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Environmental Maintenance in a Dynamic Model with Heterogenous Agents

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Abstract

We assume a population of infinitely-lived households of the economy split into two groups : one with a high discount factor (the patient) and one with a low one (the impatient). The environmental quality is deteriorated by firm's polluting emissions. The governmental policy consists in proposing households to vote for a tax aimed at environmental maintenance. We study the voting equilibrium at steady states. The resulting equilibrium maintenance is the one of the median voter. We show that (i) an increase in total factor productivity may produce effects described by the Environmental Kuznets Curve, (ii) an increase in the patience of impatient households may foster environmental quality if the median voter is impatient and maintenance positive, (iii) a decrease in inequality among the patient households leads to an increase in environmental quality if the median voter is patient and maintenance is positive. We also show that, if the median income is lower than the mean, our model predict lower level of environmental quality than the representative agent model, and that increasing public debt decreases the level of environmental quality.

Keywords: intertemporal choice and growth, discounting, government environmental policy, externalities, environmental taxes; voting equilibrium

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1 Introduction

With the growing importance of global environmental issues, such as global warming, and the emphasis put on the general question of sustainable growth and development, environmental policies and their financing have become a major subject of concern in many developing or developed countries. As a response, economic theory, and especially in macro-economics, elaborated dynamic models based on the representative agent assumption to disentangle the nexus between economic growth and pollution, or more generally environmental quality (see among many others, Gradus and Smulders (1993), Stokey (1998), or Xepapadeas (2005)). Though, it is striking to notice that the public debate about environmental policies and their financing very often focus on the distributive aspects of the policies, and more precisely on the distribution of their burden among heterogenous agents. To capture that dimension, economists must get rid of the representative agent and must start considering heterogeneous agents in their macrodynamic models. There exist several ways of introducing heterogeneity, e.g. in wealth (Kempf and Rossignol (2007)), in individual labor productivity (Jouvet *et al.* (2008)), or in age with overlapping generations (John and Pecchenino (1994), Jouvet *et al.* (2008)).

In this paper we consider heterogeneity in the agents' discount factor. [1] We assume that the population is exogenously divided into two groups, one with patient households and the other with impatient households. Each individual votes in favor, or against a public policy in environmental maintenance. Maintenance is a public policy, financed by a tax on households, and pollution flows from firm's activity. We define a voting equilibrium and the related general equilibrium of the economy at the steady state.

Our setting raises many issues. First, if the policy choice were one-dimensional, then the median-voter theorem could apply. Unfortunately, in our dynamic multidimensional setting, it cannot. We will show that, at the steady state, a voting equilibrium will coincide with the solution that would result from the median voter theorem. In other words, we provide a logically consistent definition of the median voter theorem in a dynamic setting. This establishes the applicability of the median voter theorem on steady state equilibria. This result is important because, in the literature, it is always assumed that the median voter theorem can be applied after the steady state is defined, though the steady state equilibrium itself depends on the voting equilibrium (see *e.g.* Kempf and Rossignol (2007), Corbae *et*

[1] For a general survey of the literature on models of economic growth with consumers having different discount factors, see Becker (2006).

al. (2009)). Our contribution is to prove that a dynamic voting equilibrium coincides with the application of the median voter theorem. This represents a contribution to the theoretical literature. Further, to stress the advantages of considering heterogenous agents, we compare our results with what the representative agent framework would provide. And the results differ in many respects.

Beyond the theoretical aspects, we also contribute the literature on political economy and environmental policy. With some comparative statics, we are able to show several novel results. We first show that, if the median voter is impatient, she consumes all her revenue, and maintenance will be zero. But if the median voter is patient, then maintenance will be positive, but not uniquely determined. Then we can go further and stress that there exist two channels of discount factors impact on the behavior of agents towards maintenance, a direct one and an indirect one. In our model, the higher the agent's discount factor, the larger is her desired level of maintenance. This is the direct channel. But at the same time, the richer the agent, the larger is her desired level of maintenance. It is well-known that only agents with a high discount factor have positive savings in the long run. Those with a low discount factor save nothing. Thus, the former will become wealthy in the long run and desire high levels of environmental maintenance, while the latter will become poorer and desire lower levels of maintenance. This is the indirect channel. This provides us with new insights about the relationship between economic development and environmental quality through the voting equilibrium (a new rationale for the so-called Environmental Kuznets Curve, see *e.g.* Dasgupta *et al.* (2002), Prieur (2009)). We also show (among other results) that, when the median voter is patient, then a lower inequality among agents has a positive effect on the environmental quality.

This discussion also relates to the broad debate about the discounting rate in environmental economics.[2] Even if discounting is often considered in the literature as a normative issue, it also has a positive content, as stressed by Dasgupta: "discount rates on consumption changes combine *values* with *facts*. (Dasgupta, 2008, p. 144) or by Arrow *et al.* (1995) who distinguishes *prescriptive* and *descriptive* positions. In environmental economics, a high discount rate implies relatively modest and slow environmental maintenance, while a low discount rate implies immediate and strong action. The common characteristics of all this literature is to rely on the assumption that there exists a *representative agent* in the economy whose preferences are considered

[2]Recently this debate has experienced a strong revival after the publication of the Stern Review (Stern, 2006, and Stern, 2008). Prominent economists have contributed to the debate, like Dasgupta (2008), Nordhaus (2008) or Weitzman (2007).

as given by a benevolent social planner. This agent further acts as a benevolent social planner[3]. We depart from the representative agent hypothesis by considering an economy populated with heterogeneous agents. Then, we are able to provide a microeconomic rationale to determine the implicit global discount rate in this economy. This departs from the normative discussion on what the discount rate *should* be. In our analysis, we take beforehand agents' preferences, and we scrutinize how the existence of heterogeneity shapes the policy in the global economy. This is a novel contribution to the debate on discounting based on a positive approach.

Applying the median voter theorem to dynamic models requires a suitable analytical redesign of the political settings in this model. Models of such a kind are much harder to analyze than their static counterparts or than the usual intertemporal models without political ingredients. It appears from the recent literature that the analysis of the performance of majoritarian settings in dynamic frameworks has attracted growing interest, see *e.g.* Baron (1996), Krusell *et al.* (1997), Cooley and Soares (1999), Rangel (2001) and Bernheim and Slavov (2009). The stage of development of the theory is still in its infancy, and there is no consensus about how to model dynamic majoritarian voting. Without going into detail in this introduction, it should be stressed that our approach to voting is different from the approaches used in the above-mentioned papers. We propose a novel definition of voting equilibrium, which is related to Kramer-Shepsles equilibrium concept (Kramer (1972), Shepsle (1979)). This definition will allow us to provide new theoretical results about voting equilibrium in a dynamical setting.

These results bring us to our last discussion on alternative financing schemes of the environmental maintenance. We look at the different impacts on heterogenous households and especially on the median voter, of financing maintenance both with taxes and with issuance of public bonds. We show that, under common assumption about income distribution, an increase in the public debt leads to a lower environmental quality.

The paper is organized as follows. In Section 2 we present the model, define the competitive equilibria and describe steady-state equilibria for a given policy. In Section 3 we endogenize the voting procedure on environmental maintenance, define the intertemporal and steady state voting equilibria, and show the logical consistency between the median voter theorem and the voting equilibrium in dynamic general equilibrium. In Section 4 the comparison with the representative agent framework is proposed. In Section 5 we perform some comparative statics exercises to determine under which circumstances environmental quality is positively impacted by an increase in

[3]Or the *social evaluator*, to take Dasgupta's words.

total factor productivity, an increase in patience, and a decrease in inequality. The discussion about the impact of public debt on the environmental quality is carried out in Section 6. Finally, Section 7 concludes.

2 The model, and preliminary results

Our objective in this paper is to define and to study the intertemporal competitive equilibria with voting on maintenance. We define voting equilibria in two steps. In this section, we do the first step, namely we determine the competitive equilibrium production and consumption paths for a given maintenance. In the next section, we will do the second step, namely we will select, in the class of these competitive equilibria, the ones in which there is also a voting equilibrium.

