},$$

where optimality requires that the marginal utility of consumption equals the marginal value of wealth

(8) $$U'(C)=V_K^A(K,\Delta).$$

Differentiating (7) with respect to K and using (8) yields an ordinary differential equation for the after-calamity stable manifold C^A as function of K (the Euler-Lagrange equation):

(9) $$\left[Y^A(K,d,c,A-\Delta)-C^A(K,\Delta)\right]C_K^A(K,\Delta)=\sigma\left[Y_K^A(K,d,c,A-\Delta)-\rho\right]C^A(K,\Delta),$$

where $\sigma\equiv -U'/CU''>0$ denotes the elasticity of intertemporal substitution and corresponds to the inverse of the coefficient of relative risk aversion or the inverse of the coefficient of intergenerational inequality aversion. For consumption C as a function of time, equation (9) yields the familiar Keynes-Ramsey rule (or Euler equation) which states that the growth in consumption is proportional to the difference between the net return on capital and the pure rate of time preference:

(10) $$\dot{C}(t)=\sigma\left[Y_K^A(K(t),d,c,A-\Delta)-\rho\right]C(t), \quad \sigma\equiv -U'/CU''>0.$$

The coefficient of intergenerational inequality aversion equals $1/\sigma$, so more inequality aversion means that current generations should suffer less and growth in consumption is lower. Hence, (10), after rearranging, also implies that the marginal product of capital (net of depreciation) should equal the rate of interest which equals the rate of time preference plus inequality aversion times the expected rate of consumption growth.

By integrating equation (10) from the steady state, the stable manifold $C^A(K,\Delta)$ is found and with equations (7) and (8) this gives the value function $V^A(K,\Delta)$ after the regime shift:

$$(11)\qquad V^A(K,\Delta) = \frac{U\left(C^A(K,\Delta)\right) + U'\left(C^A(K,\Delta)\right)\left[Y^A(K,d,c,A-\Delta) - C^A(K,\Delta)\right]}{\rho}.$$

The stable manifold and the associated value function do not depend on the stock of atmospheric carbon, but do depend on the size of the climate disaster Δ.

The post-disaster steady-state values of the capital stock K^{A^*} and consumption $C^A(K^*,\Delta)$ follow from

$$(12a)\qquad Y_K^A(K^*,d,c,A-\Delta) = \rho \quad \Rightarrow \quad K^{A^*} = K^{A^*}(\overset{-}{\rho},\overset{-}{d},\overset{-}{c},A\overset{+}{-}\Delta),$$

$$(12b)\qquad C^{A^*} = Y^A\left(K^*(\rho,d,c,A-\Delta),d,c,A-\Delta\right) \equiv C^{A^*}(\overset{-}{\rho},\overset{-}{d},\overset{-}{c},A\overset{+}{-}\Delta),$$

where the signs indicate the signs of the partial derivatives. After-calamity capital and consumption are thus high if society is patient, fossil fuel and the renewable are cheap, and productivity is high.

Lemma: A first-order approximation of the after-calamity stable manifold around the steady state is

$$(13)\qquad C^A(K) \cong C^{A^*}\left(\frac{K}{K^{A^*}}\right)^{\phi}, \quad \phi \equiv \left[\frac{1}{2}\rho + \frac{1}{2}\sqrt{\rho^2 - 4\sigma Y_{KK}^A C^{A^*}}\right]\frac{K^{A^*}}{C^{A^*}} > \rho.$$

Proof: (9) and l'Hôpital's Rule give $C_K^A(K^{A^*}) = \lim_{K \to K^{A^*}}\left[\frac{\sigma(Y_K^A - \rho)C^A}{Y^A - C^A}\right] = \lim_{K \to K^{A^*}}\left[\frac{\sigma Y_{KK}^A C^A}{\rho - C_K^A}\right].$ This yields the quadratic $(C_K^A)^2 - \rho C_K^A + \sigma Y_{KK}^A C^{A^*} = 0.$ The positive solution of this quadratic is given by $C_K^A = \frac{1}{2}\rho + \frac{1}{2}\sqrt{\rho^2 - 4\sigma Y_{KK}^A C^{A^*}} > \rho.$ The elasticity ϕ follows from this partial derivative.

□

It turns out that this approximation is extremely good for the calibration discussed in section 6. Using this approximation for the stable consumption manifold allows us to directly calculate from equation (11) the after-calamity value function.

4. Before the climate catastrophe: exogenous hazard rate

The question is how the prospect of climate disaster with the consequences described in section 3, affects the optimal growth path before this regime switch has actually occurred. For a constant (exogenous) hazard rate *h* the accumulation of atmospheric carbon does not matter and therefore fossil fuel and renewable use follow from the before-calamity net output function

(14) $$Y^B(K,d,c,A) \equiv \underset{E,R}{Max}\left[AF(K,E,R) - dE - cR\right] - \delta K.$$

Taking account of the imminent climate calamity implies that the before-catastrophe optimal growth path differs from the one where the imminent disaster is ignored. The before-catastrophe HJB equation in the value function V^B is (see supplement A):

(15) $$(\rho + h)V^B(K) = \underset{C}{\mathrm{Max}}\left\{U(C) + V_K^B(K)\left[Y^B(K,d,c,A) - C\right] + hV^A(K,\Delta)\right\}$$

with the familiar optimality condition

(16) $$U'(C^B) = V_K^B(K).$$

If the hazard rate *h* is zero, we have the standard Ramsey problem. If *h* > 0, we differentiate the HJB equation (15) with respect to *K* and use (16) to get the Euler-Lagrange equation:

(17) $$\begin{aligned}&\left[Y^B(K,d,c,A) - C^B(K)\right]C_K^B(K) = \\ &\sigma\left\{Y_K^B(K,d,c,A) - \rho + h\left[\frac{V_K^A(K,\Delta)}{U'\left(C^B(K)\right)} - 1\right]\right\}C^B(K), \quad \sigma \equiv -U'/CU'' > 0.\end{aligned}$$

The before-catastrophe dynamics follows from the differential equations:

(18a) $$\dot{K} = Y^B(K,d,c,A) - C, \quad K(0) = K_0,$$

(18b) $$\dot{C} = \sigma C\left[Y_K^B(K,d,c,A) + h\left[\frac{V_K^A(K,\Delta)}{U'\left(C^B(K)\right)} - 1\right] - \rho\right].$$

where the initial value *C*(0) has to adjust to place the economy on its stable manifold. The system (18a) and (18b) gives the following steady states of the capital stock and consumption:

(19a) $Y_K(K^{B^*},d,c,A)=\rho-h\left[\dfrac{V_K^A(K^{B^*},\Delta)}{U'(C^B(K^{B^*})}-1\right] \quad\Rightarrow\quad K^{B^*}=K^{B^*}(\overset{+}{h},\overset{+}{\Delta},\overset{-}{\rho},\overset{-}{d},\overset{-}{c},\overset{+}{A}),$

(19b) $C^{B^*}=Y(K^{B^*},d,c,A)\equiv C^{B^*}(\overset{+}{h},\overset{+}{\Delta},\overset{-}{\rho},\overset{-}{d},\overset{-}{c},\overset{+}{A}).$

Some conclusions can be drawn at this stage. First, we see from (15) that in case of a doomsday scenario where after the catastrophe the economy is completely destroyed ($\Delta = A$, $K^A = 0$, $V^A = 0$), the only difference with the standard case is that the discount rate is raised from ρ to $\rho + h$. Hence, the bigger the chance of a doomsday scenario, the higher the discount and thus the less the economy invests and the more it consumes.[5] However, this insight is misleading if the disaster does not lead to complete demolishment of the economy. In that case, a counteracting *precautionary* effect operates (cf., Polasky et al., 2011). We note from (8) and (16) that consumption will drop from $C^B(K)$ to $C^A(K,\Delta)$ and the marginal value at the regime shift will increase from $V_K^B(K)$ to $V_K^A(K, \Delta)$ so that the term between brackets $(1 - V_K^A(K,\Delta)/U'(C^B(K))$ is negative. Hence, if there is life after the catastrophe, the social planner should *reduce* the social rate of discount and engage in precautionary capital accumulation to be better prepared for when the climate disaster hits the economy. This suggests that the practice of raising the discount rate to account for a regime switch must be offset by precautionary capital accumulation if there is life left after the catastrophe.

Second, equation (18b) indicates that the marginal product of capital Y_K^B must be augmented with the social benefit of capital (*SBC*) or *precautionary return* of capital accumulation:

(20) $\theta=\Theta(K,\Delta,h)\equiv\left(\dfrac{V_K^A(K,\Delta)}{U'(C^B(K))}-1\right)h>0, \qquad \Theta_K>0,\Theta_\Delta>0,\Theta_h>0\,.$

This precautionary return is positive, since $V_K^A(K,\Delta)=U'(C^A)>U'(C^B)$ as $C^A<C^B.$ Society thus *aims* for a higher steady-state capital stock before the catastrophe to compensate for the drop in total factor productivity after the regime shift. The *SBC* or required precautionary return, and consequently the amount of precautionary capital accumulation, are bigger if the hazard of a climate catastrophe is higher and the eventual disaster is bigger. Furthermore, this precautionary effect is bigger if the drop in consumption after the catastrophe is bigger and the capital stock is larger (as then the economy is more vulnerable). If the market outcome ignores the possibility of a climate disaster occurring anddoes not internalize the benefits of

[5] This effect reminds one of Rowan Atkinson's 1983 film *Dead on Time*, where Bernard Fripp is told that he has thirty minutes to live and sets off on a frantic mission to life to the full whilst it still lasts.

precautionary capital accumulation, the social optimum can be realized in a decentralized market economy with a *capital subsidy* equal to the *SBC* or precautionary return (20), provided this subsidy can be financed in a non-distorting fashion with lump-sum taxes.

5. Before the climate catastrophe: stock-dependent hazard rate

If the hazard of a climate calamity is endogenous, $h = H(P)$, $H' > 0$, the hazard of a climate tipping point becomes more imminent as the atmosphere continues to accumulate carbon and global mean temperature rises. In this case we will show that to deal with the problem of climate disaster a Pigouvian tax on carbon emissions is needed alongside the precautionary saving (if necessary induced by a capital subsidy) discussed in section 4.

Since we focus at abrupt, irreversible climate disasters, the analysis after the disaster is unaffected as we abstract from marginal production losses arising from a gradual rise in the atmospheric carbon stock. The HJB equation before the climate disaster becomes (see supplement A)[6]

$$\text{(21)} \quad \begin{aligned} \rho V^B(K,P) = \underset{C,E,R}{\text{Max}} \Big\{ & U(C) + H(P)\Big[V^A(K,\Delta) - V^B(K,P)\Big] + \\ & V_K^B(K,P)\big[AF(K,E,R) - dE - cR - C - \delta K\big] + V_P^B(K,P)(\psi E - \gamma P)\Big\} \end{aligned}$$

with the following optimality conditions for consumption, fossil fuel use and renewable use:

$$\text{(22a)} \quad U'(C^B) = V_K^B(K,P),$$

$$\text{(22b)} \quad AF_E(K,E,R) = d + \tau, \qquad \tau \equiv -\psi V_P^B(K,P) / V_K^B(K,P) > 0,$$

$$\text{(22c)} \quad AF_R(K,E,R) = c.$$

Equation (22a) states that the marginal utility of consumption equals the marginal value of capital. The first part of (22b) states that the marginal product of fossil fuel must equal the cost of fossil fuel plus the social cost of carbon (*SCC*) denoted by τ. The second part of (22b) defines the social cost of carbon τ as the marginal cost of one unit of carbon emissions $-V_P^B$ times the fraction of emissions that does not return quickly to the surface of the earth ψ, converted from utility to final goods units (by dividing by the marginal utility of consumption V_K^B). The *SCC* is thus the present discounted value of all future marginal global warming

[6] We cannot substitute out energy use, since optimal fossil fuel use takes account of the higher risk of climate disaster that results from using more fossil fuel. So we do not use the net output function here.

damages. Equations (22b) imply that the social planner internalizes the present discounted value of all future marginal damages of emitting an extra unit of carbon. Equation (22c) states that the marginal product of the renewable must equal the cost of the renewable. Conditions (22b) and (22c) can be solved to give demand for fossil fuel and the renewable as increasing functions of capital and total factor productivity and decreasing functions of their costs:

$$\text{(23)} \qquad E = E^B(\overset{+}{K}, \overset{-}{d+\tau}, c, \overset{+}{A}), \quad R = R^B(\overset{+}{K}, d+\tau, \overset{-}{c}, \overset{+}{A}).$$

