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Устойчивый рост в модели с двухставочным дисконтированием
на английском языке

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In an important model of growth and pollution proposed by Stokey [Int. Econ. Rev. 39 (1998) 1] neither the rate of economic growth nor the rate of growth of emissions depends on the time preference of the representative agent, which seems somewhat paradoxical. To resolve this paradox, we introduce into Stokey's model the assumption of dual-rate discounting, prove the existence of a sustainable balanced growth optimal path, and show that the growth rates of output and emissions are increasing in the proportion between the consumption and the environmental discount factors of the representative agent.

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Sustainable Growth in a Model with Dual-Rate Discounting

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Abstract

In an important model of growth and pollution proposed by Stokey [Int. Econ. Rev. 39 (1998) 1] neither the rate of economic growth nor the rate of growth of emissions depends on the time preference of the representative agent, which seems somewhat paradoxical. To resolve this paradox, we introduce into Stokey's model the assumption of dual-rate discounting, prove the existence of a sustainable balanced growth optimal path, and show that the growth rates of output and emissions are increasing in the proportion between the consumption and the environmental discount factors of the representative agent.

Keywords: Growth; Pollution; Discounting
JEL classification: O44; C61

1 Introduction

In an important paper by Stokey (1998), a series of simple growth and pollution models to investigate the links between the limits to growth and industrial pollution was presented. In Stokey's analysis an optimizing representative agent determines saving and abatement decisions.

By adopting a CRRA utility function, she shows that along the balanced growth optimal path emissions fall if and only if the elasticity of marginal utility with respect to consumption exceeds one. Neither the rate of economic growth

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nor the rate of growth of emissions depends on the time preference of the representative agent. Even the conditions of sustainability do not depend on the discount factor.

This property of Stokey's model is somewhat paradoxical in the light of the fact that discounting plays a crucial role in the economic-environmental modeling with long time horizons, and the present debate on discounting environmental benefits and costs is centered on the inconsistency of discounting with the philosophy of sustainability. To resolve this paradox, we introduce into Stokey's model the assumption of dual-rate discounting as applied by Yang (2003).

According to the dual-rate discounting approach people have two rates of pure time preference – a consumption discount rate used to discount utility from goods consumption and an environmental discount rate used to discount utility from environmental quality (or disutility from pollution). The environmental discount rate is supposed to be lower than the consumption one. The idea of an environmental discount rate that is different from the consumption discount rate is also explored by Hasselman et al. (1997), Horowitz (2002) and Almansa Sáez and Calatrava Requena (2007).

It should be said that some authors (Tol (2005) and Weikard and Zhu (2005)) see no reason why rates of pure time preference should be different for different types of goods. They try to show formally that for purposes of cost-benefit analysis dual-rate discounting and the classical approach are equivalent.

We are not going to join the debate, but notice that in a framework of a growth model with a representative agents the environmental discount rate reflects social tastes, whereas the consumption discount rate reflects personal tastes. For ethical reasons, people may really prefer the use of lower discount rates to evaluate societal goals and objectives, even while possessing a personal high time preference rate. There is some empirical evidence indicating that the consumption discount rate and the environmental discount rate are different. For example, Lumeley (1997) reveals that Philippines upland farmers may be adopting soil conservation for ethical reasons in spite of their high individual discount rates. Taylor et al. (2003) found out that implicit discount rates are different for forest benefits of distinct nature, namely timber and recreation. Using an experimental methodology, Gintis (2000) and Farrugia (2010) arrive at the conclusion that the intertemporal preferences of individuals vary between private and environmental projects.

We consider a discrete time version of Stokey's model with dual-rate discounting. We prove the existence of a sustainable balanced growth optimal path and show that the long-run rate of economic growth is increasing in the proportion between the consumption and the environmental discount factors, whereas the rate at which pollution decreases is decreasing in this proportion.

2 The model

Potential output F_t at time t is given by the Cobb-Douglas production function:

$$F_t = A_t K_t^{\alpha},\ 0 < \alpha < 1,$$

where K_t is the stock of fully depreciating capital and A_t the total factor productivity growing at an exogenously given rate $g > 0$:

$$A_t = (1+g)^t A_0.$$

Potential output F_t and final output Y_t available for consumption and investment are related by

$$Y_t = z_t F_t,\ 0 \le z_t \le 1, \tag{1}$$

where z_t is the technique of production. It is determined endogenously. Aggregate emissions X_t are given by

$$X_t = z_t^{\rho} F_t,\ \rho > 1. \tag{2}$$

Output Y_t is available for consumption C_t and the stock of capital at next time:

$$C_t + K_{t+1} = Y_t.$$

The objective function of the central planner is

$$\sum_{t=0}^{\infty} \left[\beta_1^t u(C_t) - \beta_2^t v(X_t)\right],$$

where

$$\beta_2 > \beta_1,$$
$$v(X) := X^{\gamma}/\gamma,\ \gamma > 1;$$
$$u(C) := C^{1-\sigma}/(1-\sigma),\ 0 < \sigma \ne 1.$$