The framework of analysis used in this paper is the one with infinitely-lived consumers, supplying inelastically each time one unit of labor and with a representative globally polluting firm.

2.1 Production and pollution

Output is determined by means of a neoclassical production function $F(K_t, L_t) = Lf(k_t)$, where K_t and L_t are capital and labor at time t, $k_t = K_t/L$ is capital intensity, $f(k) = F(k, 1)$ is the production function in intensive form. Capital is assumed to depreciate completely within the period. Output can be used for consumption, investment or maintenance. Thus, the dynamics of capital is given by

$$K_{t+1} = F(K_t, L_t) - C_t - M_t,$$

where C_t is time t aggregate consumption and M_t is time t aggregate maintenance.

Pollution (the emissions level) at time t, P_t, is proportional to output:

$$P_t = \lambda F(K_t, L) = \lambda L f(k_t), \ \lambda > 0. \tag{1}$$

We denote by Q_t an index of environmental quality at time t and by M_t the maintenance of environmental quality. The dynamics of Q_t is given by

$$Q_{t+1} = \Psi(Q_t - P_t + \frac{M_t}{\mu}), \tag{2}$$

where $\Psi : \mathbb{R}_+ \to \mathbb{R}_+$ is a concave increasing function, $\mu > 0$ is exogenously given coefficient. Since "marginal environmental productivity" of maintenance, $\partial Q_{t+1}/\partial M_t = \Psi'(\cdot)/\mu$, is negatively influenced by μ, we can interpret

$1/\mu$ as the environmental efficiency of maintenance. By $\bar{Q}$ we denote a unique positive solution to the following equation: $\Psi(Q) = Q$, *i.e.* the stationary value of environmental quality in the case with no pollution and no maintenance. For example, the following particular forms of $\Psi(X)$ can be used: $\Psi(X) = X^\nu \bar{Q}^{1-\nu}$, with $0 < \nu < 1$, or $\Psi(X) = \nu X + (1-\nu)\bar{Q}$, with $0 < \nu < 1$.

Let $\Phi(\cdot) = \Psi^{-1}(\cdot)$. We can rewrite (2) as follows:

$$\mu\Phi(Q_{t+1}) = \mu(Q_t - P_t) + M_t.$$

It should be noted that $\mu\Phi'(Q)$ can be interpreted as the marginal cost of quality improvement.

The representative firm maximizes its profit π_t under the constraint of the technology $F(K_t, L_t)$ by choosing its preferred volumes of capital K_t and labor L_t, considering the real wage and interest rates, w_t and r_t, as given. The firm's problem is summarized as follows:

$$\max_{K_t, L_t} \pi_t = F(K_t, L_t) - (1 + r_t)K_t - w_t L_t, \tag{3}$$

and admits the following first-order conditions: $F'_K(K_t, L_t) = 1 + r_t$ and $F'_L(K_t, L_t) = w_t$, or in intensive terms: $f'(k_t) = 1 + r_t$ and $f(k_t) - f'(k_t)k_t = w_t$.

2.2 Consumers

Population consists of L consumers. Each consumer is endowed with one unit of labor force. For simplicity, L is integer and odd. The objective function of consumer i is

$$\sum_{t=0}^{\infty} \beta_i^t [u(c_t) + v(Q_t)],$$

where c_t is his consumption at time t, β_i is his discount factor. We assume that $u(c)$ and $v(Q)$ satisfy the following conditions:

$$u'(c) > 0, u''(c) < 0, u'(0) = \infty, v'(Q) > 0, v''(Q) < 0, v'(0) = \infty.$$

Each consumer i is patient ($\beta_i = \beta^h$) or impatient ($\beta_i = \beta^l$), $0 < \beta^l < \beta^h < 1$. We denote by H_h the set of patient consumers (with discount factor equal to β^h) and by H_l the set of impatient consumers (those with β^l).

Each consumer pays a tax $m_t = M_t/L$ to finance the public provision of environmental maintenance and the budget constraints of a consumer at time t are

$$c_t + s_t + m_t \leq w_t + (1 + r_t)s_{t-1}, \tag{4}$$

$$c_t \geq 0, \;\; s_t \geq 0,$$

where w_t is the wage rate r_t is the interest rate and s_t are her savings consumer at time t. It should be emphasized that consumers are forbidden to borrow against their future labor income and hence their savings must be non-negative.

The consumer's utility depends both on variables on which he has full control, c_t and s_t, and on the variable m_t which is determined by the voting procedure, yet to be introduced. Thus, before analyzing the result of the voting procedure, solving the consumer's objective is equivalent to choosing the optimal values of c_t and s_t, $\forall t$, considering m_t as given.

Suppose that at time τ consumer i *is given* his predetermined level of savings $\hat{s}^i_{\tau-1}$, the predetermined level of environmental quality $\hat{Q}_\tau$, *the stream of pollution* $(P_t)_{t=\tau}^{\infty}$ and *some maintenance policy which is represented by a sequence* $\mathbf{m} = (m_t)_{t=0}^{\infty}$ of non-negative numbers. Then the problem of this consumer is

$$\mathcal{P}_1(\tau) = \left\{ \begin{array}{cr} \max_{(c_t)_{t=\tau}^{+\infty}, (s_t)_{t=\tau}^{+\infty}, (Q_t)_{t=\tau}^{+\infty}} \sum_{t=\tau}^{\infty} \beta_i^t [u(c_t) + v(Q_t)], & \\ \text{subject to} & \\ \mu\Phi(Q_{t+1}) = \mu(Q_t - P_t) + Lm_t, & t = \tau, \tau+1, \ldots, \\ c_t + s_t + m_t \leq w_t + (1+r_t)s_{t-1}, & t = \tau, \tau+1, \ldots, \\ s_{\tau-1} = \hat{s}^i_{\tau-1}, Q_\tau = \hat{Q}_\tau, & \\ c_t \geq 0, \;\; s_t \geq 0, & t = \tau, \tau+1, \ldots. \end{array} \right\}$$

It should be noticed that since $\mathbf{m} = (m_t)_{t=0}^{\infty}$ is given, the sequence $(Q_t)_{t=\tau}^{+\infty}$ is in fact predetermined by $\hat{Q}_\tau$ and $\mathbf{m}$. Hence, the utility consumer i derives from environmental quality, $\sum_{t=\tau}^{\infty} \beta_i^t v(Q_t)$, does not depend on her choice, though, formally, Q_t is a control variable in problem $\mathcal{P}_1(\tau)$.

2.3 Competitive equilibrium paths and steady-state equilibria

Now we can give the definition of the equilibrium path supposing that the environmental policy is given and that no agent can change it.

Let at time 0 the environmental policy be represented by some sequence $\mathbf{m} = (m_t)_{t=0}^{\infty}$ of non-negative numbers *be given.* Let an initial state $\{(\hat{s}^i_{-1})_{i=1}^{L}, \; \hat{k}_0, \; \hat{Q}_0\}$ also be given. Here $\hat{s}^i_{-1} \geq 0$ are the initial savings of consumers $i = 1, \ldots, L$, $\hat{k}_0 > 0$ is the initial per capita stock of capital, $\sum_{i=1}^{L} \hat{s}^i_{-1} = L\hat{k}_0$, and $\hat{Q}_0 > 0$ is the initial value of environmental quality.