We now define the maximum level of before-catastrophe production, *net* of costs of input and capital depreciation, for a given level of the capital stock by

$$\text{(24)} \qquad Y^B(K, d+\tau, c, A) = \underset{E,R}{\text{Max}} \left[AF(K,E,R) - (d+\tau)E - cR \right] - \delta K.$$

Differentiating (21) with respect to K and P, using (22) and equations (2) and (3), yields the system:

$$\text{(25)} \qquad -V_{KK}^B(K,P)\dot{K} - V_{PK}^B(K,P)\dot{P} = \left[AF_K(K,E^B,R^B) - \delta - \rho - H(P) \right] V_K^B(K,P) + H(P)V_K^A(K,\Delta),$$

$$\text{(26)} \qquad V_{KP}^B(K,P)\dot{K} + V_{PP}^B(K,P)\dot{P} = \left[\rho + \gamma + H(P) \right] V_P^B(K,P) + H'(P)[V^B(K,P) - V^A(K,\Delta)].$$

This system of coupled ordinary differential equations in V_K^B and V_P^B is equivalent to the necessary conditions of the Pontryagin principle with the marginal value of capital defined as $\lambda^K \equiv V_K^B = U'(C^B)$ and the marginal social cost of the stock of atmospheric carbon as $\lambda^P \equiv -V_P^B$. These necessary conditions are (omitting the dependence on time t in the differential equations):

$$\text{(27)} \qquad -\dot{\lambda}^K = \left[AF_K(K,E^B,R^B) - \delta - \rho - H(P) \right] \lambda^K + H(P)V_K^A(K,\Delta),$$

$$\text{(28)} \qquad \dot{\lambda}^P = \left[\rho + \gamma + H(P) \right] \lambda^P - H'(P)[V^B(K,P) - V^A(K,\Delta)].$$

5.1. Implied non-marginal global warming damage function

The same set of necessary conditions results for the Ramsey growth problem if the social planner maximizes the social welfare function

$$\text{(29a)} \qquad \max_{C,E,R} \mathrm{E}\left[\int_0^\infty e^{-\rho x} \left\{ U\big(C(t)\big) - D\big(K(t),P(t)\big) \right\} dt \right]$$

with a damage function which depends on both the capital stock K and the pollution stock P:

(29b) $D(K,P)=H(P)\left[V^{B}(K,P)-V^{A}(K,\Delta)\right]>0,\quad D_{K}=H(P)(V_{K}^{B}-V_{K}^{A})<0,\quad D_{P}>0,\quad D_{PK}<0.$

The damage function decreases with the capital stock and increases with the stock of atmospheric carbon. Marginal global warming damages resulting from an extra unit of carbon emissions decrease with the capital stock which follows from diminishing marginal utility of consumption.

5.2. Before-catastrophe dynamics

The before-catastrophe saddlepoint system consists of the dynamics of the capital stock (from (2)), the dynamics of the stock of atmospheric carbon (from (3)), the modified Keynes-Ramsey rule for growth in consumption (from (27) and (22a)), and the dynamics of the social cost of carbon or the carbon tax (from (27), (28) and the definition of θ given in (30c)):

(30a) $\dot{K}=Y^{B}(K,d+\tau,c,A)-\tau Y_{d+\tau}^{B}(K,d+\tau,c,A)-C,\quad K(0)=K_{0}$

(30b) $\dot{P}=-\psi Y_{d+\tau}(K,d+\tau,c,A)-\gamma P,\quad P(0)=P_{0}$

(30c) $\dot{C}=\sigma\left[Y_{K}^{B}(K,d+\tau,c,A)+\Theta(K,P)-\rho\right]C,\qquad \theta=\Theta(K,P)\equiv H(P)\left[\dfrac{V_{K}^{A}(K,\Delta)}{U'(C)}-1\right]$

(30d)
$$\begin{aligned}\dot{\tau}&=\left[Y_{K}^{B}(K,d+\tau,c,A)+\Theta(K,P)+\gamma+H(P)\right]\tau-\frac{\psi H'(P)[V^{B}(K,P)-V^{A}(K,\Delta)]}{U'(C)}\\&=\left[Y_{K}^{B}(K,d+\tau,c,A)+\Theta(K,P)+\gamma\right]\tau-\frac{\psi D_{P}(K,P)}{U'(C)}\end{aligned}$$

where θ denotes the *SBC* or precautionary return on capital. The extra term in the capital dynamics (30a) compared with (18a) corresponds to the lump-sum rebates of the carbon tax revenues, τE. The Keynes-Ramsey rule (30c) now states that consumption growth is proportional to the net marginal product of capital *plus* the *SBC minus* the pure rate of time preference.[7] The relevant interest rate for investment decisions thus now equals the pure rate of time preference *plus* intergenerational inequality aversion times the rate of consumption growth *minus* the precautionary return to capture the ‘be prepared’ effect. This is a saddlepoint system where the initial conditions $C(0)$ and $\tau(0)$ have to adjust in such a way that the system is on its stable manifold and converges asymptotically to its steady state.

5.3. The precautionary return on capital accumulation and the optimal carbon tax

[7] With global warming damages (29b) we have $\Theta(K,P)=-D_{K}(K,P)/U'(C^{B})$.

The social optimum is sustained in a market economy if the *precautionary return* on capital or the *SBC* defined in (30c) is fully internalized by the market (else, it must be internalized via a capital subsidy) and a *carbon tax* is levied equal to the social cost of carbon implied by (30d). With a constant coefficient of intergenerational inequality aversion, $1/\sigma$, it follows from (8) that the optimal *SBC* or *precautionary return* on capital is given by:

$$(31) \qquad \theta = H(P)\left[\frac{U'(C^A)}{U'(C^B)} - 1\right] = H(P)\left[\left(\frac{C^B}{C^A}\right)^{1/\sigma} - 1\right] > 0.$$

It can be seen from (31) that the optimal precautionary return is positive as consumption jumps down as a result of the climate catastrophe. The required precautionary return is bigger if the hazard of a catastrophe *H(P)*, the size of the disaster (reflected in the downward jump in consumption) and/or the coefficient of relative intergenerational inequality aversion $1/\sigma$ are higher. The precautionary return or *SBC* induces the economy to engage in precautionary capital accumulation to be better prepared when the climate calamity strikes. As the economy proceeds along its growth path and the stock of atmospheric carbon and temperature increase, the disaster becomes more imminent and the economy has to internalize a bigger precautionary return on capital or *SBC*.

Equation (30d) implies that no carbon tax is required to replicate the social optimum if the hazard rate is exogenous and independent of the stock of atmospheric carbon (cf. section 4 with constant *h*). However, if the risk of a climate catastrophe increases with the stock of atmospheric carbon *P*, a positive carbon tax is required. By integrating (30d) forwards we get the optimal *carbon tax* (i.e., the *SCC*) as the present value of all future *non-marginal* global warming damages:

$$\tau(t) = \int_t^\infty \psi \frac{D_P\left(K(s),P(s)\right)}{U'\left(C^B(s)\right)} \exp\left(-\int_t^s \left[r(s') + \Theta\left(K(s'),P(s')\right) + \gamma\right] ds'\right) ds$$

$$= \int_t^\infty \psi H'\left(P(s)\right) \frac{V^B\left(K(s),P(s)\right) - V^A\left(K(s),\Delta\right)}{U'\left(C^B(s)\right)} \exp\left(-\int_t^s \left[r(s') + \Theta\left(K(s'),P(s')\right) + \gamma + H\left(P(s')\right)\right] ds'\right) ds,$$

where the discount rate to be used is the net marginal product of capital or the interest rate ($r = Y_K(K, d+\tau, c, A)$) *plus* the precautionary return or *SBC plus* the rate of decay of the atmospheric stock of carbon. Alternatively, we discount utility damages using the rate of time preference instead of the interest rate and convert from utility to final goods units (by integrating (28) and using $\tau = \psi\lambda^P / \lambda^K$):

$$\tau(t) = \left\{ \int_t^\infty \psi D_P \left(K(s), P(s) \right) \exp\left(-(\rho+\gamma)(s-t) \right) ds \right\} / U'\left(C^B(t) \right)$$

$$= \left\{ \int_t^\infty \psi H'\left(P(s)\right) \left[V^B\left(K(s), P(s)\right) - V^A\left(K(s), \Delta\right) \right] \exp\left(-\int_t^s \left[\rho + \gamma + H\left(P(s')\right) \right] ds' \right) ds \right\} / U'\left(C^B(t) \right).$$

If the climate disaster leads to a bigger loss in value to go, a higher carbon tax is warranted. As a result of this, there is less demand for fossil fuel and less demand for capital. If this ‘avert risk’ effect is strong enough compared with the ‘be prepared’ effect resulting from precautionary capital accumulation, capital may fall as a result of the risk of catastrophe compared with the naive outcome.

5.4. Steady state of the before-catastrophe system

Although the steady state of the before-catastrophe system will never be reached, it is affected by the hazard rate and will be relevant for steering the economy towards it before the climate disaster strikes. This disaster-distorted steady state $\left\{ K^{B*}, \theta^{B*}, P^{B*}, C^{B*}, \tau^{B*} \right\}$ differs, of course, from the steady state of the standard Ramsey model and follows from the steady state of the system (30a)-(30d) and (31):

(32a) $\quad Y_K(K^{B*}, d+\tau^{B*}, c, A) = \rho - \theta^{B*}, \qquad \theta^{B*} = H(P^{B*}) \left[\dfrac{U'\left(C^A(K^{B*}, \Delta) \right)}{U'(C^{B*})} - 1 \right],$

(32b) $\quad P^{B*} = -\psi Y_{d+\tau}(K^{B*}, d+\tau^{B*}, c, A) / \gamma,$

(32c) $\quad C^{B*} = Y^B(K^{B*}, d+\tau^{B*}, c, A) - \tau^{B*} Y_{d+\tau}(K^{B*}, d+\tau^{B*}, c, A),$

(32d) $\quad \tau^{B*} = \dfrac{\psi H'(P^{B*}) \left[V^B(K^{B*}, P^{B*}) - V^A(K^{B*}, \Delta) \right]}{\left[\rho + \gamma + H(P^{B*}) \right] U'(C^{B*})} = \dfrac{\psi H'(P^{B*}) \left[U(C^{B*}) - \rho V^A(K^{B*}, \Delta) \right]}{\left[\rho + H(P^{B*}) \right] \left[\rho + \gamma + H(P^{B*}) \right] U'(C^{B*})},$

where $V^A(K^{B*}, \Delta)$ follows from (11). This steady state depends on the gap between the before-catastrophe and the after-catastrophe value, both evaluated at the disaster-distorted steady state. The disaster-distorted steady state is calculated as in the standard Ramsey model, except that internalizing the *SBC* (i.e., θ^{B*}) pushes up the capital stock to be better prepared when the climate calamity eventually hits the global economy (from the first part of (32a)), which induces more emissions and a larger carbon stock in steady state (in the absence of a carbon tax). Consumption is higher in the distorted steady state, but in the short and medium run consumption is held back to allow for precautionary capital accumulation. In this sense, if the hazard rate is exogenous, the disaster-distorted steady state can be coined a ‘prudent’ steady state. However, if the hazard rate increases in the stock of carbon in the atmosphere, the precautionary return (possibly induced by a capital subsidy) is accompanied by a carbon tax (see (32d)). This pushes up the cost of fossil fuel and makes the economy produce less, accumulate less capital, emit less carbon and end up with a lower stock of atmospheric carbon in steady state. These negative 'avert risk' effects on the capital stock may outweigh the positive 'be prepared' effect on the capital stock in which case the capital stock falls. We will illustrate these transient and steady-state effects in more detail in the numerical simulations reported in section 7 and discuss them in the generalized model put forward in section 8, but first we discuss the calibration of our model.

