EUROPEAN UNIVERSITY AT SAINT-PETERSBURG

Sergey Ivashchenko

DSGE model estimation on base of second order approximation

Working paper Ec-07/11

Department of Economics

St. Petersburg
2011

УДК 330.43
ББК 65.01
I98

Европейский университет в Санкт-Петербурге

Сергей Иващенко

Оценка модели DSGE на основе приближения второго порядка

на английском языке

Серия препринтов; Ec-07/11; Факультет экономики
Санкт-Петербург, 2011

Ivashchenko S.

I98 DSGE model estimation on base of second order approximation / Sergey Ivashchenko : Working paper Ec-07/11; Department of Economics. — St. Petersburg : European University at St. Petersburg, 2011. — 16 p.

This article compares properties of different non-linear Kalman filters: well-known Unscented Kalman filter (UKF), Central Difference Kalman Filter (CDKF) and unknown Quadratic Kalman filter (QKF). Small financial DSGE model is repeatedly estimated by maximum quasi-likelihood methods with different filters for data generated by the model. Errors of parameters estimation are measure of filters quality. The result is that QKF has reasonable advantage in quality over CDKF and UKF with some loose in speed.

Keywords: DSGE; QKF; CDKF; UKF; quadratic approximation; Kalman filtering

JEL classification: C13, C32, E32

Sergey Ivashchenko, Saint-Petersburg State Polytechnical University, Russia, 195251, St. Petersburg, Polytechnicheskaya, 29; glucke_ru@pisem.net

DSGE model estimation on base of second order approximation

Sergey Ivashchenko

Saint-Petersburg State Polytechnical University, Russia, 195251, St. Petersburg, Polytechnicheskaya, 29; glucke_ru@pisem.net

1. Introduction

Estimation of DSGE models is very important issue. Usage of DSGE models requires knowledge about its behavior which depends on parameters values. There are different econometric techniques for models estimation but empirical literature has concentrated its attention on the estimation of first-order linearized DSGE models (Tovar (2008)).

Computation with linear approximation is much faster than higher order approximation but its behavior could differ from the models behavior (see Collard and Juillard (2001)). Second order approximation makes difference between models and approximation behavior much smaller. Asset pricing is the issue where first order approximation isn't applicable because it eliminates all risk premiums (see Tovar (2008)). That is why important to estimate DSGE models on base of second order (or higher) approximation.

Advantages of second order estimation are known. The particle filter is tool for likelihood function construction on base of nonlinear DSGE models approximations (An and Schorfheide (2006)). There are alternative filters that could be used for likelihood calculation. For example, Andreasen (2008) has shown advantage of Central Difference Kalman Filter over few versions of particle filter.

There are other nonlinear modifications of Kalman filter which could be used for DSGE models second order estimation. At this article 3 versions are used: Central Difference Kalman Filter (CDKF, see Norgaard, Poulsen and Ravn, (2000)), Unscented Kalman filter (UKF, see Julier and Uhlmann (1997)) and Quadratic Kalman filter (QKF). QKF description wasn't found at literature that is why it would be described without references. The purpose of this article is to compare different versions of second order DSGE models estimation technique.

2. Nonlinear Kalman filters

2.1 Generality

Equation (1) describes data generating process for state variables (X_t). Exogenous shocks (ε_t) have normal distribution with covariance matrix Ω_ε and mean equal zero. Measurement equation (2) describes dependence of observed variables (Y_t) on state variables and measurement errors (u_t) which have normal distribution with zero mean and covariance matrix Ω_u.