Definition 1. ***Competitive equilibrium path***

Given $\boldsymbol{m}$*, the sequence* $\mathcal{E}^{\boldsymbol{m}} = \{k_t^*, 1 + r_t^*, w_t^*, (s_{t-1}^{i*}, c_t^{i*})_{i=1}^L, P_t^*, Q_t^*\}_{t=0}^\infty$ *is called a* competitive equilibrium path *starting from* $\{(\hat{s}_{-1}^i)_{i=1}^L,\ \hat{k}_0,\ \hat{Q}_0\}$ *if*

1. *capital and labor markets clear at the following prices:* $1 + r_t = 1 + r_t^* = f'(k_t^*),\ w_t = w_t^* = f(k_t^*) - f'(k_t^*)k_t^*,\ t = 0, 1, \ldots\ ;$
2. *for each household* $i = 1, \ldots, L$ *the sequence* $(s_{t-1}^{i*}, c_t^{i*}, Q_t^*)_{t=0}^\infty$ *is a solution to problem* $\mathcal{P}_1(0)$ *at* $1 + r_t = 1 + r_t^*,\ w_t = w_t^*,\ t = 0, 1, \ldots\ ;$
3. $\sum_{i=1}^L s_{t-1}^{i*} = Lk_t^*,\ t = 0, 1, \ldots\ ;$
4. $P_t^* = \lambda L f(k_t^*),\ t = 0, 1, \ldots\ ;$
5. $\mu\Phi(Q_{t+1}^*) = \mu(Q_t^* - P_t^*) + Lm_t,\ t = 0, 1, \ldots\ .\square$

Notice that, at each time t, maintenance m_t is given and smaller than the wage rate w_t. We will not discuss the existence of equilibrium paths. Our main emphasis will be made on steady-state equilibria. Reasonably, we define steady-state equilibria under the assumption that the environmental policy is given and constant over time.

Definition 2. ***Competitive steady state equilibrium***

Let an $m \geq 0$ *be given and let* $\boldsymbol{m} = (m_t)_{t=0}^\infty$*, with* $m_t = m,\ t = 0, 1, \ldots.$ *We call a tuple* $E^m = \{k^*, 1 + r^*, w^*, (s^{i*}, c^{i*})_{i=1}^L, P^*, Q^*\}$ a competitive steady-state equilibrium *if the sequence* $\{k_t^*, 1 + r_t^*, w_t^*, (s_{t-1}^{i*}, c_t^{i*})_{i=1}^L, P_t^*, Q_t^*\}_{t=0}^\infty$ *given for all* $t = 0, 1, \ldots$ *by*

$$k_t^* = k^*,\ 1 + r_t^* = 1 + r^*,\ w_t^* = w^*, \tag{5}$$

$$(s_{t-1}^{i*}, c_t^{i*})_{i=1}^L = (s^{i*}, c^{i*})_{i=1}^L, \tag{6}$$

$$P_t^* = P^*,\ Q_t^* = Q^*. \tag{7}$$

is an equilibrium path starting from the initial state $\{(\hat{s}_{-1}^i)_{i=1}^L,\ \hat{k}_0,\ \hat{Q}_0\} = \{(s^{i*})_{i=1}^L,\ k^*,\ Q^*\}$*.* $\square$

The following proposition describing the structure of steady-state equilibria is an adaptation to our model of the well-known results by Becker (1980, 2006).

Proposition 1. ***Structure of steady state equilibrium***

A tuple $E^m = \{k^*, 1+r^*, w^*, (s^{i*}, c^{i*})_{i=1}^L, P^*, Q^*\}$ *satisfying* $m < w^*$ *is a steady-state equilibrium if and only if*

$$\beta^h = \frac{1}{1+r^*},\ 1+r^* = f'(k^*),\ w^* = f(k^*) - f'(k^*)k^*, \tag{8}$$

$$P^* = \lambda L f(k^*), \tag{9}$$

$$\mu\Phi(Q^*) = \mu(Q^* - P^*) + Lm, \tag{10}$$

$$s^{i*} = 0,\ i \in H_l, \tag{11}$$

$$s^{i*} \geq 0,\ i \in H_h, \tag{12}$$

$$\sum_{i=1}^{L} s^{i*} = \sum_{i \in H_h} s^{i*} = Lk^*; \tag{13}$$

$$c^* + s^* + m = w^* + (1+r^*)s^*. \tag{14}$$

Proof. See Appendix A.1. □

In this proposition, equation (8) shows that the steady-state capital intensity, interest rate, and the wage rate are determined by the discount factor of the patient consumer. Equations (11)-(12) tell us that impatient consumers have zero savings. It means that all the capital is own by patient consumers. As a consequence, in a steady-state equilibrium all impatient consumers are identical in terms of income and savings. In contrast, the distribution of savings among patient consumers is indeterminate in a steady state equilibrium. As shown by equation (13), only aggregate savings is determined.

3 Voting equilibria

There is no reason for heterogenous agents to agree on the desired level of environmental maintenance. One way to solve this disagreement is to choose it by majority voting. If the level of maintenance is to be determined by majority voting, which level will be chosen? If policy choices were one-dimensional, we would refer to the median voter theorem, but in our intertemporal model this theorem cannot be applied directly since, formally speaking, in such models policy choices are multi-dimensional.

However, if we constraint our consideration to steady states, median voter approach to decision-making seems to be quite reasonable. At the same time, we should not mistakenly think that mere consideration of steady states only makes policy choices one-dimensional. Fortunately, as we show in this section, for some reasonable definition of voting equilibrium, in a voting steady-state equilibrium the level of maintenance will be chosen just by the median voter.

Let $\mathbf{m} = (m_t)_{t=0}^{\infty}$ be an environmental policy. The optimal value of problem $\mathcal{P}_1(\tau)$ for consumer i is a function of $\hat{s}_{\tau-1}$, $\hat{Q}_\tau$ and $\mathbf{m}$. We will denote this optimal value by $V_{i,\tau}(\hat{s}_{\tau-1}, \hat{Q}_\tau, \mathbf{m})$.

Definition 3. ***Preferred change in maintenance***

Suppose that the environmental policy is represented by some sequence $\bar{\boldsymbol{m}} = (\bar{m}_t)_{t=0}^{\infty}$ *of non-negative numbers and that at* $\boldsymbol{m} = \bar{\boldsymbol{m}}$ *the function* $V_{i,\tau}(\hat{s}_{\tau-1}, \hat{Q}_\tau, \boldsymbol{m})$ *is differentiable in* m_τ. *We say that consumer* i *is* in favor of increasing m_τ *if* $\frac{\partial V_{i,\tau}(\hat{s}_{\tau-1}, \hat{Q}_\tau, \boldsymbol{m})}{\partial m_\tau} > 0$ *and is* in favor of decreasing m_τ *if* $\frac{\partial V_{i,\tau}(\hat{s}_{\tau-1}, \hat{Q}_\tau, \boldsymbol{m})}{\partial m_\tau} < 0$ *and* $\bar{m}_\tau > 0$. □

Let us assume that, for an equilibrium path

$$\mathcal{E}^{\bar{\mathbf{m}}} = \{k_t^*, 1 + r_t^*, w_t^*, (s_{t-1}^{i*}, c_t^{i*})_{i=1}^{L}, P_t^*, Q_t^*\}_{t=0}^{\infty}$$

the function $V_{i,\tau}(s_{\tau-1}^*, Q_\tau^*, \mathbf{m})$ is differentiable in m_τ at $\mathbf{m} = \bar{\mathbf{m}}$. We denote by $N_\tau^+(\mathcal{E}^{\bar{\mathbf{m}}})$ the number of consumers who are in favor of *increasing* $\bar{m}_\tau$, and by $N_\tau^-(\mathcal{E}^{\bar{\mathbf{m}}})$ the number of consumers who are in favor of *decreasing* $\bar{m}_\tau$.

Now we are ready to define intertemporal voting equilibria.