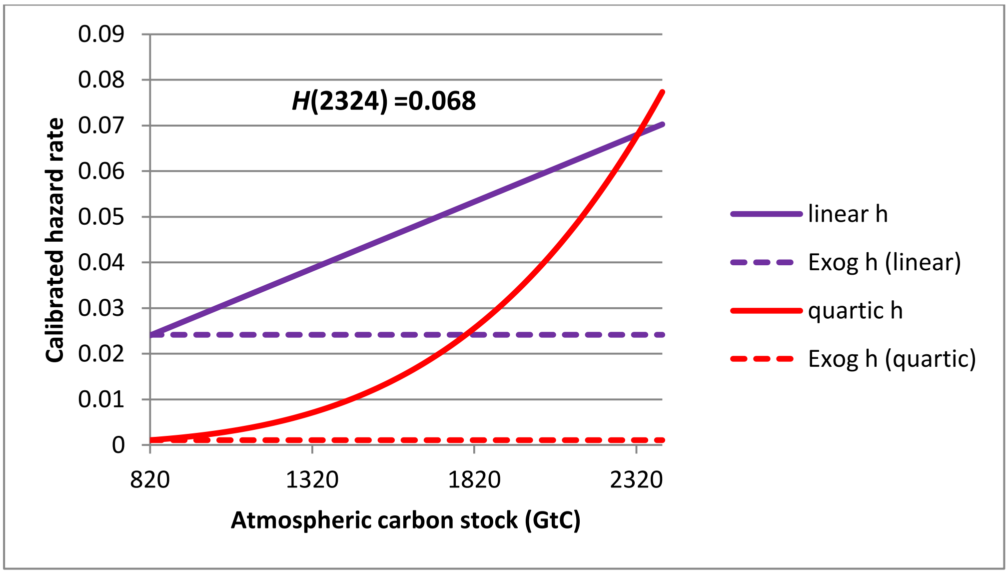
6. Calibration

We have calibrated our model to the global economy without a precautionary return or capital subsidy and without a carbon tax; GDP, consumption, capital and the carbon tax are all given in 2010 US dollars (see appendix A). We derive the before-catastrophe transient and steady-state outcomes for the situation where the global planner takes account of a potential climate disaster equal to 30 percent of total factor productivity ($\Delta = 0.3$), which depresses TFP from 11.1 to 7.8 and cuts GDP by 32%. To do this, we need the post-catastrophe steady state and stable manifold (see appendix B).

To calibrate the climate calamity hazard function, we suppose that the expected duration for this climate catastrophe to occur when the carbon concentration stays put at 2324 GtC and global mean temperature at 6 degrees Celsius is 14.7 years (cf. Nordhaus, 2008; Golosov et al., 2012). This corresponds to a hazard rate of 0.068. We thus use $H(2324) = 0.068$ to calibrate various hazard functions.

First, we calibrate a linear hazard function $H(P) = 2.926 \times 10^{-5} \times P$. Second, we use an exogenous hazard rate of 0.024 which corresponds to an expected duration for the catastrophe to occur of 42 years. This corresponds to the hazard rate predicted by this linear hazard function at the current carbon concentration. By comparing these two we can show the effect of the hazard rate increasing with the stock of atmospheric carbon. Third, we use $H(2324) = 0.068$ to calibrate the quartic hazard function $H(P) = 2.33 \times 10^{-15} \times P^4$ which implies that the initial hazard of the catastrophe is tiny (one in a thousand). The expected time for the catastrophe to occur at the initial carbon concentration is thus 1000 years instead of 42 years for the linear hazard. By comparing these two we can show the effect of a low risk initially but a strongly increasing risk at higher temperatures. These linear and convex hazard functions are plotted in fig.1.

Figure 1: Calibration of the hazard function



7. Before-catastrophe outcomes

Table 1 reports the before-calamity steady states for the constant, linear and quartic hazard rates and compares them with the no-calamity and after-calamity steady states (see supplement B). The steady-state precautionary return or *SBC* (if necessary realized by a capital subsidy) θ^{B*} pushes up the steady-state values of the capital stock, aggregate consumption and the stock of atmospheric carbon, but the steady-state carbon tax τ^{B*} pushes these levels down again. The net effect depends on the parameters of the model. The pre-catastrophe steady-state capital stock is higher than without the correcting precautionary return and carbon tax if and only if $\left(\dfrac{\delta+\rho}{\delta+\rho-\theta^{B*}}\right)^{1-\beta} > \left(\dfrac{d+\tau^{B*}}{d}\right)^{\beta\omega}$. If the hazard rate does not depend on temperature and the carbon tax is zero, this condition is always satisfied so the pre-catastrophe capital stock always increases relative to the naive outcome. However, if the hazard rate increases with temperature, the effect of the carbon tax can dominate the effect of the precautionary return in which case the above inequality is not satisfied and the capital stock falls relative to the naive outcome. Indeed, this is what happens in table 2 with the quartic hazard function (327 < 335 trillion US \$) but not with the constant hazard (486 trillion US \$) or the linear hazard function (535 trillion US \$).

The quartic hazard function leads to a much higher carbon tax in the long run, 381 US $/tC, than the linear hazard function, 136 US $/tC. The quartic hazard function pushes the stock of atmospheric carbon down substantially so that in steady state the hazard rate is very low and the carbon tax is high, conform (32d), despite a somewhat higher marginal hazard rate in the steady-state stock than in the linear case. Associated with a much more substantial reduction in fossil fuel use and reduction in global mean temperatures is a much bigger drop in GDP, a bigger drop in consumption and a fall (not a rise) in the capital stock. We also note that the long-run precautionary return or *SBC* for the quartic hazard function is much smaller (0.1%) than that for the constant hazard (1.2%) and the linear hazard function (1.6%). One reason is that the much higher carbon tax has led to a big drop in the risk of the climate calamity and has lessened the need to prepare oneself for catastrophic climate change. Also, one does not want to push up capital too much as this pushes up fossil fuel use and raises the hazard of a calamity.

Table 1: Before- and after-calamity steady states with different hazard functions

	No shock	After calamity	Before calamity			
			Exogenous h = 2.4%	Linear $H(P)$	Quartic $H(P)$	Linear $H(P)$, σ = 0.8
Capital stock, K (T US $)	356	202	486	535	346	440
Fossil fuel use, E (GtC)	10.1	5.7	11.1	8.9	5.5	8.5
Renewable use, R (TBTU)	11.3	6.4	12.5	12.7	10.8	11.9
World GDP (T US $)	75.9	43.1	83.9	85.1	72.3	80.0
Net output, Y (T US $)	52.8	30.0	53.8	52.4	49.9	52.4
Consumption, C (T US $)	52.8	30.0	53.8	53.6	52.0	53.5
Carbon stock, P (GtC)	1,679	954	1,857	1,482	911	1424
Temperature, *Temp*	4.6	2.1	5.0	4.1	1.9	3.9
Precautionary return (%)	0.0	0.0	1.2	1.6	0.1	0.95
Carbon tax (US $/tC)	0.0	0.0	0.0	136	381	123

7.1. Pre-catastrophe outcomes with constant hazard: capital subsidy, no carbon tax

We use the stable manifold $C^A(K,\Delta)$ and the corresponding value function $V^A(K,\Delta)$ for the post-catastrophe regime as input into the pre-catastrophe system (see fig. A1). The solid blue lines in fig. 2 represent the simulation outcomes for the years 2010-2070 with an exogenous hazard rate of h = 2.4% when the catastrophe hits after 20 years at the end of the year 2030. The dotted blue lines indicate what would happen to capital accumulation and aggregate consumption if the disaster would strike at a later date. The blue lines should be compared with the red lines representing the naive outcomes where the world fails to take account of the imminent climate disaster.

As a result of a modest precautionary return (if necessary internalized via a capital subsidy) starting with 1.4% and declining to 1.3% just before the calamity hits in 2030, precautionary capital accumulation leads to a capital stock of 335, instead of 299 trillion US dollar in the naive outcome, just before the catastrophe strikes. To make this possible the global economy has to initially consume 5.5 trillion US dollar less than under the naive outcome. Just before the catastrophe in 2020 the economy is still consuming 3 trillion US dollar less than under the naive outcome. As a result of this precautionary capital accumulation, the downward discrete jump in consumption at the time of the catastrophe is reduced from 13.9 trillion US dollar under the naive outcome to 9.2 trillion US dollar. This form of consumption smoothing softens the blow to welfare.

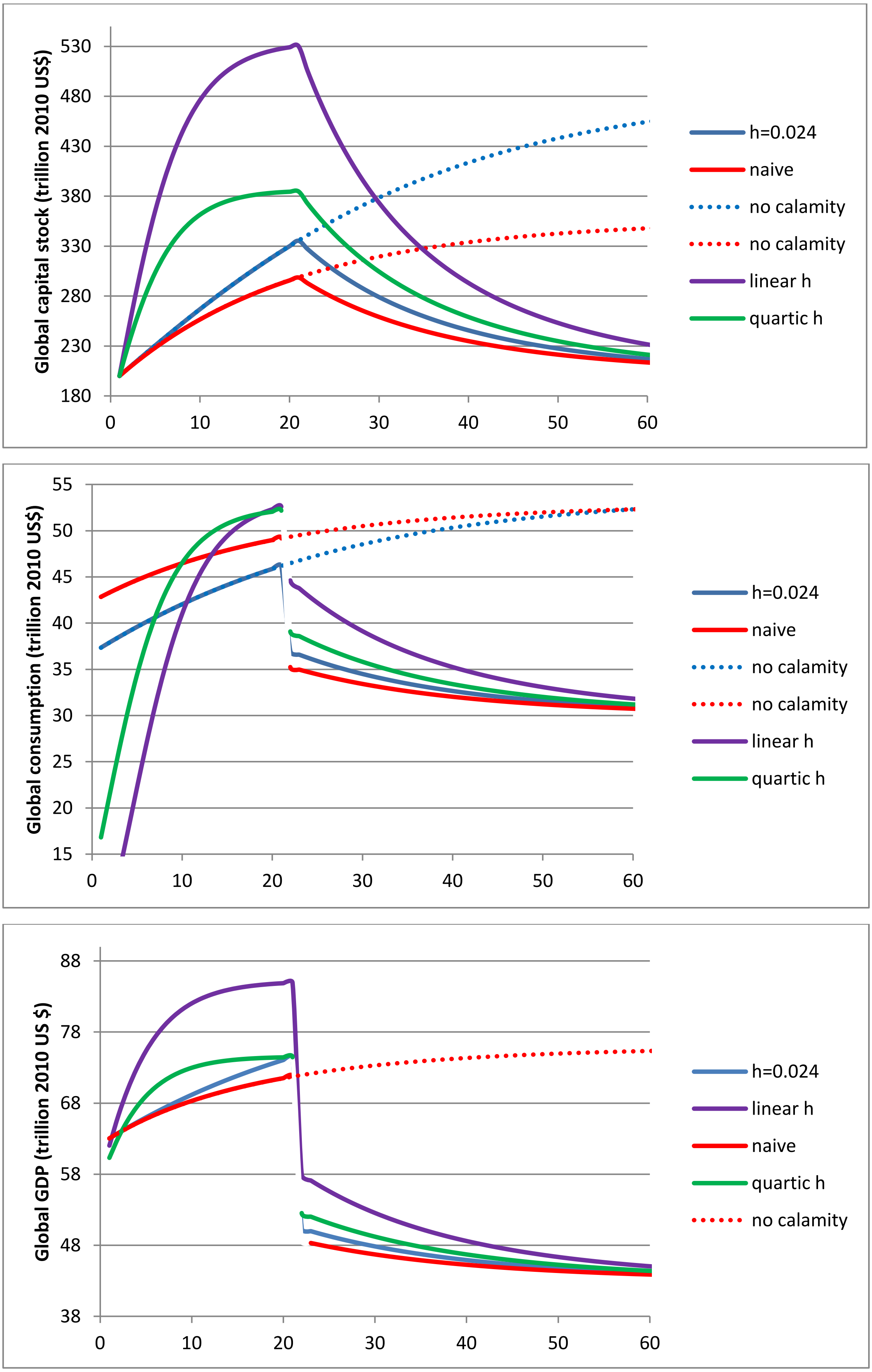
An adverse consequence of this precautionary capital accumulation is higher demand for fossil fuel (rising to 9.9 instead of 8.8 GtC just before the disaster strikes in 2030) and thus a gradual rise in the stock of atmospheric carbon and global mean temperature over and above the rise under the naive outcome (rising to 867 instead of 859 GtC just before the calamity). The effect of precautionary capital accumulation is to have slightly higher GDP than the naive outcome before the catastrophe and also slightly higher GDP after the calamity.