(1)
$$X_t = H\left(X_{t-1};\varepsilon_t\right) = C + \begin{bmatrix} B_X & B_\varepsilon \end{bmatrix} \begin{bmatrix} X_{t-1} \\ \varepsilon_t \end{bmatrix} + \begin{bmatrix} A_{xx} & A_{x\varepsilon} & 0 & A_{\varepsilon\varepsilon} \end{bmatrix} \begin{bmatrix} X_{t-1} \otimes X_{t-1} \\ X_{t-1} \otimes \varepsilon_t \\ \varepsilon_t \otimes X_{t-1} \\ \varepsilon_t \otimes \varepsilon_t \end{bmatrix}$$

(2)
$$Y_t = DX_t + u_t$$

Initialization:

(3)
$$\mu_{0,1} = E_0 X_1$$

(4) $$\Omega_{0,1} = E_0\left(X_1 - \mu\right)\left(X_1 - \mu\right)'$$

All variants of nonlinear Kalman filters have the same initialization procedure. Mean and covariance matrix of state variables unconditional distribution are calculated according to linear approximation of (1).

Updating:

(5) $$\mu_{t,t} = \mu_{t-1,t} + \Omega_{t-1,t} D\left(D\Omega_{t-1,t}D' + \Omega_u\right)^{-1}\left(Y_t - D\mu_{t-1,t}\right)$$

(6) $$\begin{aligned}\Omega_{t,t} &= \left(I - \Omega_{t-1,t} D\left(D\Omega_{t-1,t}D' + \Omega_u\right)^{-1} D\right)\\ &\times\Omega_{t-1,t}\left(I - \Omega_{t-1,t} D\left(D\Omega_{t-1,t}D' + \Omega_u\right)^{-1} D\right)'\end{aligned}$$

Updating step is the same for all variants of nonlinear Kalman filters because measurement equation (2) is linear. It is similar with Kalman filter (usual linear Kalman filter). Symbol θ_t (it would be used at following text) denotes join vector of state variables and exogenous shocks without its mean (7):

(7) $$\theta_t = \begin{bmatrix} X_{t-1} \\ \varepsilon_t \end{bmatrix} - E_{t-1}\begin{bmatrix} X_{t-1} \\ \varepsilon_t \end{bmatrix}$$

(8) $$E_{t-1}\left(\theta_t \theta_t'\right) = \begin{bmatrix} \Omega_{t-1,t-1} & 0 \\ 0 & \Omega_\varepsilon \end{bmatrix} = VV'$$

2.2 Unscented Kalman filter (UKF)

Prediction step is different across filters:

(9) $$\theta_{i,t} = \begin{cases} -\sqrt{n+\lambda}V_{-i} & i = -n : -1 \\ 0 & i = 0 \\ \sqrt{n+\lambda}V_i & i = 1 : n \end{cases}$$

Matrix V is square root of θ_t covariance (length of θ_t is n). Vector V_i is i-th column of matrix V. So, $\theta_{i,t}$ are sets of vectors around θ_t mean with dispersion dependent on θ_t covariance. For normal distribution parameters of UKF are $\alpha=10^{-3}$, $\beta=2$, $\lambda=\alpha^2(n+1)-n$.

(10)
$$X_{i,t} = C + B\left(\theta_{i,t} + \begin{bmatrix} \mu_{t-1,t-1} \\ 0 \end{bmatrix}\right) + A\left(\theta_{i,t} + \begin{bmatrix} \mu_{t-1,t-1} \\ 0 \end{bmatrix}\right) \otimes \left(\theta_{i,t} + \begin{bmatrix} \mu_{t-1,t-1} \\ 0 \end{bmatrix}\right)$$

(11)
$$\mu_{t-1,t} = \sum_{\substack{i=-n \\ i\neq 0}}^{n} \frac{X_{i,t}}{2(n+\lambda)} + \left(\frac{\lambda}{n+\lambda}\right) X_{0,t}$$

(12)
$$\Omega_{t-1,t} = \sum_{\substack{i=-n \\ i\neq 0}}^{n} \frac{\left(X_{i,t} - \mu_{t-1,t}\right)\left(X_{i,t} - \mu_{t-1,t}\right)'}{2(n+\lambda)} + \begin{pmatrix} 1-\alpha^2+\beta+ \\ +\lambda/(n+\lambda) \end{pmatrix} \left(X_{0,t} - \mu_{t-1,t}\right)\left(X_{0,t} - \mu_{t-1,t}\right)'$$

Thus, UKF sets expected state variables value equal to the weighted sum of deterministic generated trajectories (10-11). Covariance is set equal to weighted sum of deterministic generated trajectories divergence from expected value (12). Full description of UKF could be found at Julier and Uhlmann (1997).