It is convenient to eliminate z_t and F_t. To do this we can rewrite (1)-(2) as follows:

$$X_t = \Phi(Y_t, K_t, A_t),$$
$$Y_t \le A_t K_t^{\alpha},$$

where

$$\Phi(Y, K, A) := Y^{\rho} K^{\alpha(1-\rho)} A^{1-\rho}.$$

Thus, given $K_0 > 0$, the social planner's problem is

$$\max \sum_{t=0}^{\infty} \left[\beta_1^t u(C_t) - \beta_2^t v(X_t)\right], \tag{3}$$

s.t.

$$K_{t+1} + C_t = Y_t,\ t = 0, 1, ..., \tag{4}$$

$$X_t = \Phi(Y_t, K_t, A_t),\ t = 0, 1, ..., \tag{5}$$

$$Y_t \leq A_t K_t^{\alpha},\ t = 0, 1, ... \tag{6}$$

Let $(K_t^*, Y_t^*, C_t^*, X_t^*)_{t=0}^{\infty}$ be a solution to the problem (3)-(6) at $K_0 = K_0^*$. This solution is called a balanced optimal path if for some $g_K > -1$ and $g_X > -1$ and for all $t = 0, 1, ...,$

$$K_{t+1}^* = (1 + g_K)K_t^*,\ Y_{t+1}^* = (1 + g_K)Y_t^*,\ C_{t+1}^* = (1 + g_K)C_t^*,$$
$$X_{t+1}^* = (1 + g_X)X_t^*. \tag{7}$$

If, moreover, $g_K > 0$ and $g_X \leq 0$, it is called a *sustainable balanced optimal path.*

Let

$$\kappa_1 := \gamma(\rho - 1) > 0,\ \kappa_2 := \gamma + \sigma - 1 > 0.$$

Now we can prove our main result. The following theorem reads that under some conditions on the relationship between the two discount factors a sustainable balanced optimal path exists, the long-run rate of economic growth is increasing in the proportion between the consumption and the environmental discount factors, and the rate at which emissions decrease is decreasing in this proportion.

Theorem. *Suppose that*

$$\beta_2/\beta_1 < (1 + g)^{\kappa_1}.$$

If either

$$0 < \sigma < 1 \text{ and } (1 + g)^{\kappa_1} \leq (\beta_2/\beta_1)^{\frac{\gamma[\rho(1-\alpha)+\alpha]}{1-\sigma}},$$

or

$$\sigma > 1,$$

then for any sufficiently high A_0 a sustainable balanced optimal path exists. On this path,

$$g_K = (\beta_1/\beta_2)^{\frac{1}{(1-\alpha)\kappa_1+\kappa_2}}(1 + g)^{\frac{\kappa_1}{(1-\alpha)\kappa_1+\kappa_2}} - 1,$$

$$g_X = (\beta_1/\beta_2)^{\frac{\rho-\alpha(\rho-1)}{(1-\alpha)\kappa_1+\kappa_2}}(1 + g)^{\frac{\kappa_1-(\rho-1)\kappa_2}{(1-\alpha)\kappa_1+\kappa_2}} - 1.$$

Proof. For a sustainable balanced optimal path, constraint (6) is not binding. Therefore, to prove the theorem it is sufficient to show that for any $A_0 > 0$ there is a solution $(K_t^*, Y_t^*, C_t^*, X_t^*)_{t=0}^{\infty}$ to the problem (3)-(5) (at $K_0 = K_0^*$) which satisfies

(7) for some $g_K > 0$ and $g_X \leq 0$ and check that for sufficiently high A_0 this solution satisfies (6).