Definition 4. ***Intertemporal voting equilibrium***

Let $\boldsymbol{m}^* = (m_t^*)_{t=0}^{\infty}$ *be a maintenance policy and* $\mathcal{E}^{\boldsymbol{m}^*}$ *be an equilibrium path constructed at this policy. We call the couple* $(\boldsymbol{m}^*, \mathcal{E}^{\boldsymbol{m}^*})$ an intertemporal voting equilibrium path *if at* $\boldsymbol{m} = \boldsymbol{m}^*$ $\forall \tau = 0, 1, \ldots$ *the function* $V_{i,\tau}(s_{\tau-1}^*, Q_\tau^*, \boldsymbol{m})$ *is differentiable in* m_τ, *and*

$$N_\tau^+(\mathcal{E}^{\boldsymbol{m}^*}) < \frac{L}{2},\ N_\tau^-(\mathcal{E}^{\boldsymbol{m}^*}) < \frac{L}{2}, \forall \tau = 0, 1, \ldots.$$

□

This definition is in line with the usual way of defining intertemporal equilibrium, as articulated by Hicks (1937) and, more recently, by Grandmond (1983). In our model, any intertemporal voting equilibrium can be seen as a sequence of temporary voting equilibria in which agents correctly anticipate the whole future, including voting results. Indeed, let $(\mathbf{m}^*, \mathcal{E}^{\mathbf{m}^*})$ be an intertemporal voting equilibrium. Suppose that at time τ the agents are asked to vote on m_τ and that they correctly anticipate m_t^* for all $t = \tau+1, \tau+2, ...$ Then it is clear that all the conditions for the median voter theorem hold in this one-dimensional voting, and that the preferred value of m_τ for the median voter coincides with m_τ^*. A key implication is that intertemporal voting equilibria are time consistent.

In the rest of the paper we shall focus on steady state voting equilibria. Consider a couple (m^*, E^{m^*}), where $m^* \geq 0$ and $E^{m^*} = \{k^*, 1 + r^*, w^*, (s^{i*}, c^{i*})_{i=1}^L, P^*, Q^*\}$ is a steady-state equilibrium constructed at the maintenance policy $\mathbf{m}^* = (m_0^*, m_1^*, ...)$, $m_t^* = m^*, t = 0, 1, ...$ Let $\mathcal{E}^{m^*}$ be an equilibrium path corresponding to E^{m^*}.

Definition 5. ***Steady state voting equilibrium***

We call the couple (m^*, E^{m^*}) a steady state voting equilibrium *if the couple* $(m^*, \mathcal{E}^{m^*})$ *is an intertemporal voting equilibrium path.* □

To answer the question of whether a couple (m^*, E^{m^*}) is a steady state voting equilibrium or not it is sufficient to know which consumers are in favor of increasing of $m_0^* = m^*$ at time 0 and which ones are in favor of its decreasing.

We know that for each i the sequence $(\tilde{s}_{t-1}^i, \tilde{c}_t^i, \tilde{Q}_t)_{t=0}^\infty$ given by

$$\tilde{s}_{t-1}^i = s^{i*}, \; \tilde{c}_t^i = c^{i*}, \; \tilde{Q}_t = Q^*, \tag{15}$$

is a solution to

$$\max_{(c_t)_{t=0}^{+\infty}, (Q_t)_{t=0}^{+\infty}} \sum_{t=0}^{\infty} \beta_i^t [u(c_t) + v(Q_t)], \tag{16}$$

$$\mu\Phi(Q_{t+1}) = \mu(Q_t - P^*) + Lm_t^*, \; t = 0, 1, \ldots, \tag{17}$$

$$c_t + s_t + m_t^* \leq w^* + (1 + r^*)s_{t-1}, \; t = 0, 1, \ldots, \tag{18}$$

$$s_{-1}^i = \hat{s}_{-1}^i, \; Q_0 = \hat{Q}_0, \tag{19}$$

$$c_t \geq 0, \; s_t \geq 0, \; Q_t \geq 0, \; t = 0, 1, \ldots \tag{20}$$

at $\hat{s}_{-1}^i = s^{i*}$, $\hat{Q}_0 = Q^*$.

Lemma 1. ***Differentiability of value function w.r.t. maintenance and sign of derivative***

Let for some i the sequence $(\tilde{s}_{t-1}^i, \tilde{c}_t^i, \tilde{Q}_t)_{t=0}^\infty$ given by (15) be a solution to problem (16)-(20) at given $m_t^* = m^* \in [0, w^*)$, $t = 0, 1, ...$ and at $\hat{s}_{-1}^i = s^{i*}$, $\hat{Q}_0 = Q^*$. Then $V_{i,0}(s^{i*}, Q^*, \boldsymbol{m}^*)$ is differentiable in m_0^* and

$$\frac{\partial V_{i,0}(s^{i*}, Q^*, \boldsymbol{m}^*)}{\partial m_0^*} \gtreqless 0 \Leftrightarrow \beta_i L v'(Q^*) \gtreqless \mu u'(c^{i*})(\Phi'(Q^*) - \beta_i). \tag{21}$$

Proof. See appendix A.2 □

The interpretation of Lemma 1 runs as follows. Consider the first inequality of equation (21) at a given maintenance m_0^* and suppose that the left-hand side is higher than the right-hand side . In this case, out of a marginal change in maintenance, the induced marginal utility of environmental quality, *i.e.* the LHS of equation (21), is larger than the induced marginal utility of consumption, *i.e.* the RHS of (21). This is likely to happen when the given maintenance level m_0^* is low. This entails that the consumer is in favor of an increase in maintenance. In the opposite case, the given maintenance m_0^* is likely to be large so that the induced marginal utility of consumption is higher than the induced marginal utility of quality and the consumer is in favor of decreasing maintenance.

To check whether a couple (m^*, E^{m^*}) is a voting steady-state equilibrium or not, consider the following problem in which household i is free to determine her preferred level of maintenance m_t:

$$\mathcal{P}_2 = \left\{ \begin{array}{lr} \max_{(c_t)_{t=0}^{+\infty},(s_t)_{t=0}^{+\infty},(m_t)_{t=0}^{+\infty},(Q_t)_{t=0}^{+\infty}} \sum_{t=0}^{\infty} \beta_i^t [u(c_t) + v(Q_t)], & \\ \text{subject to} & \\ \mu\Phi(Q_{t+1}) \leq \mu(Q_t - P^*) + Lm_t, & t = 0, 1, \ldots, \\ c_t + s_t + m_t \leq w^* + (1 + r^*)s_{t-1}, & t = 0, 1, \ldots, \\ s_{-1} = \hat{s}_{-1}, \ Q_0 = \hat{Q}_0, & \\ c_t \geq 0, \ s_t \geq 0, \ m_t \geq 0, \ Q_t \geq 0, & t = 0, 1, \ldots \end{array} \right\}$$

We say that $(\tilde{s}, \tilde{c}, \tilde{m}, \tilde{Q}) \in \mathbb{R}_+^4$ *determines a steady-state solution* to this problem if the sequence $(\tilde{s}_{t-1}, \tilde{c}_t, \tilde{m}_t, \tilde{Q}_t)_{t=0}^{\infty}$ given by

$$\tilde{s}_{t-1} = \tilde{s}, \ \tilde{c}_t = \tilde{c}, \ \tilde{m}_t = \tilde{m}, \ \tilde{Q}_t = \tilde{Q} \tag{22}$$

is its solution at $\hat{s}_{-1} = \tilde{s}$ and $\hat{Q}_0 = \tilde{Q}$.

Prior to formulating the following lemma, remind that $\beta^h(1+r^*) = 1$ and hence that $\beta_i(1 + r^*) < 1, \ \forall i \in H_l$, and $\beta_i(1 + r^*) = 1, \forall i \in H_h$.