2.3 Central Difference Kalman Filter (CDKF)

Similarly to UKF, $\theta_{i,t}$ are sets of vectors around θ_t mean with dispersion dependent on θ_t covariance. For normal distribution parameter of CDKF is $h^2=3$.

(13) $$\theta_{i,t} = \begin{cases} -hV_{-i} & i=-n:-1 \\ 0 & i=0 \\ hV_i & i=1:n \end{cases}$$

(14) $$\begin{aligned} X_{i,t} &= C + B\left(\theta_{i,t} + \begin{bmatrix}\mu_{t-1,t-1} \\ 0\end{bmatrix}\right) \\ &+ A\left(\theta_{i,t} + \begin{bmatrix}\mu_{t-1,t-1} \\ 0\end{bmatrix}\right) \otimes \left(\theta_{i,t} + \begin{bmatrix}\mu_{t-1,t-1} \\ 0\end{bmatrix}\right) \end{aligned}$$

CDKF is based on approximation (15) which is each dimension second order finite difference approximation of function (1):

(15) $$\begin{aligned} X_t &\approx X_{0,t} + \frac{1}{2h}\left[X_{1,t} - X_{-1,t} \quad \dots \quad X_{n,t} - X_{-n,t}\right]\theta_t + \\ &+ \frac{1}{2h^2}\left[X_{1,t} + X_{-1,t} - 2X_{0,t} \quad \dots \quad X_{n,t} + X_{-n,t} - 2X_{0,t}\right]\theta_t^{\bullet 2} = \\ &= X_{0,t} + \frac{1}{2h}\hat{B}\theta_t + \frac{1}{2h^2}\hat{A}\theta_t^{\bullet 2} \end{aligned}$$

It should be noted that second order approximation includes component (16):

(16) $$\frac{1}{8h^2}\sum_{i=1}^{n}\sum_{\substack{j=1 \\ j\neq i}}^{n}\{\theta_t\}_i\{\theta_t\}_j\left(X_{i,t} - X_{-i,t}\right)\left(X_{j,t} - X_{-j,t}\right),$$

where $\{\theta_t\}_i$ is i-th element of vector θ_t and $\theta_t^{\bullet 2}$ is vector of $\left(\{\theta_t\}_i\right)^2$, but its expected value is zero and its influence on covariance of X_t depends on third and fourth moments of θ_t distribution.

(17) $$\begin{aligned} \mu_{t-1,t} &= X_{0,t} + \frac{1}{2h^2}\sum_{i=1}^{n}\left(X_{i,t} + X_{-i,t} - 2X_{0,t}\right) = \\ &= \sum_{\substack{i=-n \\ i\neq 0}}^{n}\frac{X_{i,t}}{2h^2} + \left(\frac{h^2 - n}{h^2}\right)X_{0,t} \end{aligned}$$

(18) $$\Omega_{t-1,t} = \frac{1}{4h^2}\hat{B}\hat{B}' + \frac{h^2-1}{4h^2}\hat{A}\hat{A}'$$

Mean and covariance are set according to (17)-(18). Full description of CDKF could be found at Norgaard, Poulsen and Ravn, (2000).

2.4 Quadratic Kalman filter (QKF)

The first step of QKF is transformation of (1) into (19) which is dependence of state variables X_t on vector θ_t.