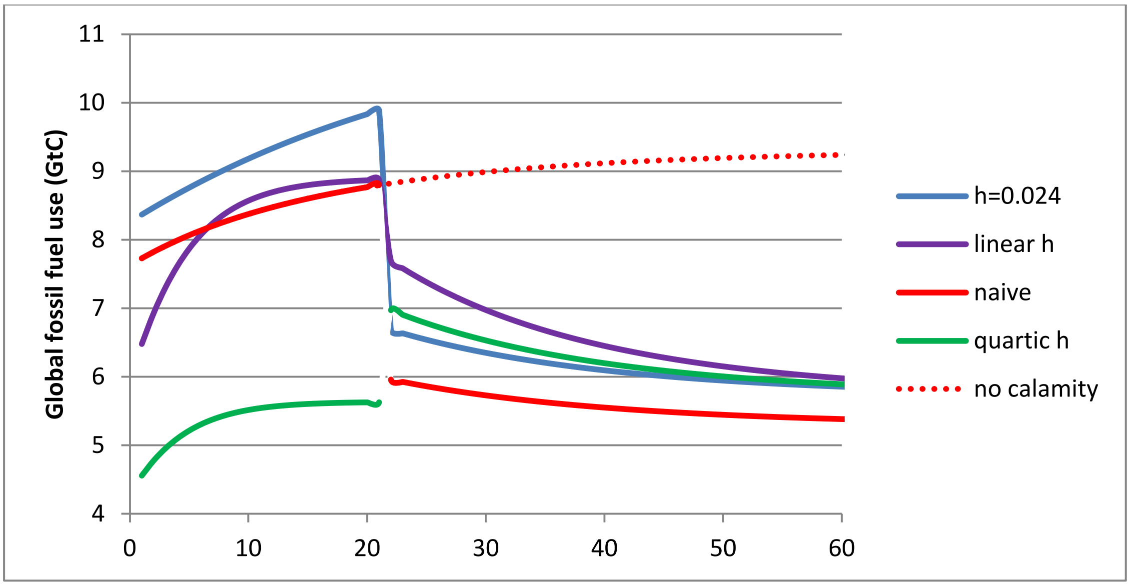
7.2. Pre-catastrophe outcomes with linear hazard function

The solid purple lines in fig. 3 correspond to the optimal policy simulations when the hazard rate starts with the hazard rate of 2.4% (see the blue lines) and then rises linearly, passing through the point $H(2324) = 6.8\%$ which corresponds to the hazard at 6 degrees Celsius.[8] The transient paths for the optimal precautionary return on capital or *SBC* starts at 2.42% in 2010 and grows a little to 2.49% in 2030 just before the disaster strikes. The need for precautionary capital accumulation is higher, since the risk of disaster occurring now rises steadily as the

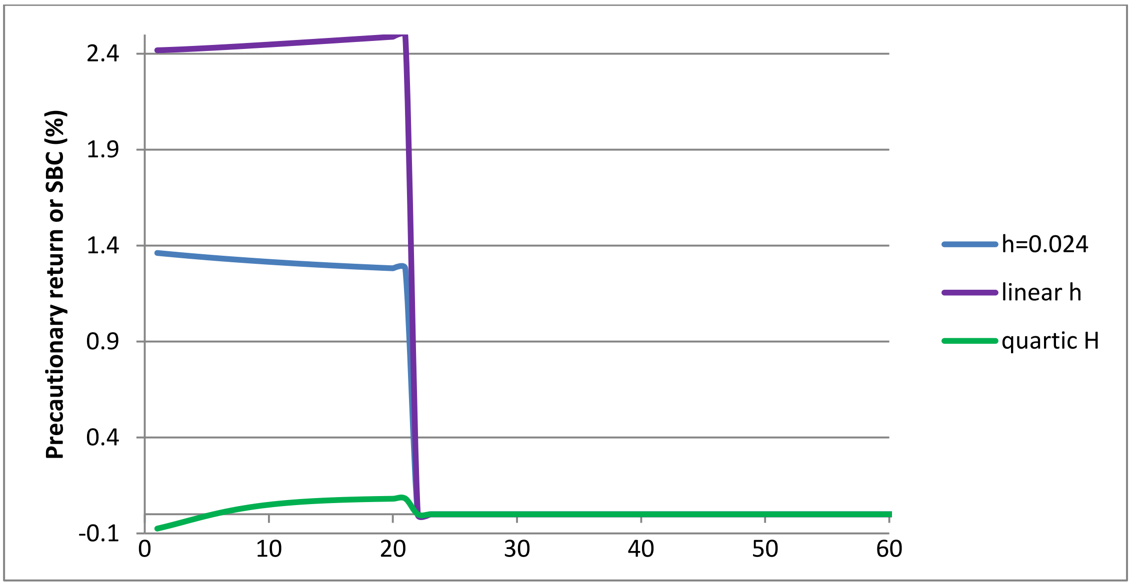
[8] If the hazard was constant and equal to the average hazard over the range 826-2324 GtC, i.e., 4.6%, the dynamic paths of capital and consumption would be indiscernible from results with the linear hazard function. Because with this average hazard there is no rationale for a carbon tax, the capital subsidy starts much higher at 3.6% and then tapers off to 1.9% just before the catastrophe. Fossil fuel use and GDP are therefore higher in the period before the calamity and the economy warms up more than with the linear hazard function.

Figure 2: Preparing for a climate disaster at some future unknown time

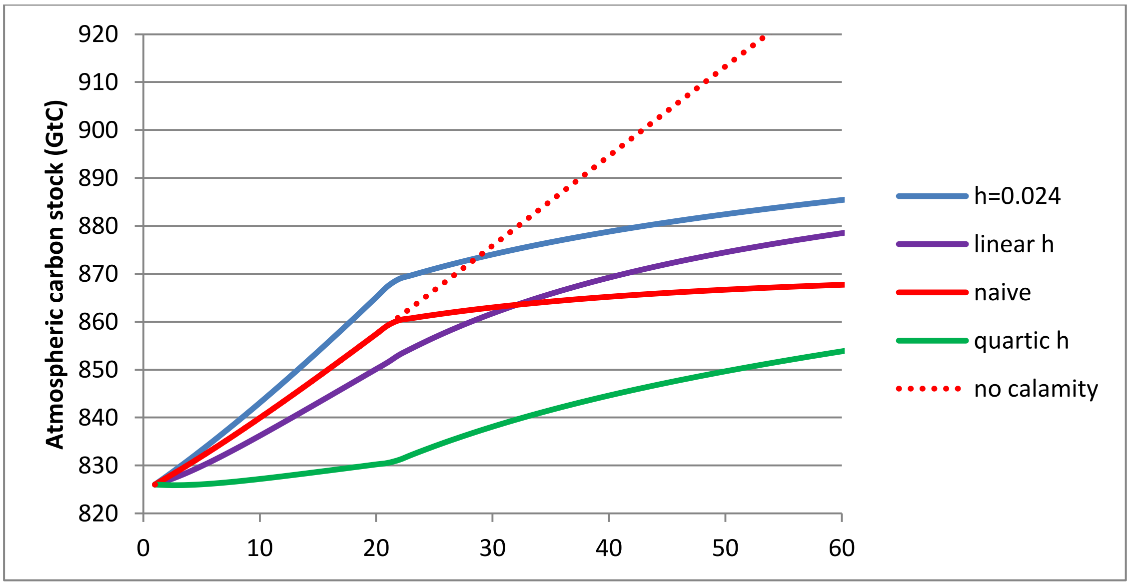




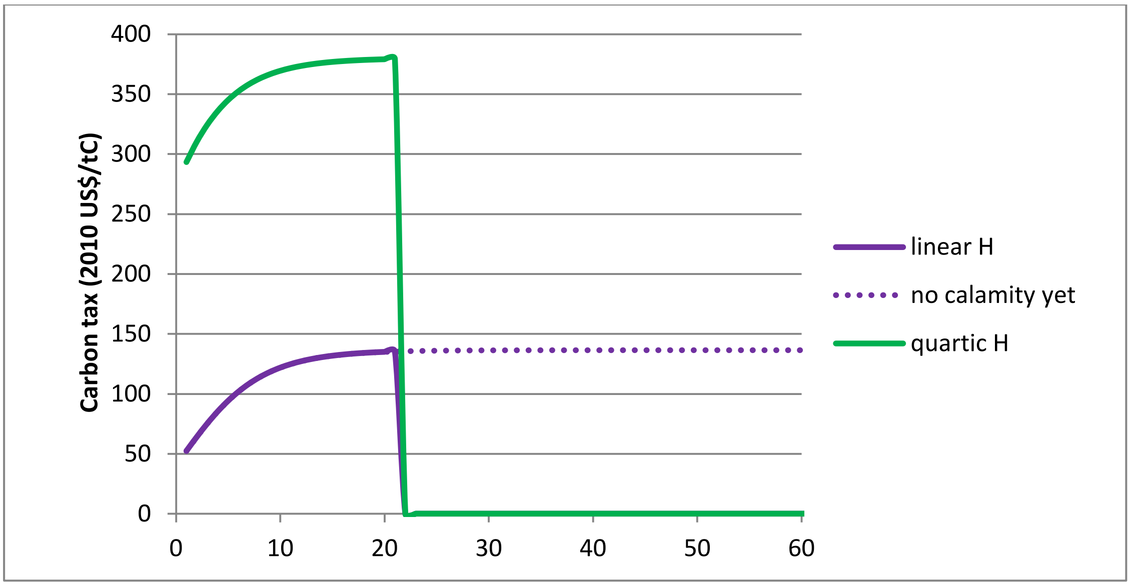
Global fossil fuel use (GtC)
11
10
9
8
7
6
5
4
0
10
20
30
40
50
60
h=0.024
linear h
naive
quartic h
no calamity



Precautionary return or SBC (%)
2.4
1.9
1.4
0.9
0.4
-0.1
0
10
20
30
40
50
60
h=0.024
linear h
quartic H



Atmospheric carbon stock (GtC)
920
910
900
890
880
870
860
850
840
830
820
0
10
20
30
40
50
60
h=0.024
linear h
naive
quartic h
no calamity



atmospheric stock of carbon and temperature rise. This is why the precautionary return on capital is higher than the one which prevails if the hazard rate stays put at the initial level of 2.4%: capital is 200 trillion US dollar higher than with constant hazard just before the catastrophe hits the economy. To make this possible, consumption has to fall dramatically to 5.5 trillion US dollar in 2010 although it rises steeply and reaches 52.6 trillion US dollar, overtaking consumption under the naive and a fortiori the constant hazard outcome.

To avoid the global economy burning more fossil fuel, emitting more carbon, warming up the earth and increasing the risk of a catastrophe, a carbon tax rising from 52 US $/tC in 2010 to 135 US $/tC just before the catastrophe in 2030 has to be levied. This carbon tax is also needed to curb the inevitable rise in fossil demand associated with the very substantial amount of precautionary capital accumulation that occurs in this scenario. The net effect is a relatively low fossil fuel demand of 6.8 GtC in 2010, then a steep rise, and flattening off to 8.8 GtC in 2030 just before the calamity. Fossil fuel demand is, thanks to the carbon tax, lower than under the constant hazard throughout the period before the climate catastrophe. As a result, the stock of atmospheric carbon and global mean temperature are lower than under the constant hazard. In fact, they are lower than under the naive outcome. Due to the very substantial amount of precautionary capital accumulation, global GDP is much higher than under the constant hazard. Global GDP is pushed up to 84.9 trillion US dollar, notwithstanding the lower levels of fossil fuel demand. The precautionary saving also cushions the blow to consumption to 8.0 trillion US dollar, which is less than the discrete fall in consumption at the time of the calamity under the constant hazard outcome and the naive outcome (9.2 and 13.9 trillion US dollar, respectively).

7.3. Pre-catastrophe outcomes with quartic hazard function

The solid green lines in fig. 3 show what happens if the hazard rate is described by a quartic function, starting with a hazard rate of only 0.1% and also passing through $H(2314)$ = 6.8%. This yields a global carbon tax which starts at 293 US $/tC and rises to 379 US $/tC just before the catastrophe hits. The carbon tax is initially about six times and eventually almost three times bigger than with a linear hazard. The quartic hazard gives rise to a small negative precautionary return, quickly rising to a tiny positive return of about 0.08%. This precautionary return does not vary much with time. The return is tiny, because it is no longer needed to prepare so much for disaster as the much higher carbon tax has curbed fossil fuel use and the rise in temperature and has thus cut the risk of climate disaster.[9] The price one pays for reduced fossil fuel use and reduced precautionary capital accumulation is a much lower GDP before the catastrophe strikes compared with the linear hazard function. Consumption is initially higher compared with the linear hazard function (16.8 instead of 5.5 trillion US dollar in 2030) as less precautionary capital accumulation is required with the quartic hazard than with the linear hazard (but more than under the naive outcome). As a result of the substitution from fossil fuel towards the renewable and of the reduced precautionary capital accumulation, the stock of atmospheric carbon and temperature are always below the level for the linear hazard and a fortiori the level for the constant hazard.