Lemma 2. ***Characterization of steady state solution to*** $\mathcal{P}_2$

The tuple $(\tilde{s}, \tilde{c}, \tilde{m}, \tilde{Q}) \in \mathbb{R}_+^4$ *determines a steady-state solution to* $\mathcal{P}_2$ *if and only if*

$$\beta_i(1+r^*) < 1 \Rightarrow \tilde{s} = 0 \tag{23}$$

$$\beta_i L v'(\tilde{Q}) \leq \mu u'(\tilde{c})(\Phi'(\tilde{Q}) - \beta_i) \ (= \mathit{if}\ \tilde{m} > 0) \tag{24}$$

$$\tilde{c} = w^* + r^*\tilde{s} - \tilde{m} \tag{25}$$

$$\mu(\Phi(\tilde{Q}) - \tilde{Q} + P^*) = L\tilde{m} \tag{26}$$

Proof. *See appendix A.3.* □

To simplify the presentation we can get rid of $\tilde{m}$ by noticing that $\tilde{m} > 0 \Leftrightarrow \tilde{c} < w^* + r^*\tilde{s}$ and rewriting conditions (24)-(25) as follows:

$$\tilde{c} = (w^* + r^*\tilde{s} - \frac{\mu}{L}P^*) + \frac{\mu}{L}(\tilde{Q} - \Phi(\tilde{Q})), \tag{27}$$

$$\tilde{c} \leq w^* + r^*\tilde{s}, \tag{28}$$

$$\beta_i L v'(\tilde{Q}) \leq \mu u'(\tilde{c})(\Phi'(\tilde{Q}) - \beta_i) \ (= \text{if } \tilde{c} < w^* + r^*\tilde{s}). \tag{29}$$

Equation $\beta_i L v'(Q) = \mu u'(c)(\Phi'(Q) - \beta_i)$ implies an increasing dependence of c on Q. As for equation $c = (w^* + r^*\tilde{s} - \frac{\mu}{L}P^*) + \frac{\mu}{L}(Q - \Phi(Q))$, for any given $\tilde{s}$, it specifies a dependence of c on Q which is simply decreasing, or is first increasing ($\Phi'(Q) < 1$) and then decreasing ($\Phi'(Q) > 1$).

Suppose we are given $m^* \in [0, w^*)$, where w^* is given by (8). Let $E^{m^*} = \{k^*, 1+r^*, w^*, (s^{i*}, c^{i*})_{i=1}^{L}, P^*, Q^*\}$ be a steady-state equilibrium constructed at the maintenance policy $\mathbf{m}^* = (m^*, m^*, ...)$. Put all households in ascending order of their savings and take the median one, i_m.[4] Lemmas 1 and 2 lead to the following theorem.

Theorem 1. ***Steady state voting equilibrium and median voter***
The couple (m^*, E^{m^*}) is a steady-state voting equilibrium if and only if for $i = i_m$, the tuple $(s^{i*}, c^{i*}, m^*, Q^*)$ is a steady-state solution to problem $\mathcal{P}_2$. □

This theorem reads that, in the long-run, the capital stock depends on the discount factor of the patient households, while maintenance and environmental quality depend on the median discount factor and the median savings.

[4]More formally, we can put the set of households in an order such that, if $\beta_i < \beta_j$ and if $s^{i*} < s^{j*}$, then i precedes j. Such an order exists because the impatient consumers do not save in a steady-state equilibrium. Now take the household median in the sense of the introduced order, i_m.

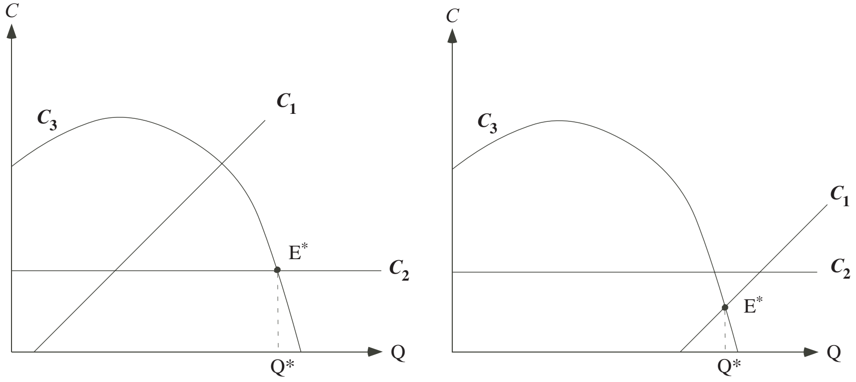

Figure 1: Left: Zero-Maintenance Equilibrium (Regime 1) - Right: Positive Maintenance Equilibrium (Regime 2)

It follows from this theorem that, in equilibrium, there exist two possible cases, depending on whether $c^{i_m*} = w^* + r^* s^{i_m*}$ ($\Leftrightarrow m^* = 0$) or $c^{i_m*} < w^* + r^* s^{i_m*}$ ($\Leftrightarrow m^* > 0$). They are illustrated by the left and right panel of Fig. 1, on which we take s^{i_m*} as given. On these graphs the three curves $\mathcal{C}_1$, $\mathcal{C}_2$ and $\mathcal{C}_3$ are defined as follows:

$$\text{Curve } \mathcal{C}_1: \quad \beta_{i_m} L v'(Q) = \mu u'(c)(\Phi'(Q) - \beta_{i_m}) \tag{30}$$

$$\text{Curve } \mathcal{C}_2: \quad c = w^* + r^* s^{i_m*} \tag{31}$$

$$\text{Curve } \mathcal{C}_3: \quad c = (w^* + r^* s^{i_m*} - \frac{\mu}{L} P^*) + \frac{\mu}{L}(Q - \Phi(Q)) \tag{32}$$

Let us describe more precisely these two regimes:

Regime 1 - Zero-maintenance. The equilibrium point (Q^*, c^{i_m*}) is at the intersection of the $\mathcal{C}_2$ curve and the $\mathcal{C}_3$ curve (see figure 1a) and, as far as curve $\mathcal{C}_1$ is concerned, we have $\beta_{i_m} L v'(Q^*) < \mu u'(c^{i_m*})(\Phi'(Q^*) - \beta_{i_m})$.

Regime 2 - Positive-maintenance. The equilibrium point (Q^*, c^{i_m*}) is at the intersection of the $\mathcal{C}_1$ curve with the $\mathcal{C}_3$ curve (see figure 1b) and, as far as curve $\mathcal{C}_2$ is concerned, we have $c^{i_m*} < w^* + r^* s^{i_m*}$.

In combination with the above-mentioned two regimes ($m^* > 0$ and $m^* = 0$), two cases must be distinguished:

Case 1 - Impatient median voter: $\beta_{i_m} = \beta^l$ and savings of the median voter are determined uniquely, $s^{i_m *} = 0$.

Case 2 - Patient median voter: $\beta_{i_m} = \beta^h$ and the savings of the median voter, $s^{i_m *}$, are not determined uniquely; they can take any value in the interval $[0, \frac{2}{L+1} L k^*]$.

In both cases, the regime of equilibrium maintenance can be nil or positive. In *Case 1, the equilibrium levels of maintenance and environmental quality are determined uniquely.* As for *Case 2*, if there exists at least one equilibrium with positive maintenance, *the equilibrium levels of maintenance and environmental quality are indeterminate since there is a continuum of these.*

Several words on the existence of steady-state voting equilibria are in order. It is clear that if the majority of consumers is impatient, then steady-state voting equilibria exist for any distribution of savings among patient consumers because in this case the solution to problem $\mathcal{P}_2$ for the median voter, $(\tilde{s}, \tilde{c}, \tilde{m}, \tilde{Q})$, unconditionally satisfies $\tilde{m} < w^*$. If if the majority of consumers is patient, steady-state voting equilibria exists for any distribution of savings among patient consumers where the savings of the median voter are nil or sufficiently small.

However, if the majority of consumers is patient, and the savings of the median voter are sufficiently high, it may be that she will vote for maintenance which exceeds the wage rate and hence steady-state equilibrium does not exist.

4 How agents' heterogeneity shapes environmental maintenance

In this section we compare the level of environmental quality in voting steady-state equilibria of our model with that in steady-state equilibria of a similar economy populated with identical agents. We constraint our consideration to the case where the equilibrium values of capital stock and hence output is the same in both models. The question we raise is the following: what is the effect of agents' heterogeneity in discount factor and wealth on environmental maintenance when agents are asked to vote?[5]

[5] Note that this is different from the question raised by Caselli and Ventura (2000) : under which condition does a model with heterogenous agents "admits" a representative agent model, namely a model with homogenous agents displaying the same aggregate and

We will identify the homogenous population model as a particular case of our model where the discount factors of all consumers are the same and equal to β^h. Moreover, by steady-state equilibria in the homogenous population model we will mean symmetrical voting steady-state equilibria in this particular case of our model *i.e.* equilibria where the savings of all consumers are the same and hence consumption of all agents is the same. To be precise, for symmetrical equilibria voting is somewhat irrelevant because in such equilibria voting is unanimous.