(19)
$$\begin{aligned}
X_t &= C + B\left(\theta_t + \begin{bmatrix}\mu_{t-1,t-1}\\ 0\end{bmatrix}\right) + A\left(\theta_t + \begin{bmatrix}\mu_{t-1,t-1}\\ 0\end{bmatrix}\right)\otimes\left(\theta_t + \begin{bmatrix}\mu_{t-1,t-1}\\ 0\end{bmatrix}\right) = \\
&= C + B\begin{bmatrix}\mu_{t-1,t}\\ 0\end{bmatrix} + A\begin{bmatrix}\mu_{t-1,t}\\ 0\end{bmatrix}\otimes\begin{bmatrix}\mu_{t-1,t}\\ 0\end{bmatrix} + \\
&+\left(B + A\left(I_n \otimes \begin{bmatrix}\mu_{t-1,t-1}\\ 0\end{bmatrix} + \begin{bmatrix}\mu_{t-1,t-1}\\ 0\end{bmatrix}\otimes I_n\right)\right)\theta_t + A\theta_t\otimes\theta_t = \\
&= \bar{C} + \bar{B}\theta_t + \bar{A}\theta_t\otimes\theta_t
\end{aligned}$$

After that expected value of X_t could be find easily. Formula (20) shows result which is based on zero expected value of θ_t.

(20) $$\mu_{t-1,t} = \bar{C} + Avec(\Omega)$$

The last step is calculation of X_t covariance matrix. The formula (21) expresses true value of X_t covariance matrix if vector θ_t has normal distribution. It is based on following properties of normal distribution: third-order central moments is zero, fourth-order central moments is function of covariance matrix (22).

(21)
$$\begin{aligned}&vec\left(\Omega_{t-1,t}\right)=E_{t-1}\left(X_t-\mu_{t-1,t}\right)\otimes\left(X_t-\mu_{t-1,t}\right)=E_{t-1}\left(\bar{B}\theta_t\right)\otimes\left(\bar{B}\theta_t\right)+\\&+E_{t-1}\left(A\left(\theta_t\otimes\theta_t-E_t\theta_t\otimes\theta_t\right)\right)\otimes\left(A\left(\theta_t\otimes\theta_t-E_t\theta_t\otimes\theta_t\right)\right)=\\&=\left(B\otimes B\right)vec\left(\Omega\right)+\left(A\otimes A\right)\left(vec\left(\Omega\otimes\Omega\right)+vec\left(vec\left(\Omega\right)\otimes\Omega\right)\right)\end{aligned}$$

(22)
$$\begin{aligned}E_{t-1}\left(\theta_t\otimes\theta_t\otimes\theta_t\otimes\theta_t\right)=vec\left(\Omega\otimes\Omega\right)+vec\left(vec\left(\Omega\right)\otimes\Omega\right)\\+vec\left(\Omega\right)\otimes vec(\Omega)\end{aligned}$$

It should be noted that true distribution of θ_t is differ from normal (even if ε_t is normal distributed) because function (19) isn't linear. Even if θ_t is normal then distribution of X_t and θ_{t+1} wouldn't be normal. It means that QKF is approximation for calculation of conditional moments (as UKF and CDKF). The quality of QKF, CDKF and UKF would be compared by maximum quasi-likelihood estimation of small DSGE model.

3. DSGE model

Finance is one of areas where DSGE models linear approximation is unsuitable. That is why finance model is used for comparison of different Kalman filters. Householders maximize expected utility function (23) with budget constraint (24). Budget constraint talks that householders spent money for consumption (C_t) with exogenous price ($Z_{P,t}$), bonds (B_t) and stocks (X_t) which price is S_t. Sources of money are exogenous income, bonds and stocks bought in previous period.

(23)
$$E_0\sum_{t=0}^{\infty}\beta^t\frac{\left(C_t\right)^{\gamma}}{\gamma}\rightarrow\max_{C;B;X}$$

(24)
$$Z_{P,t}C_t+B_t+X_tS_t=R_{t-1}B_{t-1}+X_{t-1}\left(S_t+D_t\right)+S_tZ_{I,t}$$

The model suggests that dividend growth is exogenous (25), bond amount is set by government exogenous (26), amount of stocks is equal to 1 (27).