7.4. Role of intergenerational inequality aversion

Much attention has been given to the role of time preference for precautionary climate policy. This should not be deduced from market outcomes, but should be the result of ethical considerations (e.g., Stern, 2007) or prudence considerations (e.g., Gollier, 2012). We add to this debate a new rationale for precaution which results from the need to be better prepared when a large-scale climate disaster eventually hits the world. An important other parameter in society's attitude to intergenerational welfare comparisons is the coefficient of relative intergenerational inequality aversion (*CRIA*) which has been set to $1/\sigma = 2$ in the simulations. In fact, this key parameter also corresponds to the coefficient of relative risk aversion (*CRRA*). A higher value for σ thus corresponds to both a lower *CRIA* and a lower *CRRA*, and thus has two opposite effects on the *SCC* and the optimal carbon tax:

[9] Lemoine and Traeger (2013, appendix) indicate that tipping has tiny effects on capital accumulation too. Pre-tipping there is an incentive to accumulate capital to mitigate future welfare losses and an incentive to reduce capital to avoid carbon emissions and temperature rises that increase the hazard of tipping. We find that these offsetting effects are especially strong with the quartic hazard function.

1. Less precautionary saving resulting from a lower precautionary return (see equation (31)). Effectively, society has a lower *CRRA* and is therefore less risk averse and engages in less capital accumulation to be prepared for the eventual disaster. As a result of this, fossil fuel demand is less and there is less need for a carbon tax to curb carbon emissions. This depresses the optimal level of the global carbon tax.
2. More ambitious climate policy, since a lower *CRIA* implies that society is more prepared to make sacrifices to improve the climate for future generations (see $U'(C)$ in the numerator of the expression for the social cost of carbon). This suggests a higher carbon tax.

Of course, in general equilibrium the effects will be more intricate (see, for example, equation (32d′) in supplement B). To illustrate these two effects in general equilibrium, we consider the experiment where σ is increased from 0.5 to 0.8 and consequently the *CRIA* and the *CRRA* are reduced from 2 to 1.25 (see supplement C for the simulations of the transient paths). With the constant hazard the long-run *SBC* is lower (0.7% < 1.2%) and thus long-run precautionary capital accumulation is smaller (417 < 486 trillion US dollar), despite the lower *CRIA*. Since there is less precautionary capital accumulation, fossil fuel use is less too and consequently the long-run carbon stock is lower (1768 GtC instead of 1857GtC). With the linear hazard the higher value of σ gives a smaller carbon tax (123 US $/tC instead of 136 US $/tC) and a smaller precautionary return or *SBC* (0.95% < 1.6%) whilst the capital stock is lower (440 instead of 535 trillion US dollar); see the last column of table 1. Hence, in our experiment the first 'be prepared' effect dominates the second effect which is driven by intergenerational inequality aversion. This is why society sacrifices less consumption upfront (i.e., with a linear hazard initial consumption is higher, 11.6 instead of 5.5 trillion US dollar).[10]

With a quartic hazard function, a higher value of σ still leads to a precautionary return of 0.1% and to a lower carbon tax (342 US $/tC instead of 381 US $/tC) and a slightly lower capital accumulation in the pre-catastrophe steady state (343 instead of 346 trillion US dollar).

7.5. Alternative climate calamities

It is straightforward to allow for other types of climate calamities. For example, one could consider a disaster which destroys part $0 < \Omega < K$ of the capital stock rather than part of total factor productivity. In that case, the relevant after-calamity value function to use is

[10] With a constant hazard initial consumption is somewhat higher too (36.4 versus 35.0 trillion dollar).

$V = V^A(K-\Omega,0)$ rather than $V = V^A(K,\Delta)$ but the analysis of the optimal precautionary return (or if necessary the capital subsidy) and carbon tax is not changed in an essential way. If there is a bigger fall in consumption at the time of the disaster, the marginal value of capital is higher and thus a bigger *SBC* must be internalized.

Another type of climate catastrophe is a sudden release of greenhouse gases and carbon Ξ into the atmosphere for which the risk increases with temperature (e.g., Naevdal, 2006). Now the after-calamity value function does depend on the stock of atmospheric carbon. In general, the after-calamity value function is then $V = V^A(K-\Omega,\Delta,P-\Xi).$ Alternatively, there may be a sudden unleashing of positive feedback loops which alter the dynamics of the carbon cycle and the climate system. This may occur with the sudden melting of ice sheets. The warming up causes more ice to melt and sets in motion even more global warming.

8. Gradual global warming damages and the social cost of carbon

So far, we have assumed that a climate disaster destroys part of total factor productivity and ignored that global warming may directly and gradually reduce total factor productivity. To remedy this, we allow for marginal damages by assuming that total factor productivity is $A = \bar{A}(Temp),\ \bar{A}' < 0, \bar{A}'' \le 0,$ or $A(P) \equiv \bar{A}(3\ln(P/581)/\ln(2)),\ A' = 3\bar{A}'/[P\ln(2)] < 0.$ Since damages are a convex function of temperature and temperature a concave function of the carbon stock, $A(P)$ can be concave or convex. If A depends on P, $A'FV_K^B + HV_P^A$ has to be subtracted from the right hand-side of (26) and added to the right-hand side of (28), so that using $\tau = \psi\lambda^P/\lambda^K$ and $\lambda^K \equiv V_K^B = U'(C^B)$ and integrating the new (28) yields the following generalized expression for the before-catastrophe social cost of carbon (see supplement D):

$$\tau(t) = \underbrace{\frac{-\psi}{U'(C(t))}\int_t^\infty A'(P(s))F(K(s),E(s),(s))U'(C(s))e^{-\int_t^s[\rho+\gamma+H(P(s'))]ds}ds}_{\text{conventional Pigouvian social cost of ca rbon}}$$

(35a)
$$+\underbrace{\frac{-\psi}{U'(C(t))}\int_t^\infty H(P(s))V_P^A(K(s)-\Omega,\Delta,P(s)+\Xi)e^{-\int_t^s[\rho+\gamma+H(P(s'))]ds}ds}_{\text{'raising the stakes' effect}}$$

$$+\underbrace{\frac{\psi}{U'(C(t))}\int_t^\infty H'(P(s))\left\{V^B(K(s),P(s))-V^A(K(s)-\Omega,\Delta,P(s)+\Xi)\right\}e^{-\int_t^s[\rho+\gamma+H(P(s'))]ds}ds}_{\text{'risk averting' effect}},\ 0 \le t < T.$$

After the catastrophe, the optimal carbon tax reduces to the standard Pigouvian social cost of carbon:

(35b) $$\tau(t)=\underbrace{\frac{-\psi}{U'\left(C(t)\right)}\int_t^\infty A'\left(P(s)\right)F\left(K(s),E(s),(s)\right)U'\left(C(s)\right)e^{-(\rho+\gamma)(s-t)}ds}_{\text{Pigouvian social cost of carbon}},\quad t\geq T.$$

The after-tax value function now depends on both the capital stock, the stock of atmospheric carbon and the size of the various possible climate catastrophes discussed at the end of section 7 (i.e., a sudden destruction in the capital stock Ω, a sudden fall in total factor productivity Δ, or a sudden release of greenhouse gases into the atmosphere Ξ).

Expression (35b) for the after-calamity carbon tax simply equals the conventional Pigouvian tax, i.e., the present discounted value of all future marginal global warming damages. This is the optimal carbon tax which is calculated in most integrated assessment studies of the social cost of carbon. In the analysis so far, we focused on the *non-marginal* regime switch induced by an impending climate catastrophe and ignored the *marginal* global warming damages which are highlighted by the integrated assessment literature. The optimal Pigouvian carbon tax in the main body of our paper thus equals zero.

Expression (35a) shows that our expression for the before-disaster carbon tax reduces to the conventional Pigouvian carbon tax if there is no hazard of a climate catastrophe. However, in general there is a stock-dependent hazard rate and our catastrophe analysis adds the following considerations to the optimal climate policy:

1. The '*be prepared*' effect which is captured by relative risk aversion and is reflected in society engaging in precautionary capital accumulation to be better prepared for when the climate disaster eventually hits the globe (if necessary induced by a capital subsidy). In general, there will also be a shift from growth-promoting capital towards adaptation capital (see section 9).
2. The '*make hay while the sun shines*' or the '*Mr Fripp*' effect which increases consumption resulting from the belief that good times may come to an end. As a result of the higher discount rate ($\rho+\gamma+H(P)>\rho+\gamma$), society places less value on future utility of consumption and future global warming damages. This effect depresses the Pigouvian component in (35a) (not in (35b)) and also depresses the 'raise the stakes' and 'avert risk' effects in (35a), and it is stronger if the risk of catastrophe is higher.

3. The '*raise the stakes*' effect is a consequence of burning more fossil fuel increasing the stock of atmospheric carbon and increasing marginal global warming damages. Since the value after the calamity is reduced (see the second term of (35a)), this raises the stakes and boosts the social cost of carbon before the calamity.
4. The '*avert risk*' effect which arises from reacting to the unpleasant fact that burning fossil fuel leads to a higher stock of atmospheric carbon and temperature which increases the risk of climate catastrophe and the chance of a *non-marginal* catastrophic loss in value resulting from the eventual regime switch (see the third term of (35a)). This introduces a discontinuity into the marginal damage (cf., the 'hazard' effect in Lemoine and Traeger (2012)).

Without gradual global warming damages, equations (35a) and (35b) become:

(35a′)

$$\tau(t) = \underbrace{\frac{\psi}{U'(C(t))}\int_t^\infty H'(P(s))\left\{V^B(K(s),P(s)) - V^A(K(s)-\Omega,\Delta)\right\} e^{-\int_t^s [\rho+\gamma+H(P(s'))]ds} ds}_{\text{'risk averting' effect}}, \; 0 \le t < T,$$

(35b′) $$\tau(t) = \underbrace{0}_{\text{Pigouvian social cost of carbon}}, \quad t \ge T.$$

There are two main effects on capital accumulation: the 'be prepared' effect which boosts capital (and offsets the 'make hay' effect) and the 'avert risk' effect which cuts demand for capital (as capital and fossil fuel are cooperative production factors) via the fall in fossil fuel demand induced by the carbon tax. Without gradual damages the optimal carbon tax is pushed above the (zero) no-calamity carbon tax.

Allowing for gradual global warming damages in the production process implies a need for a strictly positive carbon tax (see the first term of (35a) or (35b)) and introduces in addition the 'raise the stakes' effect (the second term). Both the 'raise the stakes' and the 'avert risk' effect lower demand for fossil fuel and (with cooperative production factors) the demand for capital too. Capital is thus depressed compared with the naive outcome if the 'be prepared' effect is dominated by the combined effects of 'make hay', 'raise the stakes' and 'avert risk'. The risk of a climate disaster pushes up the optimal carbon tax in the absence of gradual marginal damages, but with gradual damages the conventional Pigouvian component of the carbon tax is depressed due to the higher discount rate associated with the 'make hay' effect. The 'raise the stakes' and the 'avert risk' effects push the carbon tax above its conventional

Pigouvian component. The risk of climate disaster thus generally leads to a bigger gap between the disaster-distorted optimal carbon tax and the conventional Pigouvian carbon tax.

9. Investing in adaptation capital

Our model highlights that the risk of climate catastrophe induces precautionary capital accumulation so as to be better prepared when the disaster eventually hits. Unfortunately, this leads to burning more fossil fuel and a higher risk of catastrophe. To be better prepared for climate disaster, one could invest in adaptation capital (e.g., dykes and other water defences) L as well as productive capital K. Suppose both types of capital are *ex ante* perfect substitutes, ignore depreciation, $\delta = 0$, and focus at a disaster which destroys a big chunk of total factor productivity (as in sections 2-7).