Let $\{k_S^*, 1 + r_S^*, w_S^*, (s_S^{i*}, c_S^{i*})_{i=1}^L, P_{S*}, Q_{S*}\}$ be a symmetric steady-state voting equilibrium of our model with $\beta_i = \beta^h$, $i = 1, \ldots, L,$, and $\{k^*, 1 + r^*, w^*, (s^{i*}, c^{i*})_{i=1}^L, P^*, Q^*\}$ be a steady-state voting equilibrium in our model with arbitrarily chosen discount factors. By symmetric we mean that $s_S^{1*} = \ldots = s_S^{L*}$. It should be noticed that

$$k_S^* = k^*,\ r_S^* = r^*,\ w_S^* = w^*$$

and that, by assumption,

$$s_S^{i*} = k^*,\ i = 1, \ldots, L.$$

The last equation says that the savings of agents in the symmetric steady-state voting equilibrium with $\beta_i = \beta^h,\ i = 1, \ldots, L$, are equal to the mean of the savings in the model in the heterogeneous agent case. We assume that in the former model the discount factor shared by all consumer is β^h but not β^l, because otherwise equilibrium stocks of capital and output would be different in the two models.

Let

$$m^* = w^* + r^* s^{i_m *} - c^{i_m *},$$

$$m_S^* = w_S^* + r_S^* k_S^* - c_S^* (= w^* + r^* k^* - c_S^*),$$

where $c_S^* = c_S^{1*} (= \ldots = c_S^{L*})$.

The following proposition can be proved by means of the argument analogous to those in the previous section.

Proposition 2. ***Homogenous vs. heterogeneous population equilibria***

average behavior. Indeed, in our case, by assumption, we fix capital intensity to be the same in both models. On the other hand we do not fix maintenance, nor do we look at the representative agent version of the model which would yield the same maintenance.

1) Suppose that $\beta_{i_m} = \beta^l$ and hence $s^{i_m*} = 0$ in the heterogenous agent economy. In this case:

1. if $m_S^* = 0$, then $m^* = 0$ and $Q^* = Q_S^*$;
2. if $m_S^* > 0$, then $m^* < m_S^*$ and $Q^* < Q_S^*$.

2) Suppose that $\beta_{i_m} = \beta^h$ in the heterogenous agent economy. In this case:

1. if $s^{i_m*} \leq s_S^{i*} = k^*$, then:
   (a) if $m_S^* = 0$, $m^* = 0$ and $Q^* = Q_S^*$
   (b) if $m_S^* > 0$, $m^* < m_S^*$ and $Q^* < Q_S^*$
2. if $s^{i_m*} \geq s_S^{i*} = k^*$, then:
   (a) if $m^{i*} = 0$, $m_S^* = 0$ and $Q^* = Q_S^*$
   (b) if $m^* > 0$, then $m^* > m_S^*$ and $Q^* > Q_S^*$

□

We will see in the next section that under some reasonable assumptions it is natural to expect that in less developed countries there is no maintenance and that it is positive in developed countries. Thus, the above proposition reads that for less developed countries the predictions of both models are the same: there is no maintenance in steady-state equilibria irrespective of whether the median voter is patient or impatient.

For developed countries the predictions of the models differ. If the majority of agents in our model is impatient, then the equilibrium levels of maintenance and environmental quality in our model are lower than those predicted by the homogenous population model. If the majority of agent is patient, then it is necessary to compare the median saving or income with the mean ones. If the median savings are lower than the mean, or, equivalently, the median income is lower then the mean income, then the equilibrium levels of maintenance and environmental quality in our model are lower that those in the homogenous population model. Otherwise, the situation is the opposite. The case where the median income is lower than the mean is usually considered as typical.

Thus our model suggests that, in most cases in the real world, lower levels of maintenance and environmental quality should be observed than what the homogenous agents population model would predict.

5 Some comparative statics on preferences, income inequality, and technology

As stressed above, if the median voter is patient, in a steady state the savings of the median voter, s^{i_m*}, are not determined uniquely. They can take any value in the interval $[0, \frac{2}{L+1}Lk^*]$. Therefore when making a comparative statics exercise we should remember that a change in a parameter will have an indeterminate effect on the savings of the median voter. To circumvent this problem, in this section, we first assume that k^* is kept unchanged wheras s^{i_m*} changes and then we assume that the ratio $s^{i_m*}/(\sum_{i=1}^{L} s^{i*})$ and hence the ratio s^{i_m*}/k^* remain intact when a parameter changes (notice that since $k^* = (\sum_{i=1}^{L} s^{i*})/L$ shows the mean savings, s^{i_m*}/k^* shows the proportion between the median and mean savings).

5.1 An increase in s^{i_m*} other things equal

First, we carry out a comparative statics exercise relevant only in Case 2, where the median voter is patient and, consequently, his savings can be positive. Assume that k^* is kept unchanged and s^{i_m*} increases. This means that the increase in s^{i_m*} reflects a change in the distribution of savings among the patient consumers only. Consequently, it leads to a another income distribution (more precisely, an increase in the median income relative to the mean).

- under *Zero-Maintenance Equilibrium* (Regime 1), a small increase in s^{i_m*}, other things equal, will shift $\mathcal{C}_2$ and $\mathcal{C}_3$ upwards by the same magnitude. Hence, consumption of the median voter c^{i_m*} will increase, but the environmental quality Q^* will remain unchanged. A larger increase in s^{i_m*} may lead the economy to Regime 2.

- under *Positive-Maintenance Equilibrium* (Regime 2), a small increase in s^{i_m*}, other things equal, will shift $\mathcal{C}_3$ upwards, while letting $\mathcal{C}_1$ untouched. Hence the environmental quality Q^* will increase.

Following the politico-economic literature about income inequality (see e.g. Meltzer and Richard (1981)), an income distribution is called more equal, the higher the median income is relative to the mean (this is only reasonable in the case where the median income does not exceed the mean, which is considered as a typical situation). For our model this implies that in developed economies, where maintenance is positive, lower inequality has a positive effect on environmental quality, whereas in less developed economies, where there is no maintenance, inequality itself does not effect environmental quality.

5.2 An increase in total factor productivity

In the following sub-sections we shall assume that the production function is Cobb-Douglas, $f(k) = k^{\alpha}$, $0 < \alpha < 1$, and that the fraction of output necessary to eliminate emissions is lower than the labor share in output, $1 - \alpha > \mu\lambda$. Geometrically, the latter assumption implies that the curve $\mathcal{C}_3$ shifts upwards after an increase in capital intensity. In the following we will clearly indicate which of our conclusions rely on this assumption.

Let us first assume an increase in the total factor productivity by introducing a scale parameter a in the production function, which one becomes $aF(K, L) = Laf(k)$, where a represents the total factor productivity. The impact of an increase in total factor productivity will depend on the regime the economy follows in equilibrium.

Regime 1. Zero-maintenance Equilibrium

In this regime, a small *increase* in a will lead to an increase in k^*, w^* and $w^* + r^* s^{i_m *}$. Hence, it will also increase the output level $Lf(k^*)$ and pollution P^* but will not make maintenance positive. As a consequence, the environmental quality Q^* *will decrease*. Graphically (see Figure 2, left panel), $\mathcal{C}_2$ will shift upwards due to the increase in $w^* + r^* s^{i_m *}$. $\mathcal{C}_3$ will also shift upwards, but to a smaller extent, since both w^* and P^* increase. If the increase in a becomes too large, then the economy switches to Regime 2, the Positive-maintenance Equilibrium.