Let $(K_t^*, Y_t^*, C_t^*, X_t^*)_{t=0}^{\infty}$ be a solution to the problem (3)-(5) at $K_0 = K_0^*$ and let λ_t and μ_t be the costate variables corresponding to constraints (4) and (5) respectively. Necessary first-order conditions for this problem are:

$$\beta_1^t u'(C_t^*) = \lambda_t,$$

$$\beta_2^t v'(X_t^*) = \mu_t,$$

$$\mu_t \Phi_K(Y_t^*, K_t^*, A_t) = -\lambda_{t-1},$$

$$\mu_t \Phi_Y(Y_t^*, K_t^*, A_t) = \lambda_t.$$

More specifically they can be rewritten as follows:

$$\beta_1^t (C_t^*)^{-\sigma} = \lambda_t, \tag{8}$$

$$\beta_2^t (X_t^*)^{\gamma-1} = \mu_t, \tag{9}$$

$$\mu_t \alpha(1-\rho)(Y_t^*)^{\rho} A_t^{1-\rho} (K_t^*)^{\alpha(1-\rho)-1} = -\lambda_{t-1}, \tag{10}$$

$$\mu_t \rho (Y_t^*)^{\rho-1} A_t^{1-\rho} (K_t^*)^{\alpha(1-\rho)} = \lambda_t. \tag{11}$$

If $(K_t^*, Y_t^*, C_t^*, X_t^*)_{t=0}^{\infty}$ satisfies (7), then there are $g_\lambda > -1$ and $g_\mu > -1$ such that, for all $t = 0, 1, ...,$

$$\lambda_{t+1} = (1 + g_\lambda)\lambda_t, \ \mu_{t+1} = (1 + g_\mu)\mu_t. \tag{12}$$

It is not difficult to check that

$$\begin{aligned} 1 + g_\lambda &= \beta_1 (1 + g_K)^{-\sigma}, \\ 1 + g_\mu &= \beta_2 (1 + g_X)^{\gamma-1}, \\ 1 + g_\lambda &= (1 + g_\mu)(1 + g_K)^{(1-\alpha)(\rho-1)}(1 + g)^{1-\rho}, \\ 1 + g_X &= (1 + g_K)^{\rho+\alpha(1-\rho)}(1 + g)^{1-\rho}. \end{aligned} \tag{13}$$

It follows that

$$\frac{1 + g_\lambda}{1 + g_\mu} = \frac{1 + g_X}{1 + g_K} \tag{14}$$

and

$$(\beta_1/\beta_2)(1 + g)^{\kappa_1} = (1 + g_K)^{(1-\alpha)\kappa_1+\kappa_2}. \tag{15}$$

Therefore

$$g_K > 0 \ \Leftrightarrow \ (\beta_2/\beta_1)^{1/\kappa_1} < (1 + g).$$

Let us now prove that if $0 < \sigma < 1$, then

$$g_X \leq 0 \;\Leftrightarrow\; 1 + g \leq (\beta_2/\beta_1)^{\frac{\rho-\alpha(\rho-1)}{(\rho-1)(1-\sigma)}}, \tag{16}$$

and if $\sigma > 1$, then

$$g_X \leq 0 \; \forall g > 0. \tag{17}$$

Indeed, it follows from (15) that

$$1 + g_K = (\beta_1/\beta_2)^{\frac{1}{(1-\alpha)\kappa_1+\kappa_2}} (1 + g)^{\frac{\kappa_1}{(1-\alpha)\kappa_1+\kappa_2}}.$$

Therefore,

$$\begin{aligned} 1 + g_X &= (\beta_1/\beta_2)^{\frac{\rho-\alpha(\rho-1)}{(1-\alpha)\kappa_1+\kappa_2}} (1 + g)^{\frac{[\rho-\alpha(\rho-1)]\kappa_1}{(1-\alpha)\kappa_1+\kappa_2}+1-\rho} \\ &= (\beta_1/\beta_2)^{\frac{\rho-\alpha(\rho-1)}{(1-\alpha)\kappa_1+\kappa_2}} (1 + g)^{\frac{\kappa_1-(\rho-1)\kappa_2}{(1-\alpha)\kappa_1+\kappa_2}}. \end{aligned}$$

Hence,

$$g_X \leq 0 \;\Leftrightarrow\; (\beta_2/\beta_1)^{\rho-\alpha(\rho-1)} \geq (1 + g)^{\kappa_1-(\rho-1)\kappa_2} = (1 + g)^{(\rho-1)(1-\sigma)}.$$

Since $\beta_2/\beta_1 > 1$, it follows that $\sigma > 1$ implies (17) and $0 < \sigma < 1$ implies (16).