Adaptation capital L reduces the disaster shock $\Delta(L)$, so $\Delta'(L) < 0$. From the calamity onwards L is constant as there is no need to cover for depreciation or to destroy it as *ex post* it cannot be turned into productive capital (due to the assumption of 'putty clay' technology). With these extensions and abstracting from gradual global warming damages the post-calamity problem yields the consumption manifold $C^A = C^A(K,\Delta(L))$ and the value function:

(36)

$$V^A\left(K,\Delta(L)\right) = \left[U\left(C^A(K,\Delta(L))\right) + U'\left(C^A\left(K,\Delta(L)\right)\right)\left\{Y\left(K-L, A-\Delta(L)\right) - C^A\left(K,\Delta(L)\right)\right\}\right] / \rho.$$

With an exogenous hazard rate, the HJB equation for the before-calamity problem becomes:

$$(37)\quad \begin{aligned}(\rho+h)V^B(K,P) = \underset{C,L}{\text{Max}} \Big\{ & U(C) + V_K^B(K,P)\left[Y(K-L,c,d+\tau,A) - \tau Y_{d+\tau}(K-L,c,d+\tau,A) - C\right] \\ & + V_P^B(K,P)\left[-\psi Y_{d+\tau}(K-L,c,d+\tau,A) - \gamma P\right] + hV^A\left(K,\Delta(L)\right)\Big\}.\end{aligned}$$

Apart from $U'(C) = V_K^B(K,P)$ and the expression for the optimal social cost of carbon τ, the marginal product of capital in production must equal the expected marginal product of capital in adaptation:

$$(38)\qquad Y_{K-L}(K-L,c,d+\tau,A) = \frac{hV_\Delta^A\left(K,\Delta(L)\right)\Delta'(L)}{U'(C)},$$

where $V_\Delta^A\left(K,\Delta(L)\right) < 0.$ The bigger the hazard of the climate calamity, the bigger the expected marginal return on adaptation capital. The left-hand side of (38) gives, for given aggregate K, the marginal product of productive capital as increasing function of adaptive capital L. The right-hand side of (38) gives the expected marginal return on adaptation capital as decreasing function of L. Hence, the optimal level of adaptation capital increases with the hazard rate and aggregate consumption (via the lower marginal utility of consumption boosting the return on adaptation capital):

$$(39) \qquad L = L(\overset{+}{C}, \overset{+}{h}, \overset{(+)}{K}, \overset{+}{c}, d \overset{+}{+} \tau).$$

Higher costs of energy curb (via the factor price frontier) the return on capital in production and thus also boost the allocation of capital to adaptation. Adaptation capital typically increases with the stock of aggregate capital if the effect of a lower return on productive capital dominates the effect of a lower expected return on adaptation capital. This can be seen as insurance, since more adaptation capital is warranted if there is more productive capital which is vulnerable to a climate calamity. With an endogenous hazard rate, things become more complicated as the HJB equation also needs to consider the effects of the change in the stock of atmospheric carbon on adaptation capital.

The hazard of a climate catastrophe typically boosts the stock of adaptation capital L (the 'be prepared' effect) but depresses productive capital $K - L$ (the 'avert risk' effect via less burning of fossil fuel). If the hazard rate is exogenous, the latter effect disappears and the aggregate capital stock K always increases as the result of the risk of a climate calamity. However, if the hazard rate increases with temperature, the 'avert risk' effect kicks in which attenuates the increase in the capital stock. If the hazard is very sensitive to temperature, the 'avert risk' effect is strong and the aggregate capital stock can outweigh the 'be prepared' effect in which case the aggregate capital stock falls. More realistic simulation scenarios than the ones discussed in section 7 must thus allow different types of capital stocks and the possibility that the endogenous risk of a climate calamity leads, on the one hand, to a fall in capital used in the production process, and, on the other hand, an increase in adaptation capital.

10. Concluding remarks

A non-marginal analysis of first-best optimal climate policy within the context of a Ramsey growth model with a possible regime switch has been put forward. The analysis shows how to cope with and reduce the risk of various types of climate catastrophe varying from a sudden destruction of productive capacity (e.g., from a sudden fall in total factor productivity) to a release of greenhouse gases into the atmosphere. Apart from a carbon tax which is set equal to the social cost of carbon, there must be precautionary capital accumulation which arises from a precautionary return on capital (if necessary realized via a capital subsidy).

On top of the conventional Pigouvian component of the optimal carbon tax, corresponding to the present value of all future gradual marginal global warming damages, two extra terms have to be added to the social cost of carbon. The first one is the '*avert risk*' term which increases in the marginal change in the expected gap between welfare before and after the catastrophe arising from burning an additional unit of fossil fuel. This captures that an increase in the stock of atmospheric carbon or global mean temperature pushes up the risk of a climate catastrophe and the loss in welfare that this may cause. This term disappears if the hazard of a climate calamity is exogenous. The second one is the '*raise the stakes*' term which captures that burning an additional unit of fossil fuel raises global mean temperature and thus leads to lower after-calamity welfare. This means that the fall in welfare following a climate catastrophe is bigger.

With non-marginal damages of global warming, the global economy needs in addition precautionary capital accumulation, which results from a precautionary return on capital (if necessary induced by a capital subsidy). This precautionary return reflects the '*be prepared*' effect and increases in the hazard of a catastrophe and the gap between the marginal utility of consumption after and before the catastrophe. In contrast to the carbon tax, this precautionary return is required even if the hazard rate is exogenous. If the hazard depends on temperature, there is a carbon tax which curbs fossil fuel demand and capital demand, thereby offsetting precautionary capital accumulation. We have also shown that the risk of climate disaster leads to an '*adaptation relocation*' effect which implies that capital that is used as engine of growth is shifted from the productive process to precautionary adaptation capital with the aim to mitigate the adverse effects of potential climate disasters (water defences etc.). Hence, as a result of the hazard of a climate catastrophe, adaptation capital increases and capital used in the production process falls, especially if the hazard increases with temperature.

To illustrate our results, we have calibrated a simple Ramsey growth model of the global economy with both fossil fuel and renewable use. With a linear hazard function calibrated to an expected duration of 15 years for a 32% drop in global GDP to occur when global mean temperature stays put at 6 degrees Celsius of global warming, our policy simulations suggest that the first-best non-marginal optimal global policy requires a precautionary return on capital of 1.6% (or if necessary a capital subsidy) to best prepare the global economy for a catastrophic climate event and a global carbon tax of 136 US $/tC to curb the risk of a climate catastrophe. With a much more convex hazard function (a quartic) calibrated to the same catastrophe the carbon tax can be as high as 381 US $/tC whilst the precautionary return drops to 0.1%. With this more convex hazard function the 'be prepared' effect is eventually wiped out as long-run capital accumulation falls compared with the naive outcome.

A key role is played by the elasticity of intertemporal substitution. A higher value implies less risk aversion and thus less precautionary capital accumulation. As a result, fossil fuel demand and carbon emissions will be less and there is less need for a high carbon tax. A higher value for this elasticity also implies less intergenerational inequality aversion, so society is more prepared to sacrifice consumption. This pushes up the carbon tax. Interestingly, our simulations suggest that the first effect dominates so that a higher elasticity of intertemporal substitution leads to a lower carbon tax.

Although the Stern Review claims that uncertainty about impacts strengthens the argument for mitigation and that it deals with the economics of the management of very large risks (Stern, 2007), it does not analyze the relevant probabilities and sizes of potential climate disasters, the degree of risk aversion and their effect on the social cost of carbon or on investment in adaptation capital. Indeed, Weitzman (2007, pp. 704-705) argues that "… spending money now to slow global warming should not be conceptualized primarily as being about optimal consumption smoothing so much as an issue about how much insurance to buy to offset the small chance of a ruinous catastrophe …".

Calibrations based on existing models of rare macroeconomic disasters with substantial relative risk aversion (a coefficient of about 3) and fat-tailed uncertainty in line with the observed equity premium suggest that optimal environmental investment can be a significant share of GDP even when not relying on unrealistically low discount rates (Barro, 2013). Although our analysis is based on small Poisson-type risks of a catastrophic regime switch rather than on fat-tailed uncertainty, we hope it offers complementary general equilibrium estimates of the social cost of carbon to the ones based on fat-tailed risk. We find a rationale for substantial carbon taxes even for relatively low risks of catastrophe at high levels of global warming.

We hope that our analysis helps to gain a better understanding of the non-marginal drivers of optimal climate policy. The challenge for future empirical research is to pin down the effect of adaptation investment and curbing fossil fuel demand on the probability of climate disaster as well as the baseline probability of climate disaster.

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Appendix A: Functional forms and calibration

We adopt a utility function with a constant coefficient of intergenerational inequality aversion of 2 (i.e., an elasticity of intertemporal substitution equal to $\sigma = 0.5$) and use a pure rate of time preference of 1.4 percent per annum ($\rho = 0.014$).

We use a Cobb-Douglas production function $\tilde{A}F(K,E,R) = \tilde{A}K^{\alpha}(E^{\omega}R^{1-\omega})^{\beta},\ 0<\alpha,\beta,\omega<1,$ so optimal renewable and fossil fuel inputs are $R = \beta(1-\omega)\tilde{A}F/c$ and $E = \beta\omega\tilde{A}F/(d+\tau),$ and net output is $Y(K,d+\tau,c,\tilde{A}) = (1-\beta)\left[\tilde{A}K^{\alpha}\beta^{\beta}\left(\frac{\omega}{d+\tau}\right)^{\beta\omega}\left(\frac{1-\omega}{c}\right)^{\beta(1-\omega)}\right]^{\frac{1}{1-\beta}} - \delta K,$ where $\tau = 0$ after the climate calamity. We use a share of capital in value added of 30 percent ($\alpha = 0.3$) and a depreciation rate of 5 percent per year ($\delta = 0.05$). Labour supply is inelastic and normalized to unity.

Table A1 presents these calibration parameters and also gives the stylized facts for a rough, illustrative calibration of our model to market outcomes based on figures for the world economy for the year 2010. Data sources are the BP Statistical Review and the World Bank Development Indicators. We set the initial capital stock to 200 trillion US \$, which is well below the steady-state level consistent with the 2010 figure for world GDP of 63 trillion US \$ (i.e., $\alpha Y_0/(\rho+\delta)$ = 291 trillion US \$) to reflect that the global economy is still developing. Bio-fuels production in 2010 was 1.45% of total oil production. Counting also nuclear and other renewable sources of energy might triple this figure. Hydro-electricity production is tiny. So as a ballpark, we take 2% for the share of renewable energy output (9.4 million GBTU) in total fossil fuel energy output (468.3 million GBTU) in 2010.

Prices of oil, natural gas and coal are roughly 14 US \$, 6.5 US \$ and 4 US \$, respectively, per million BTU, so we set the average initial cost of fossil fuel to 9 US \$ per million BTU. We set the cost of the renewable at double that figure. Hence, the budget share of fossil fuel in total energy is 96.1 percent, since $\omega = \frac{468.3\times 9}{468.3\times 9 + 9.4\times 18} = 0.961.$ The share of fossil fuel in value added is 6.7 percent, since $\beta\omega = \frac{9\times 468.3}{63} = 0.0669.$ It thus follows that the share of energy in value added is 7 percent, since $\beta = \frac{0.0669}{0.961} = 0.0696.$ It follows that the share of labour in value added is 63 percent. We calibrate total factor productivity (TFP) to match 2010 world GDP: $A = 63 \times 200^{-\alpha} \times 8.3^{-\beta\omega} \times 9.4^{-\beta(1-\omega)} = 11.1$.

To convert we use a conversion factor for fossil fuel of 1 GtC = 56 million GBTU. We measure fossil fuel in GtC, so that the carbon-emission ratio equals one. The corresponding market price can then be expressed as 504 US $ per ton of carbon. The share of carbon that does not return quickly to the surface of the earth is half ($\psi = 0.5$). To calculate the global mean temperature *Temp*, we follow IPPC (2007) and assume an equilibrium climate sensitivity of 3 so that $Temp = 3\ln(P/581)/\ln(2)$.