Regime 2. Positive-maintenance Equilibrium

In that regime an *increase* in a will shift $\mathcal{C}_3$ upwards, as shown in Figure 2, right panel, and hence to an *increase* in Q^* (this is not necessarily true if Assumption A does not fulfil).

To sum up, if the economy starts under Regime 1, then an increase in a from 0 to $+\infty$ first leads to a decrease in the environmental quality Q^*, and then to an increase, as shown in Figure 2. If one considers that less developed countries most likely correspond to Regime 1 and wealthy countries to Regime 2, then this conclusion means that technological progress first goes with a decrease in environmental quality, and after some stage of development to an increase in environmental quality. This result provides a new rationale for an Environmental Kuznets Curve (see *e.g.* Stockey, 1998, Dasgupta *et al.*

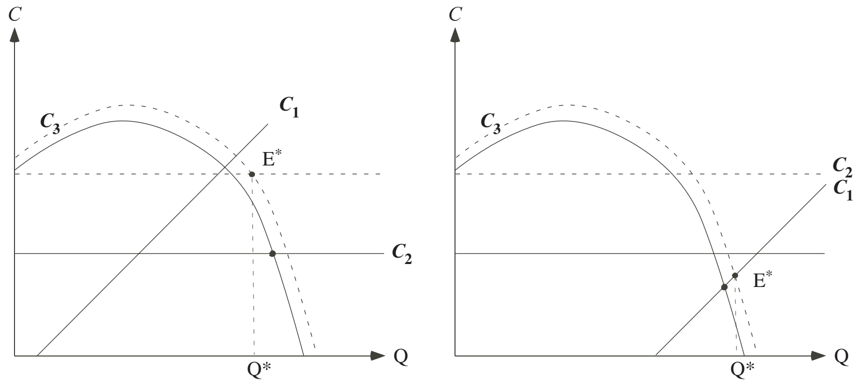

Figure 2: An increase in total factor productivity in Regime 1 (Left) and Regime 2 (Right)

(2002) or Prieur (2009)) to exist in the presence of heterogeneous consumers and voting.

Let us now turn to two comparative statics related to households' preferences.

5.3 Patient agents become more patient: an increase in β^h

We first consider an increase in β^h, meaning that patient agents become even more patient. The effects on the environmental quality will depend on which regime the economy experiences.

Under *Zero-Maintenance Equilibrium* (Regime 1), a small *increase* in β^h leads to an increase in capital intensity k^*, wage rate w^*, output $Lf(k^*)$ and pollution P^*, but it cannot make maintenance positive. Hence Q^* *decreases* as β^h increases under Regime 1. Graphically (see Fig. 2, left panel), $\mathcal{C}_2$ shifts upwards due to the increase in w^*; $\mathcal{C}_3$ also shifts upwards, but to a smaller extent (w^* will increase but P^* will also increase). If moreover the median voter is patient, Case 2, then, $\mathcal{C}_1$ shifts to the right. As a consequence the economy may switch the economy to the Positive-maintenance regime (regime 2).

Under *Positive-Maintenance Equilibrium* (Regime 2, see Fig. 2b) an *in-*

crease in β^h will lead to an upward shift of $\mathcal{C}_3$ and, in Case 2, to a shift of $\mathcal{C}_1$ to the right. Hence Q^* will *increase* (this is not necessarily true if Assumption A does not fulfil).

5.4 Impatient agents become less impatient: an increase in β^l

Now, let us consider an increase in β^l, which means that impatient agents become less impatient. The effect on Q^* will depend on whether the median consumer is impatient or patient, what we referred to as Case 1 and Case 2, respectively.

In the case where the median voter is impatient (Case 1, $\beta_{i_m} = \beta^l$), then the two regimes must be considered.

- under *Zero-Maintenance Equilibrium* (Regime 1), a small increase in β^l does not change k^*, w^*, $Lf(k^*)$ or P^*. It neither changes Q^*. This case results in a shift of $\mathcal{C}_1$ to the right, as illustrated in Fig. 3. Still, if the increase in β^l becomes large enough, then the economy switches to Regime 2;

- under *Positive-Maintenance Equilibrium* (Regime 2), a small increase in β^l does not change k^*, w^*, $Lf(k^*)$ or P^*, but it does increase Q^*, as illustrated in Fig. 3.

In the case where the median voter is patient (Case 2, $\beta_{i_m} = \beta^h$), then it is clear that changing β^l has no effect on Q^*.

6 Debt-financed versus tax-financed maintenance policy

We now turn to the analysis of some alternative policy scenarios. Up to now we assumed that the maintenance expenditure was financed by a a *pay-as-you-go* tax τ_t. Such a scheme is called *tax-finance*. A *debt-finance* scheme would mean that the government can also issue public bonds to finance its expenditure. It is of interest for our approach since heterogenous households are likely to be hit differently by the taxes needed to finance public debt and by the interest earned on public bonds. The median voter could thus be changed in this alternative scenario. On the side of the government, financing environmental maitenance to a lesser extent by taxes may be a way

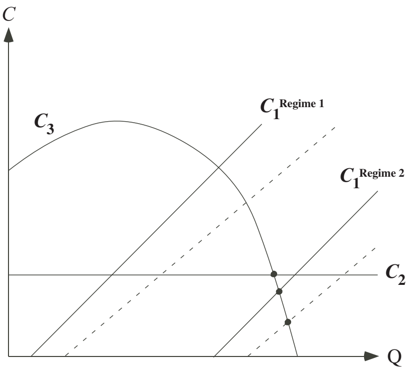

Figure 3: Impatient agents become less impatient

to increase acceptability of its abatement policy. Moreover, the introduction of public debt in our infinitely-lived agent model has no impact on the equilibrium steady state capital intensity, so that we can focus on its impact on environmental quality.

In this section we shall assume that the government can both raise the voted taxe τ_t and issue new one-period public bonds d_{t+1} to finance maintenance m_t. As a result, an additional expenses appears in its budget constraint, namely the repayment of interests and principal of public bonds. It is also assumed that public bonds and physical capital are perfect substitute and bear the same interest rate r_t.

Let $d_t \geq 0$ be the per capita public debt and $\tau_t \geq 0$ be the lump-sum tax at time t. These must satisfy the following government budget constraint:

$$\tau_t + d_{t+1} = m_t + (1 + r_t)d_t.$$

The budget constraint of a consumer (4) now becomes:

$$c_t + s_t + \tau_t \leq w_t + (1 + r_t)s_{t-1}, s_t \geq 0.$$

One can easily modify the definitions of competitive equilibrium path for this case. The only thing deserving attention is that condition 3 (equilibrium on the capital market) now becomes

$$\sum_{i=1}^{L} s_{t-1}^{i*} = L(k_t^* + d_t),\ t = 0, 1, \ldots.$$

Suppose that public debt is constant over time, $d_t = d,\ t = 0, 1, \ldots$. Then we can naturally define competitive steady-state equilibrium. Consider such an equilibrium, (m^*, E^{m^*}), where $E^{m^*} = \{k^*, 1 + r^*, w^*, (s^{i*}, c^{i*})_{i=1}^{L}, P^*, Q^*\}$. As in Mankiw (2000), government debt does not affect the steady-state capital stock and national income. Namely, as in the case with no government debt,

$$\beta^h = \frac{1}{1 + r^*},\ 1 + r^* = f'(k^*),\ w^* = f(k^*) - f'(k^*)k^*.$$

At the same time, government debt does influence the distribution of income. A higher level of debt means a higher level of taxation to pay for the interest payments on the debt. The taxes fall on both patient and impatient consumers, but the interest payments go entirely to the patient consumers because only patient consumers save in a steady-state equilibrium.