(25) $$\frac{D_t}{D_{t-1}} = Z_{D,t}$$

(26) $$B_t = Z_{B,t} S_t$$

(27) $$X_t = 1$$

The model (23)-(27) is transformed to (28)-(32) where stable variables are used. Table 1 shows dependence between initial and stable variables.

(28) $$E_0 \sum_{t=0}^{\infty} \beta^t e^{\gamma \sum_{i=0}^{t} (s_i - z_{P,i})} \frac{\left(e^{c_t}\right)^{\gamma}}{\gamma} \to \max_{c;b;x}$$

(29) $$e^{c_t} + b_t + x_t = e^{r_{t-1} - s_t} b_{t-1} + x_{t-1}\left(1 + e^{d_t}\right) + z_{I,t}$$

(30) $$d_t - d_{t-1} + s_t = z_{D,t}$$

(31) $$b_t = z_{B,t}$$

(32) $$x_t = 1$$

TABLE 1

DSGE model variables

Variable	Description	Stationary variable
B_t	value of bonds bought by householders at period t	$b_t = B_t / S_t$
C_t	consumption at time t	$c_t = \ln\left(Z_{P,t} C_t / S_t\right)$
D_t	dividends at time t	$d_t = \ln\left(D_t / S_t\right)$
R_t	interest rate at time t	$r_t = \ln\left(R_t\right)$
S_t	price of stocks at time t	$s_t = \ln\left(S_t / S_{t-1}\right)$

Variable	Description	Stationary variable
X_t	amount of stocks bought by householders at period t	$x_t = X_t$
Λ_t	Lagrange multiplier corresponding to budget restriction of householders at period t	$\lambda_t = \Lambda_t$
$Z_{A,B,t}$	exogenous process corresponding to near-rationality of householders with its bond position	$z_{A,B,t} = Z_{A,B,t}$
$Z_{A,C,t}$	exogenous process corresponding to near-rationality of householders with its consumption	$z_{A,C,t} = Z_{A,C,t}$
$Z_{A,S,t}$	exogenous process corresponding to near-rationality of householders with its stocks position	$z_{A,C,t} = Z_{A,C,t}$
$Z_{B,t}$	exogenous process corresponding to bond amount sell by government	$z_{B,t} = Z_{B,t}$
$Z_{D,t}$	exogenous process corresponding to dividends growth	$z_{D,t} = Z_{D,t}$
$Z_{I,t}$	exogenous process corresponding to householders income	$z_{T,t} = Z_{T,t}$
$Z_{P,t}$	exogenous process corresponding to price level	$z_{P,t} = \ln\left(Z_{P,t}/Z_{P,t-1}\right)$

The optimal conditions of (28)-(29) problems with additional exogenous process ($z_{A,S,t}$, $z_{A,B,t}$, $z_{A,C,t}$) are following:

(33) $$e^{\lambda_t + z_{A,S,t}} = E_t e^{\lambda_{t+1} + \ln(\beta) + \gamma\left(s_{t+1} - z_{P,t+1}\right)}\left(1 + e^{d_{t+1}}\right)$$

(34) $$e^{\lambda_t + z_{A,B,t}} = E_t e^{\lambda_{t+1} + r_t - s_{t+1} + \ln(\beta) + \gamma\left(s_{t+1} - z_{P,t+1}\right)}$$

(35) $$\gamma c_t = \lambda_t + c_t + z_{A,C,t}$$

Additional exogenous process could be interpreted as near-rational householders (this processes have zero mean). Another interpretation is compensation of approximation errors (this processes allows to use linear approximation for parameter estimation). All exogenous processes are AR(1) with following parameterization:

(36) $$z_{*,t} = \eta_{0,*,t}(1 - \eta_{1,*,t}) + \eta_{1,*,t} z_{*,t-1} + \varepsilon_{*,t}$$

The model parameters are estimated by maximum quasi-likelihood with QKF method. The monthly data (Average rate on 1-month certificates of deposit, MSCI USA price return, MSCI USA gross return) from December 1969 till December 2010 are used. The estimated values are used for observations generating by the model.