Table A1: Calibration of the Green Ramsey model with a tipping point[11]

Variable/parameter	
Pure rate of time preference, ρ	0.014
Elasticity of intertemporal substitution, σ	0.5
Share of capital in value added, α	0.3
Share of fossil fuel (oil, gas, coal) in value added, $\beta\,\omega$	0.0669
Share of fossil fuel in total energy, ω	0.961
Share of energy in value added, β	0.0696
Share of labour in value added, $1-\alpha-\beta$	0.63
Depreciation rate of manmade capital, δ	0.05
Initial level of GDP, Y_0	63 trillion US $
Initial capital stock, K_0	200 trillion US $
Initial fossil fuel use, E_0	468.3 million G BTU = 8.3 GtC
Initial renewable use, R_0	9.4 million G BTU
Total factor productivity, A	11.1
Cost of fossil fuel, d	9 US $/million BTU = 504 US $/tC
Cost of renewable, c	18 US $/million BTU
Initial stock of carbon, P_0	826 GtC = 388 ppm by vol. CO2
Pre-industrial carbon stock	581 GtC = 273 ppm by vol. CO2
Fraction of carbon that stays up in atmosphere, ψ	0.5
Eventual climate shock, Δ	$0.3\,A = 3.33$
Equilibrium climate sensitivity	3
Exogenous hazard rate, h	0.0242
Hazard sensitivity: linear hazard, h_1	$2.926 \times 10^{-5} \times P$
Hazard sensitivity: quartic hazard, h_2	$2.33 \times 10^{-15} \times P^4$

Appendix B: Post-catastrophe steady state and stable manifold

The steady-state after-calamity capital stock and consumption follow from (12a) and (12b):

[11] Conversion factors: 1 ppm by volume CO2 = 2.13 GtC and 1 kg carbon = 3.664 kg CO2.

$$K^{A^*} = \left[\frac{Q(\Delta)}{1-\beta} \left(\frac{\alpha}{\delta+\rho} \right) \right]^{\frac{1-\beta}{1-\alpha-\beta}} = 202 \text{ and } C^A(K^*, \Delta) = Q(\Delta) K^{*^{\frac{\alpha}{1-\beta}}} - \delta K^* = 30.0$$

with $Q(\Delta) \equiv (1-\beta) \left[(A-\Delta)\beta^{\beta} \left(\frac{\omega}{d} \right)^{\beta\omega} \left(\frac{1-\omega}{c} \right)^{\beta(1-\omega)} \right]^{\frac{1}{1-\beta}} = 7.24.$

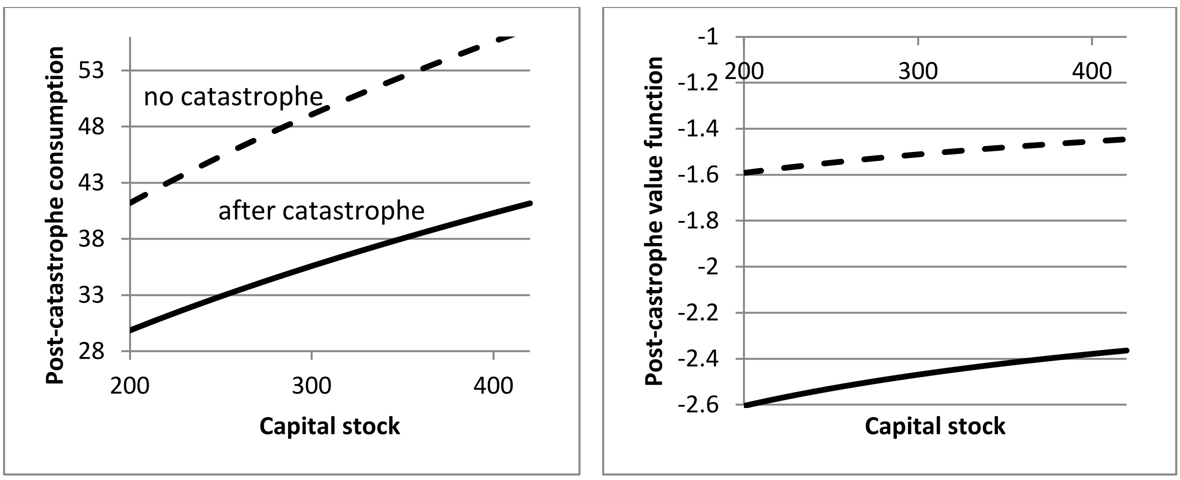
The stable manifold is approximated by (13) with

$$\phi = \left[\frac{1}{2}\rho + \frac{1}{2}\sqrt{\rho^2 + 4\sigma(\delta+\rho)\left(\frac{1-\alpha-\beta}{1-\beta} \right) \frac{C^{A^*}}{K^*}} \right] \frac{K^{A^*}}{C^{A^*}} = 0.432.$$

To find a more accurate estimate of the stable manifold $C^A(K, \Delta)$ we could integrate backwards in time from a point close to the steady state, in the right direction given by l'Hôpital's Rule in the proof of the lemma in section 2. Integrating backwards gives a starting point and is more stable than integrating forwards in the vector field of this system. We can use the Euler-Lagrange equation (9) for this, but the numerical algorithm is more stable if we use instead the system of two differential equations in time consisting of the dynamics of capital accumulation (2) and the Keynes-Ramsey rule (10). In the range of interest, the result we get is numerically indistinguishable from the approximate manifold (13), hence we use (13) as this gives closed-form expressions for the after-calamity consumption manifold and value function. We can obtain similar expressions for the no-calamity consumption manifold and value function. These expressions are plotted in fig. 3.

Figure 3: Post-catastrophe outcomes

(a) **Stable manifold $C^A(K, \Delta)$** **(b) Value function $V^A(K, \Delta)$**



SUPPLEMENTARY MATERIAL

Supplement A: Derivation of the HJB equations (15) and (21)

Since the probability of a regime shift in an infinitesimally small time period Δt is $h(P)\Delta t$, the Principle of Optimality at time t, from the perspective of time zero, can be written as:

$$e^{-\rho t}V^B\left(K,P\right)=\underset{C,E,R}{\text{Max}}\left\{\int_t^{t+\Delta t}e^{-\rho s}U(C(s))ds+\left[1-h(P)\Delta t\right]e^{-\rho(t+\Delta t)}V^B\left(K(t+\Delta t),P(t+\Delta t)\right)\right.$$
$$\left.+h(P)\Delta te^{-\rho(t+\Delta t)}V^A\left(K(t+\Delta t),\Delta\right)\right\}.$$

Multiplying both sides by $e^{\rho t}$, rearranging and dividing by Δt, we obtain:

$$\underset{C,E,R}{\text{Max}}\left\{\frac{\int_t^{t+\Delta t}e^{-\rho(s-t)}U\left(C(s)\right)ds}{\Delta t}-h(P)e^{-\rho\Delta t}\left[V^B\left(K(t+\Delta t),P(t+\Delta t)\right)-V^A\left(K(t+\Delta t),\Delta\right)\right]\right.$$
$$\left.+\left[\frac{\left(e^{-\rho\Delta t}-1\right)V^B\left(K(t+\Delta t),P(t+\Delta t)\right)}{\Delta t}+\frac{V^B\left(K(t+\Delta t),P(t+\Delta t)\right)-V^B\left(K,P\right)}{\Delta t}\right]\right\}=0.$$

Evaluating the integral for infinitesimally small Δt and taking the limit as $\Delta t\to 0$ whilst using (2) and (3) and l'Hôpital's Rule for $\lim_{\Delta t\to 0}\frac{\exp(-\rho\Delta t)-1}{\Delta t}=-\rho$, we get the stationary HJB equation:

(A1)
$$\rho V^B\left(K,P\right)=\underset{C,E,R}{\text{Max}}\left\{U(C)+h(P)\left[V^A(K,\Delta)-V^B(K,P)\right]+\right.$$
$$\left.V_K^B(K,P)(AF(K,E,R)-dE-cR-C-\delta K)+V_P^B(K,P)(\psi E-\gamma P)\right\}.$$

Supplement B: Pre-calamity steady state of calibrated model

Pre-calamity net output is $Y^B=Q^B(\tau)K^{\frac{\alpha}{1-\beta}}-\delta K$,

$$Q^B(\tau)\equiv(1-\beta)\left[A\beta^\beta\left(\frac{\omega}{d+\tau}\right)^{\beta\omega}\left(\frac{1-\omega}{c}\right)^{\beta(1-\omega)}\right]^{\frac{1}{1-\beta}}.$$

We use these expressions to calculate the pre-calamity steady state from (32a)-(32d):

(32a′)
$$K^{B*}=\left[A\beta^\beta\left(\frac{\alpha}{\delta+\rho-\theta^{B*}}\right)^{1-\beta}\left(\frac{\omega}{d+\tau^{B*}}\right)^{\beta\omega}\left(\frac{1-\omega}{c}\right)^{\beta(1-\omega)}\right]^{\frac{1}{1-\alpha-\beta}},$$
$$\theta^{B*}=H(P^{B*})\left[\left(\frac{C^{B^*}}{C^A(K^{B*},\Delta)}\right)^{1/\sigma}-1\right],$$

(32b′) $$P^{B*}=\psi\left(\frac{\beta\omega}{d+\tau^{B*}}\right)\left(\frac{\rho+\delta-\theta^{B*}}{\alpha\gamma}\right)K^{B*},$$

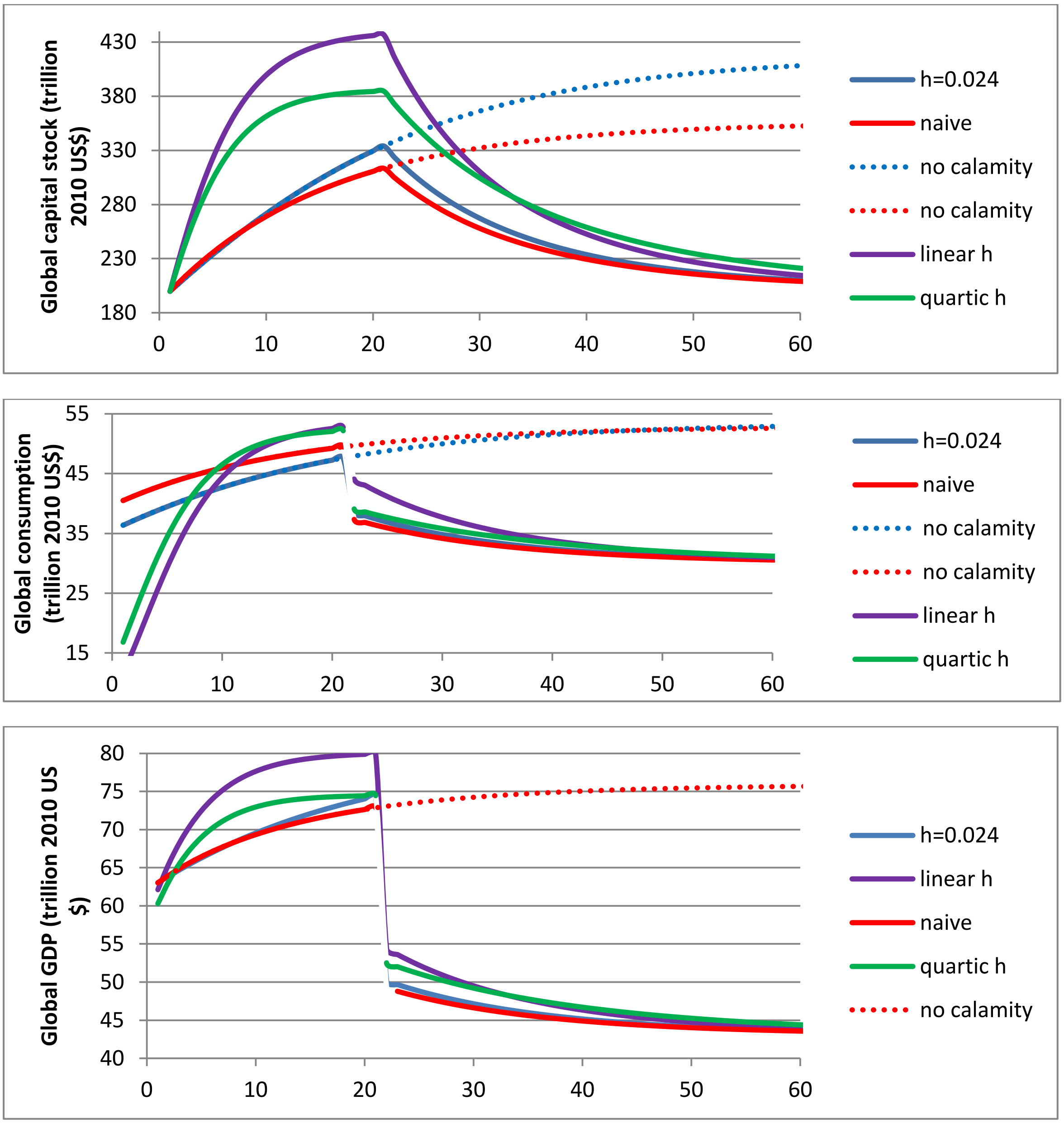
$$(32c') \quad C^{B*} = \left[\left(1 - \beta + \frac{\beta\omega\tau^{B*}}{d + \tau^{B*}} \right) \left(\frac{\rho + \delta - \theta^{B*}}{\alpha} \right) - \delta \right] K^{B*},$$