In the steady-state equilibrium the budget constraint of the government becomes

$$\tau_t + d = m^* + (1 + r^*)d.$$

Hence, $\tau_t = \tau(d),\ t = 0, 1, \ldots,$ where

$$\tau(d) = m^* + r^*d.$$

Therefore, the budget constraint of a consumer in the steady-state equilibrium is as follows:

$$c_t + s_t \leq w^* - \tau(d) + (1 + r^*)s_{t-1}, s_t \geq 0.$$

If the median voter is impatient, in a steady-state equilibrium we have $s^{i_m*} = 0$ and hence

$$c^{i_m*} + m^* = w^* - r^*d.$$

Therefore, for the median voter, an increase in d is practically equivalent to a decrease in the post-tax wage rate. It follows that in the case where maintenance is positive, $m^* > 0$, *if the majority of agents is impatient, an increase in public debt unambiguously leads to a decrease in maintenance and environmental quality in the voting steady-state equilibrium.*

If the median voter is patient, in a steady state the savings of the median voter, s^{i_m*}, are not determined uniquely and hence a change in d will have an indeterminate effect on the savings of the median voter. Let us assume that the ratio $s^{i_m*}/(\sum_{i=1}^{L} s^{i*})$ does not change. Since, in equilibrium, $(\sum_{i=1}^{L} s^{i*})/L = k^* + d$, this implies that the ratio $\gamma = s^{i_m*}/(k^* + d)$, which shows the proportion between the median and the mean savings, stays unchanged. Under this assumption, the parameter γ plays the crucial role, because in this case in a steady-state voting equilibrium we have

$$c^{i_m*} + m^* = w^* + r^*s^{i_m*} - r^*d = w^* + r^*(\gamma k^* + (\gamma - 1)d).$$

It is clear that an increase in d leads to a decrease in $c^{i_m*} + m^*$, if $\gamma < 1$, and to an increase in $c^{i_m*} + m^*$, if $\gamma > 1$.

Thus, in the case where maintenance is positive, $m^* > 0$, *if the median savings and income are lower than the mean ($\gamma < 1$), an increase in public debt leads to a decrease in maintenance and environmental quality and if the median savings and income are higher than the mean ($\gamma > 1$), an increase in public debt results in an increase in maintenance and environmental quality.* As noticed above, the case where the median savings and income are lower than the mean is usually considered as common.

7 Conclusion

In this paper, we assume that the population is exogenously divided into two groups: one with patient households and the other with impatient households. The environmental maintenance is voted by the households. We introduce the notion of voting equilibrium, look for steady state voting equilibria and find that for them the median voter theorem applies. If the majority of households is impatient the equilibrium levels of maintenance and environmental quality is determined uniquely, but if the majority of households is patient, there can be a continuum of these.

We fulfil comparative statics analysis for steady state voting equilibria and show that (i) an increase in total factor productivity may produce effects described by the Environmental Kuznets Curve, (ii) an increase in the patience of impatient households may improve the environmental quality if the median voter is impatient and maintenance positive, (iii) in the case where the median voter is patient and maintenance is positive, if the median income is lower than the mean, a decrease in inequality can lead to an increase in the environmental quality .

We also compare our model with a representative agent economy which is identified with the particular case of our model where all consumers are patient and savings are distributed evenly across agents. In the case of impatient median voter, the level of environmental quality predicted by our model is lower than the one predicted by a representative agent economy. The same holds true if the median voter is patient but the median income lower that the mean, which is the common case.

Finally, some policy implications of our model are discussed. In this purpose we introduce public debt as an alternative source of financing environmental maintenance. We show that, if the median income is lower than the mean, then an increase in public debt leads to a lower environmental quality in the long run.

8 References

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A Appendices

A.1 Proof of Proposition 1

It is sufficient to notice that since in a steady-state equilibrium we have

$$\mu\Phi(Q^*) = \mu(Q^* - P^*) + L\bar{m},$$

and for each i, the sequence

$$(\tilde{s}^i_{t-1}, \tilde{c}^i_t)^\infty_{t=0}$$

given by

$$\tilde{s}^i_{t-1} = s^{i*},\ \tilde{c}^i_t = c^{i*},$$

is a solution to

$$\max \sum_{t=0}^{\infty} \beta_i^t u(c_t),$$

$$c_t + s_t \leq (w^* - \bar{m}) + (1 + r^*)s_{t-1},$$
$$s^i_{-1} = s^{i*},$$
$$c_t \geq 0,\ \ s_t \geq 0,$$

and to refer to Becker (1980, 2006). □

A.2 Proof of Lemma 1

We have:

$$\frac{\partial V_{i,0}(s^{i*}, Q^*, \mathbf{m}^*)}{\partial m^*_0} = \frac{\partial \Lambda_{i,0}(Q^*, \mathbf{m}^*)}{\partial m^*_0} + \frac{\partial \Gamma_{i,t}(s^{i*}, \mathbf{m}^*)}{\partial m^*_0},$$

where the functions $\Lambda_{i,0}$ and $\Gamma_{i,0}$ are defined as follows:

$$\Lambda_{i,0}(Q_0, \mathbf{m}^*) = \max_{(Q_t)_{t=1}^{\infty}} \{\sum_{t=0}^{\infty} \beta_i^t v(Q_t) \mid \mu\Phi(Q_{t+1}) \leq \mu(Q_t - P^*) + Lm^*_t,\ Q_{t+1} \geq 0,\ t = 0, 1, \ldots\},$$

$$\Gamma_{i,0}(s_{-1}, \mathbf{m}^*) = \max_{(c_t)_{t=0}^{\infty}, (s_t)_{t=0}^{\infty}} \{\sum_{t=0}^{\infty} \beta_i^t u(c_t) \mid c_t + s_t + m^*_t \leq w^* + (1 + r^*)s_{t-1},\ c_t \geq 0,\ s_t \geq 0,\ t = 0, 1, \ldots\}.$$

It is not difficult to check that

$$\frac{\partial \Lambda_{i,0}(Q^*, \mathbf{m}^*_t)}{\partial m^*_t} = \beta_i \frac{Lv'(Q^*)}{\mu(\Phi'(Q^*) - \beta_i)}$$

and

$$\frac{\partial \Gamma_{i,0}(s^{i*}, \mathbf{m}^*_t)}{\partial m^*_t} = -u'(c^*).$$

Therefore,

$$\frac{\partial V_{i,0}(s^{i*}, Q^*, \mathbf{m}^*)}{\partial m^*_t} = \beta_i \frac{Lv'(Q^*)}{\mu(\Phi'(Q^*) - \beta_i)} - u'(c^*),$$

which implies (21). □

A.3 Proof of Lemma 2

Using a traditional argument (see e.g. McKenzie (1986)) we can prove that a sequence $(\tilde{s}_{t-1}, \tilde{c}_t, \tilde{m}_t, \tilde{Q}_t)_{t=0}^{\infty}$ given by (22) is a steady-state solution to problem $\mathcal{P}_2$ if and only if there exist q and p such that for

$$p_t = \beta_i p_{t-1} = ... = \beta_i^t p,$$

$$q_{t+1} = \beta_i q_t = ... = \beta_i^{t+1} q.$$

the following relationships hold:

$$\beta_i^t u'(\tilde{c}_t) = p_t,$$

$$\beta_i^t v'(\tilde{Q}_t) + q_{t+1}\mu - q_t \mu \Phi'(\tilde{Q}_t) = 0,$$

$$(1 + r^*) p_t \leq p_{t-1} (= \text{if } \tilde{s}_{t-1} > 0),$$

$$q_{t+1} L - p_t \geq 0 \ (= \ \text{if } \tilde{m}_t > 0),$$

$$q_{t+1} \tilde{Q}_t + p_t \tilde{s}_{t-1} \to_{t \to \infty} 0,$$

or, equivalently,

$$u'(\tilde{c}) = p,$$

$$v'(\tilde{Q}) = \mu q (\Phi'(\tilde{Q}) - \beta_i),$$

$$\beta_i \leq \frac{1}{1 + r^*} \ (= \text{if } \tilde{s} > 0),$$

$$\beta_i L q - p \geq 0 \ (= \ \text{if } \tilde{m} > 0).$$

The existence of such q and p is equivalent to conditions (23)-(24). □