4. Results

The following procedure is used for comparison of different Kalman filters:

1. Generation of 400 observations from second order approximation of model.
2. Parameters estimation by quasi-maximum likelihood method based on second order approximation with different Kalman filters (QKF, CDKF, UKF). The true values of parameters are used as initial one.
3. Parameters estimation by quasi-maximum likelihood method based on first order approximation. The true values of parameters are used as initial one.
4. Repeating 100 times of steps 1-3

Results presented at table 2. First order approximation produces the worst quality of parameters estimators. Only estimation of $\varepsilon_{A,C}$ standard deviation is better than for second order approximation. Trace of estimator errors is 32%-38% higher than for UKF and CDKF. But line approximation is much faster than quadratic (time for likelihood calculation is 49.5 times smaller than for UKF and CDKF). These results are expectable. They are showed as benchmark for quadratic approximations. It should be noted that dynare is used for line approximation. Self-made code on base of dynare is used for second order approximation. It means that line code is efficient and code for second order could be optimized.

TABLE 2

Results of filters comparison.

RMSE	QKF	UKF	CDKF	Line
st. dev. $\varepsilon_{A,B}$	2.90*10^{-05}	5.46*10^{-05}	4.98*10^{-05}	6.33*10^{-05}
st. dev. $\varepsilon_{A,C}$	6.76*10^{-03}	2.66*10^{-02}	2.68*10^{-02}	5.26*10^{-03}
st. dev. $\varepsilon_{A,S}$	3.09*10^{-04}	3.86*10^{-04}	3.32*10^{-04}	1.34*10^{-03}
st. dev. ε_{B}	1.03*10^{-02}	1.09*10^{-02}	1.09*10^{-02}	5.22*10^{-01}
st. dev. ε_{D}	1.91*10^{-03}	2.50*10^{-03}	2.39*10^{-03}	3.96*10^{-03}
st. dev. ε_{I}	4.52*10^{-03}	8.41*10^{-02}	7.02*10^{-02}	4.21*10^{+00}
st. dev. ε_{P}	2.73*10^{-04}	2.37*10^{-04}	2.10*10^{-04}	8.30*10^{-04}
ln(β)	3.14*10^{-04}	3.53*10^{-04}	3.59*10^{-04}	3.34*10^{-04}
γ	1.46*10^{-03}	2.86*10^{-03}	2.14*10^{-03}	1.36*10^{-02}
$\eta_{0,B}$	4.09*10^{+00}	7.61*10^{+00}	7.58*10^{+00}	7.22*10^{+00}
$\eta_{0,D}$	1.74*10^{-04}	1.23*10^{-04}	1.16*10^{-04}	4.47*10^{-04}
$\eta_{0,I}$	4.37*10^{+00}	4.08*10^{+00}	4.63*10^{+00}	5.67*10^{+00}
$\eta_{0,P}$	6.80*10^{-05}	7.64*10^{-05}	7.07*10^{-05}	3.76*10^{-04}
$\eta_{1,AB}$	1.03*10^{-02}	1.01*10^{-02}	1.07*10^{-02}	1.12*10^{-02}
$\eta_{1,AC}$	8.45*10^{-05}	2.69*10^{-01}	3.03*10^{-01}	1.01*10^{-03}
$\eta_{1,AS}$	1.58*10^{-02}	1.25*10^{-02}	1.00*10^{-02}	4.00*10^{-02}
$\eta_{1,B}$	6.53*10^{-01}	8.06*10^{-01}	8.92*10^{-01}	7.04*10^{-01}
$\eta_{1,D}$	2.32*10^{-01}	2.39*10^{-01}	2.30*10^{-01}	3.58*10^{-01}
$\eta_{1,I}$	7.98*10^{-01}	9.31*10^{-01}	9.03*10^{-01}	1.15*10^{+00}
$\eta_{1,P}$	8.88*10^{-01}	9.99*10^{-01}	1.18*10^{+00}	9.75*10^{-01}
Trace of parameters estimators covariance matrix	2.61*10^{+01}	4.45*10^{+01}	4.66*10^{+01}	6.16*10^{+01}
Trace of square parameters estimators matrix (sum of MSE)	3.77*10^{+01}	7.72*10^{+01}	8.20*10^{+01}	1.05*10^{+02}
Time for likelihood calculation(sec)*	4.14	2.97	2.97	0.06
Number of parameters were QKF is better		15	16	19