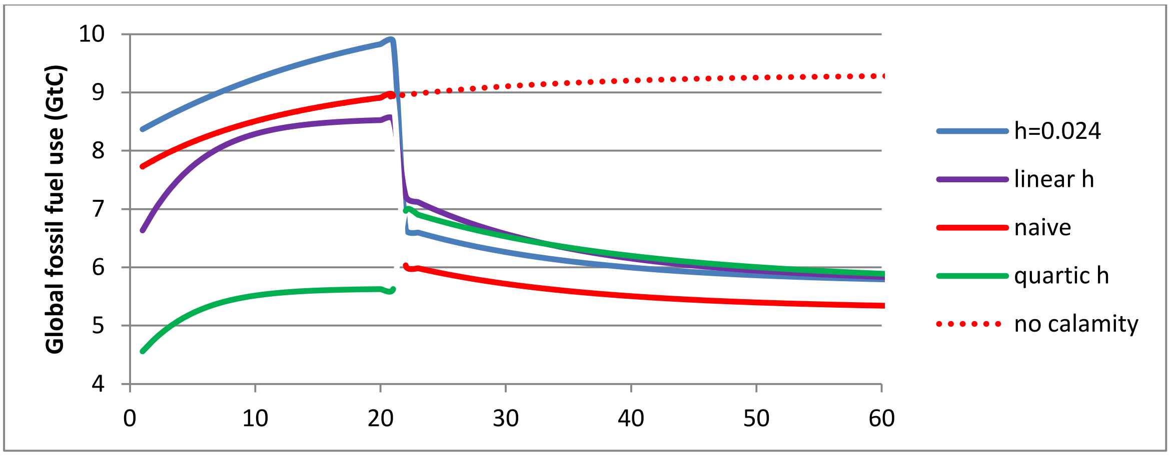
$$(32d') \quad \tau^{B*} = \frac{\psi H'(P^{B*}) C^{B*^{1/\sigma}} \left[\frac{C^{B*^{1-1/\sigma}}}{1-1/\sigma} - \rho V^{A}(K^{B*}, \Delta) \right]}{\left[\rho + H(P^{B*}) \right] \left[\rho + \gamma + H(P^{B*}) \right]}.$$

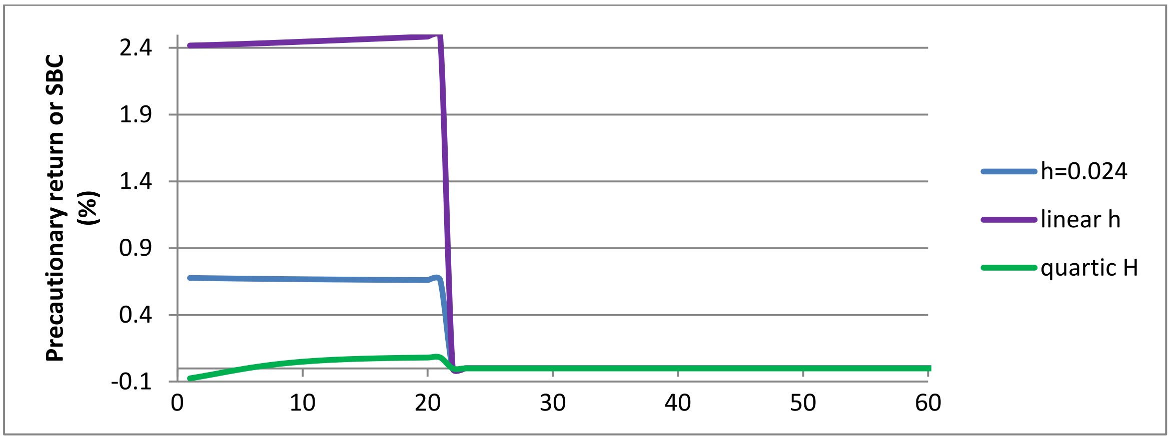
Supplement C: Before-catastrophe outcomes with $\sigma = 0.8$

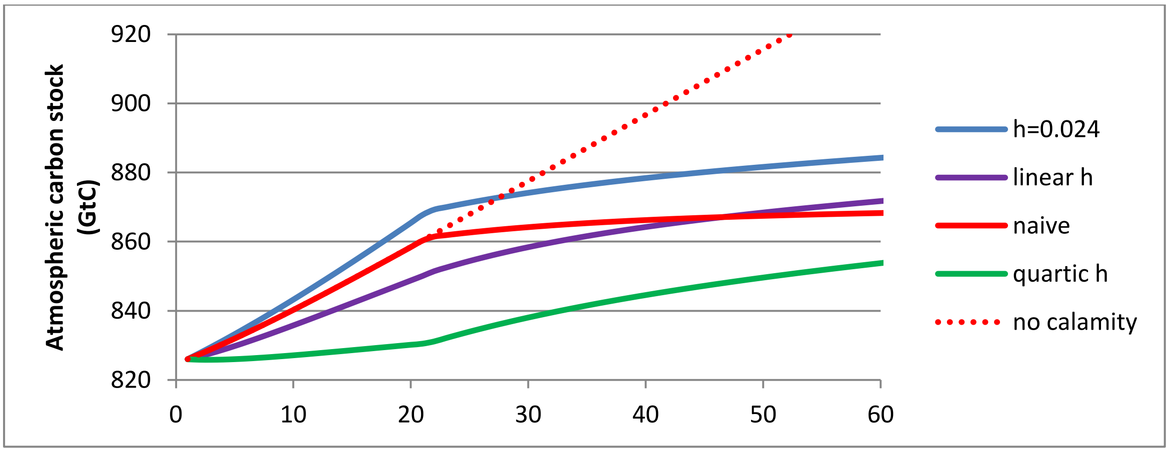
The optimal responses to the possible catastrophe in 2030 and the before- and after-catastrophe outcomes with σ increased from 0.5 to 0.8 are presented in fig. 4.

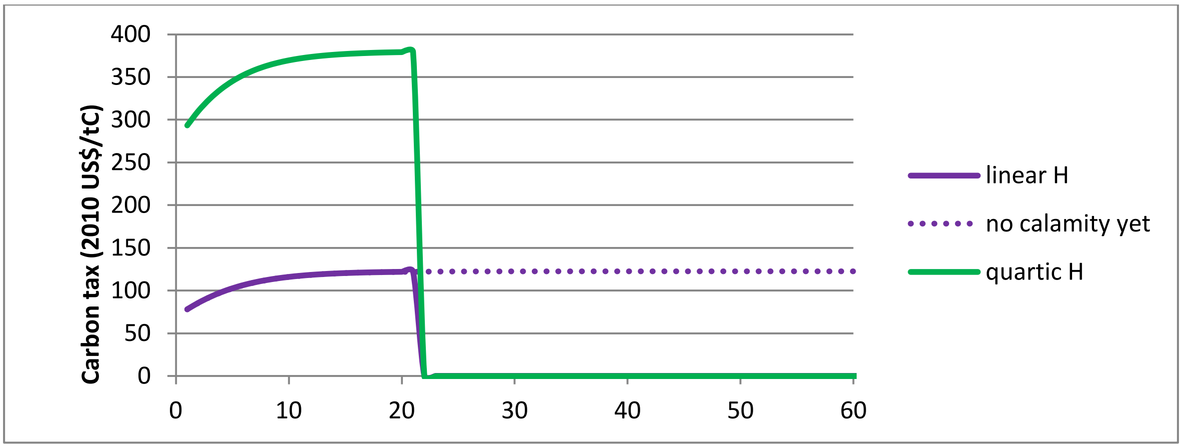
Figure 4: Before-catastrophe outcomes with $\sigma = 0.8$




Global fossil fuel use (GtC)
10
9
8
7
6
5
4
0
10
20
30
40
50
60
h=0.024
linear h
naive
quartic h
no calamity


Precautionary return or SBC (%)
2.4
1.9
1.4
0.9
0.4
-0.1
0
10
20
30
40
50
60
h=0.024
linear h
quartic H


Atmospheric carbon stock (GtC)
920
900
880
860
840
820
0
10
20
30
40
50
60
h=0.024
linear h
naive
quartic h
no calamity


Carbon tax (2010 US$/tC)
400
350
300
250
200
150
100
50
0
0
10
20
30
40
50
60
linear H
no calamity yet
quartic H

Supplement D: Marginal and non-marginal components of social cost of carbon

After the catastrophe

The value function now depends on *K* and *P*. The HJB equation for the post-catastrophe phase is:

(A2) $$\rho V^A(K,P) = \underset{C,E,R}{\text{Max}} \Big\{ U(C) + V_K^A(K,P)\Big[\big(A(P)-\Delta\big)F(K,E,R) - cR - dE - \delta K - C\Big] + V_P^A(K,P)\big[\psi E - \gamma P\big]\Big\}.$$

Defining $\tau \equiv -\psi V_P^A / V_K^A$, the optimality conditions for *E* and *R* are $AF_E = d + \tau$ and $AF_R = c.$ We also have $U'(C) = V_K^A$. Differentiating the HJB equation with respect to time, we obtain in the usual way the Euler-Lagrange equation and we get:

(A3) $$\dot{C} = \sigma C\Big[Y_K\big(A(P)-\Delta, K, \tau\big) - \rho\Big], \quad t \geq T,$$

(A4) $$\dot{\tau} = \Big[Y_K\big(A(P)-\Delta, K, \tau\big) + \gamma\Big]\tau + \psi A'(P) Y_A\big(A(P)-\Delta, K, \tau\big), \quad t \geq T,$$

where net output is defined $Y\big(A(P)-\Delta, K, \tau\big) \equiv \underset{E,R}{\text{Max}}\Big[\big(A(P)-\Delta\big)F(K,E,R) - cR - (d+\tau)E\Big] - \delta K$ with $Y_{A-\Delta} = F > 0, Y_K = (A-\Delta)F_K - \delta > 0$ and $Y_\tau = -E < 0$ (i.e., suppressing for notational convenience the effects of *c* and *d*). We also have:

(A5) $$\dot{K} = Y\big(A(P)-\Delta, K, \tau\big) - \tau Y_\tau\big(A(P)-\Delta, K, \tau\big) - C, \quad K(T+) = K(T-) - \Omega, \quad t \geq T,$$

(A6) $$\dot{P} = -\psi Y_\tau\big(A(P)-\Delta, K, \tau\big) - \gamma P, \quad P(T+) = P(T-) + \Xi, \quad t \geq T.$$

The saddlepoint system (A3)-(A6) with predetermined variables *K* and *P* and non-predetermined variables *C* and τ can be solved to yield the stable manifolds and (using (A2)) the value function:

(A7) $$C = C^A(K-\Omega, \Delta, P+\Xi), \quad \tau = \tau^A(K-\Omega, \Delta, P+\Xi), \quad V = V^A(K-\Omega, \Delta, P+\Xi).$$

Note that we have added the climate disaster shocks arising from a sudden destruction of the capital stock, a sudden drop in total factor productivity and a sudden release of carbon as arguments.

Backward integration of (A4) yields (35b), which gives the social cost of carbon as the present value of *marginal* global warming damages only. It corresponds to the Pigouvian carbon tax.

Before the catastrophe

The HJB equation before the catastrophes have occurred is given by:

(A8) $$\rho V^B(K,P) = \underset{C,E,R}{\text{Max}} \Big\{ U(C) + V_K^B(K,P)\big[A(P)F(K,E,R) - cR - dE - \delta K - C\big] + V_P^B(K,P)\big[\psi E - \gamma P\big] - H(P)\Big[V^B(K,P) - V^A(K-\Omega, \Delta, P+\Xi)\Big]\Big\}.$$

This yields $AF_E = d + \tau$, $AF_R = c$ and $U'(C) = V_K^B$. Using the same procedure with the Euler-Lagrange equation as before, we obtain:

(A9) $\dot{C} = \sigma C\left[Y_K\left(A,K,\tau\right) + \theta - \rho\right], \quad \theta = H(P)\left[\frac{V_K^A(K-\Omega,\Delta,P+\Xi)}{U'(C)} - 1\right], \quad 0 \le t < T,$

(A10)
$$\dot{\tau} = \left[Y_K\left(A(P),K,\tau\right) + \gamma + \theta + H(P)\right]\tau + \psi A'(P) Y_A\left(A(P),K,\tau\right) +$$
$$\psi H(P)\left(\frac{V_P^A(K-\Omega,\Delta,P+\Xi)}{U'(C)}\right) - \psi H'(P)\left(\frac{V^B(K,P) - V^A(K-\Omega,\Delta,P+\Xi)}{U'(C)}\right), \quad 0 \le t < T.$$

These equations can be solved as a four-dimensional saddlepoint system together with:

(A11) $\dot{K} = Y\left(A(P),K,\tau\right) - \tau Y_\tau\left(A(P)-\Delta,K,\tau\right) - C, \quad K(0) = K_0, \quad 0 \le t < T,$

(A12) $\dot{P} = -\psi Y_\tau\left(A(P),K,\tau\right) - \gamma P, \quad P(0) = P_0, \quad 0 \le t < T.$

Backward integration of (A10) yields (35a).