We can summarize by formulating

Lemma 1. *If* $0 < \sigma < 1$, *then*

$$g_K > 1 \textit{ and } g_X \leq 0 \;\Leftrightarrow\; (\beta_2/\beta_1) < (1 + g)^{\kappa_1} \leq (\beta_2/\beta_1)^{\frac{\gamma[\rho(1-\alpha)+\alpha]}{1-\sigma}}.$$

If $\sigma > 1$, *then*

$$g_K > 1 \textit{ and } g_X \leq 0 \;\Leftrightarrow\; (\beta_2/\beta_1) < (1 + g)^{\kappa_1}.$$

Now we can prove

Lemma 2. *If* $g_X \leq 0$, *then any sequence* $(K_t^*, Y_t^*, C_t^*, X_t^*)_{t=0}^{\infty}$ *satisfying* (7)-(11) *is a solution to the problem* (3)-(5) *at* $K_0 = K_0^*$.

Proof of Lemma 2. Let $(K_t, Y_t, C_t, X_t)_{t=0}^{\infty}$ be a sequence satisfying (4)-(5) and $K_0 = K_0^*$. We claim that

$$\sum_{t=0}^{\infty} \left[\beta_1^t u(C_t^*) - \beta_2^t v(X_t^*)\right] \geq \sum_{t=0}^{\infty} \left[\beta_1^t u(C_t) - \beta_2^t v(X_t)\right].$$

We have for all $t = 0, 1, \ldots$

$$\beta_1^t u(C_t^*) - \lambda_t C_t^* \geq \beta_1^t u(C_t) - \lambda_t C_t,$$

$$\mu_t X_t^* - \beta_2^t v(X_t^*) \geq \mu_t X_t - \beta_2^t v(X_t),$$

$$\lambda_t Y_t^* - \lambda_{t-1} K_t^* - \mu_t \Phi(Y_t^*, K_t^*, A_t) \geq \lambda_t Y_t - \lambda_{t-1} K_t - \mu_t \Phi(Y_t, K_t, A_t).$$

Therefore for any T,

$$\sum_{t=0}^{T} \left[\beta_1^t u(C_t^*) - \beta_2^t v(X_t^*)\right] - \sum_{t=0}^{T} \left[\beta_1^t u(C_t) - \beta_2^t v(X_t)\right]$$

$$= \sum_{t=0}^{T} \left[\beta_1^t u(C_t^*) - \beta_1^t u(C_t)\right] - \sum_{t=0}^{T} \left[\beta_2^t v(X_t^*) - \beta_2^t v(X_t)\right]$$

$$\geq \sum_{t=0}^{T} (\lambda_t C_t^* - \lambda_t C_t) + \sum_{t=0}^{T} (\mu_t X_t - \mu_t X_t^*)$$

$$\geq \sum_{t=0}^{T} [\lambda_t (Y_t^* - K_{t+1}^*) - \lambda_t (Y_t - K_{t+1})]$$

$$+ \sum_{t=0}^{T} [\mu_t \Phi(Y_t, K_t, A_t) - \mu_t \Phi(Y_t^*, K_t^*, A_t)]$$

$$\geq \sum_{t=0}^{T} [\lambda_t Y_t^* - \lambda_{t-1} K_t^* - \mu_t \Phi(Y_t^*, K_t^*, A_t)]$$

$$- \sum_{t=0}^{T} [\lambda_t Y_t - \lambda_{t-1} K_t - \mu_t \Phi(Y_t, K_t, A_t)] - \lambda_T K_{T+1}^* + \lambda_T K_{T+1}$$

$$\geq -\lambda_T K_{T+1}^* + \lambda_T K_{T+1} \geq -\lambda_T K_{T+1}^*.$$

To complete the proof it is sufficient to notice that from (13), (14) and $g_X \leq 0$ we obtain

$$(1 + g_\lambda)(1 + g_K) = (1 + g_X)(1 + g_\mu) = \beta_2 (1 + g_X)^\gamma < 1$$

and hence $\lambda_T K_{T+1}^* \to 0$ as $T \to \infty$. □

Suppose that $A_0 > 0$ is given. On the one hand, it is clear that if sequences $(K_t^*, Y_t^*, C_t^*, X_t^*)_{t=0}^{\infty}$, $(\lambda_t)_{t=0}^{\infty}$ and $(\mu_t)_{t=0}^{\infty}$ satisfy (7), (8)-(11) and (12), then $(K_0^*, Y_0^*, C_0^*, X_0^*, \lambda_0, \mu_0)$ is a solution to the following system of equations:

$$X = A_0^{1-\rho} Y^\rho K^{\alpha(1-\rho)}, \tag{18}$$

$$C = Y - (1 + g_K) K, \tag{19}$$

$$\lambda = C^{-\sigma}, \tag{20}$$

$$\mu = X^{\gamma-1}, \tag{21}$$

$$\mu\alpha(1-\rho)A_0^{1-\rho}Y^{\rho}K^{\alpha(1-\rho)-1} = -\lambda/(1+g_\lambda), \tag{22}$$