*PC used: Intel core 2 Duo E8400 3 GHz, 1 Gb RAM, Windows XP.

The main result is that QKF is slower but better then CDKF and UKF. The loose in speed is 28%. The gain in quality (trace) is 70%-78%. Why does it happen? QKF uses analytical formulas for mean and variance calculation while CDKF and UKF use approximation. Approximation is less accurate than analytical formulas. CDKF and UKF require many calculation of function (1) while QKF require smaller number of more complicated functions calculations.

What are disadvantages of QKF? QKF is based on normality of variable distribution. If distribution of shocks or variables would be far from normal QKF performance would be worse. CDKF and UKF could be modified for non-normality easily. But usual DSGE model uses normal distribution for shocks and its variables are close to normal distribution. Another disadvantage of QKF is that QKF is based on second order approximation. CDKF and UKF could be used for quasi-likelihood calculations with higher order approximation.

5. Conclusion

This article describes three nonlinear Kalman filters. Those performances are compared on small financial DSGE model. Well known CDKF and UKF quality is similar (32%-38% smaller errors but 49.5 times slower then linear approximation). QKF performance is even better (136% smaller errors and 69 times slower then linear approximation). The gain of QKF over CDKF and UKF (70%-78% in quality with 28% loose in speed and) is result of specialization for common DSGE models (normal shocks and second order approximation).

References

An S. and F. Schorfheide (2006): “Bayesian analysis of DSGE models”, Working Papers from Federal Reserve Bank of Philadelphia No 06-5

Andreasen, M. M. (2008): “Non-Linear DSGE Models, the Central Difference Kalman Filter, and the Mean Shifted Particle Filter,” CREATES Research Paper 2008-33. Available at SSRN: http://ssrn.com/abstract=1148079

Collard F.and M. Juillard (2001): “Accuracy of stochastic perturbation methods: The case of asset pricing models”, Journal of Economic Dynamics and Control, 25 (6-7), 979-999.

Julier S. J. and J. K. Uhlmann (1997): “A new extension of the Kalman filter to nonlinear systems,” in Proc. AeroSense: 11th Int. Symp. Aerospace/Defense Sensing, Simulation and Controls, pp. 182–193.

Norgaard, M., N. K. Poulsen and O. Ravn, (2000): “New developments in state estimation for nonlinear systems”, Automatica, 36 (11), 1627-1638.

Tovar C. E. (2008): “DSGE models and central banks,” BIS Working Papers from Bank for International Settlements. No 258.

Иващенко С.М. Оценка модели DSGE на основе приближения второго порядка. — СПб. : Европейский университет в Санкт-Петербурге, 2011. — 16 с. — (Серия препринтов; факультет экономики; Ec-07/11). — На английском языке.

Отпечатано с оригинал-макета, предоставленного авторами

Подписано в печать 20.09.2011

Формат 60x88 1/6. Тираж 50 экз.

Факультет экономики

Европейский университет в Санкт-Петербурге

191187, Санкт-Петербург, ул. Гагаринская, д. 3

http://www.eu.spb.ru/education/departments/econ

econ@eu.spb.ru