$$\mu\rho A_0^{1-\rho}Y^{\rho-1}K^{\alpha(1-\rho)} = \lambda. \tag{23}$$

On the other hand, if $(K_0^*, Y_0^*, C_0^*, X_0^*, \lambda_0, \mu_0)$ is a solution to the system of equations (18)-(23) and the sequences $(K_t^*, Y_t^*, C_t^*, X_t^*)_{t=0}^\infty$, $(\lambda_t)_{t=0}^\infty$ and $(\mu_t)_{t=0}^\infty$ are given by (7) and (12), then these sequences satisfy (8)-(11) and hence $(K_t^*, Y_t^*, C_t^*, X_t^*)_{t=0}^\infty$ is a solution to the problem (3)-(5) at $K_0 = K_0^*$.

Let

$$\eta := \frac{(1+g_\lambda)\alpha(\rho-1)}{\rho},$$

$$\xi := 1-(1+g_K)\eta.$$

Lemma 3. *If* $(K_0^*, Y_0^*, C_0^*, X_0^*, \lambda_0, \mu_0)$ *is the solution to the system of equations* (18)-(23), *then*

$$Y_0^* = \left((\rho\xi^{\sigma})^{1/\gamma}\eta^{\alpha(1-\rho)}\right)^{\frac{1}{(1-\sigma)/\gamma-\rho+\alpha(\rho-1)}} A_0^{\frac{1-\rho}{(1-\sigma)/\gamma-\rho+\alpha(\rho-1)}},$$

$$K_0^* = \eta Y_0^*.$$

Proof. It follows from (20)-(23) that

$$\rho[\alpha(\rho-1)]^{-1}Y^{-1}K = 1+g_\lambda$$

and hence

$$K = \eta Y.$$

From (18) we obtain

$$X = \eta^{\alpha(1-\rho)}A_0^{1-\rho}Y^{\rho+\alpha(1-\rho)} \tag{24}$$

and from (19)

$$C = \xi Y,$$

We can rewrite (22) as

$$\xi^{-\sigma}Y^{-\sigma} = \rho A_0^{1-\rho}X^{\gamma-1}Y^{\rho-1}K^{\alpha(1-\rho)}.$$

By dividing this equation by (18) we get

$$\xi^{-\sigma}Y^{-\sigma}X^{-1} = \rho X^{\gamma-1}Y^{-1}.$$

Therefore

$$\xi^{-\sigma}Y^{1-\sigma} = \rho X^{\gamma}.$$

After substituting (24) we obtain

$$(\rho\xi^{\sigma})^{-1/\gamma}Y^{(1-\sigma)/\gamma} = \eta^{\alpha(1-\rho)}A_0^{1-\rho}Y^{\rho+\alpha(1-\rho)}$$

and hence

$$Y = \left((\rho\xi^{\sigma})^{1/\gamma}\eta^{\alpha(1-\rho)}\right)^{\frac{1}{(1-\sigma)/\gamma-\rho+\alpha(\rho-1)}} A_0^{\frac{1-\rho}{(1-\sigma)/\gamma-\rho+\alpha(\rho-1)}}. \square$$

It is not difficult to notice that if $(K_t^*, Y_t^*, C_t^*, X_t^*)_{t=0}^{\infty}$ is a solution to the problem (3)-(5) at $K_0 = K_0^*$ and

$$Y_0^* \leq A_0(K_0^*)^{\alpha}, \qquad (25)$$

then

$$Y_t^* \leq A_t(K_t^*)^{\alpha},\ t = 0, 1, ...$$

It follows from Lemma 3 that (25) can be rewritten as

$$(Y_0^*)^{1-\alpha} \leq \eta^{\alpha}A_0,$$

or, equivalently, as

$$\left((\rho\xi^{\sigma})^{1/\gamma}\eta^{\alpha(1-\rho)}\right)^{\frac{1-\alpha}{(1-\sigma)/\gamma-\rho+\alpha(\rho-1)}} A_0^{\frac{(1-\rho)(1-\alpha)}{(1-\sigma)/\gamma-\rho+\alpha(\rho-1)}} \leq \eta^{\alpha}A_0.$$

Since $0 < \frac{(1-\rho)(1-\alpha)}{(1-\sigma)/\gamma-\rho+\alpha(\rho-1)} < 1$, there is an $\bar{A} > 0$ such that (25) is true if and only if $A_0 \geq \bar{A}$. This completes the proof of the theorem.

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