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Growth and distribution in models with endogenous impatience

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Kirill Borissov[1] *and Stephane Lambrecht*[2]. **Growth and Distribution in Models with Endogenous Impatience**[3]**. Working paper Ec-03/05.**

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Abstract

This paper combines two strands of the literature on inequality and distribution issues: the classical approach, which insists on the division of society into classes characterized by different saving propensities, and the social conflict approach, which considers that inequality inflicts direct and indirect costs to economic development. Along the paper two simple models are studied: an exogenous and an endogenous growth model. In these two models, we assume that each consumer's subjective discount factor is determined endogenously and depends on economic inequality through the following two channels. On the one hand, it is positively related to the individual consumer's relative wealth. On the other hand, it is negatively affected by a simple aggregate measure of social conflict. We show that, unlike models with exogenously given discount rates, steady state equilibria in our two models are indeterminate and that the set of all equilibria is a continuum which can be parameterized by a simple index of income inequality. Economic development, proxied by national output or by the growth rate, is ambiguously related to the inequality index. However, under some reasonable assumptions, the dependence of economic development on this index has an inverted U-shaped form.

Keywords: wealth distribution, intertemporal choice, growth, development

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1. Introduction

Despite the reliance of mainstream growth theory on 'representative' agents and long disregard of problems of inequality and distribution, societies are patently not homogeneous, whether in incomes, wealth, or any other dimension. A concern with the importance of distribution was central to the thinking of classical economists such as David Ricardo and Karl Marx. However, up until recently, the mainstream economics profession seemed to have little to say about the impact that the distributions of income and wealth might have on efficiency of an economy and its growth rate.

Some interest in the relationship between distribution and growth restarted in the 1950s with the work of Kaldor (1956) and Pasinetti (1962) and the last decade has seen its new revival. Existing theories about the effect of income distribution on the process of development can be classified into two broad categories distinguished by their conflicting predictions. The classical approach originated by Smith (1776) and further interpreted and developed by Keynes (1920), Lewis (1954), Kaldor (1957), and Bourguignon (1981) suggests that inequality stimulates capital accumulation and thus promotes economic growth, whereas the modern approach argues in contrast that for sufficiently wealthy economies equality stimulates investment in human capital and hence may enhance economic growth.

The modern paradigm is represented by three complementary approaches. The capital markets imperfections approach, developed by Galor and Zeira (1993), Aghion and Bolton (1997) and others, has argued that, in the presence of credit markets imperfection, equality in sufficiently wealthy economies stimulates investment in human capital and in individual specific projects and hence enhances economic growth. The political economy approach initiated by Alesina and Rodrik (1994), Bertola (1993) and Persson and Tabellini (1994) has argued that equality diminishes the tendency for distortionary redistribution and hence stimulates investment and economic growth.

Another strand of the literature emphasizes the importance of social conflict as a link between inequality and efficiency. Alesina and Perotti (1996) argue that inequality can lead to less political stability, and this in turn can lead to sub-optimal investment levels. Social conflict may also have high opportunity costs caused by violence. Violence levels have recently increased sharply in both of the most unequal regions in the world (Latin America and sub-Saharan Africa), and in the one where its growth has been fastest (Eastern Europe, Russia and Central Asia). Fajnzylber et. al. (1998) documented these global trends, and find evidence to suggest that income inequality is significantly associated with violence levels, across countries. Bourguignon (2001) and others have documented the growing importance of the social and economic burden imposed on society by this rising violence, both in terms of the direct costs in lives and medical resources, and in terms of the opportunity costs of (both public and private) resources diverted from other activities towards preventing and fighting crime.

This paper proposes two closely interrelated models (exogenous- and endogenous-growth ones) combining the classical approach and the approach that considers social conflict and socio-political instability as links between inequality and development. These models are based on the following twofold assumption.

First, it is assumed that the subjective discount factor of a consumer is not given exogenously but is determined endogenously. When Uzawa (1968) noted that in an economy populated by infinitely-lived agents with different time-preference rates, the entire capital stock will eventually be owned by the most patient agent, he cast doubt on the assumption of a constant rate of time preference and assumed that a higher level of consumption increases the subjective discount rate (this assumption is needed for stability). Blanchard and Fischer (1989) argue, however, that this assumption "is difficult to defend *a priori*; indeed, we usually think it is the rich who are more likely to be patient" (p. 73). In this paper, we assume that the rich are more patient than the poor. In this respect our models agree closely with a model developed by Schlicht (1975) and Bourguignon (1981) and do not contradict to the empirical evidence showing (see, e.g., Browning and Lusardi (1996) and Soulels (1999)) that the marginal propensity to consume is substantially higher for consumers with low wealth or low income than for consumers with high wealth or income. They are also akin to several dynamic macroeconomic models with heterogeneous consumers (see e.g. Becker (1980), Michel and Pestieau (1998), Smetters (1999), Mankiw (2000)), but, unlike those models, we assume that all consumers are identical in their exogenous parameters.

Secondly, it is assumed that inequality increases the impatience of economic agents. The extremely simplifying reasoning behind this assumption is as follows. Income inequality increases social discontent and the probability of coups, revolutions, mass violence, etc. (see Alesina and Perotti (1996) and Perotti (1996) for empirical evidence). For simplicity, we reduce all risks emerging from high income inequality to the threat of a revolution leading to the total disruption of the ordinary economic order with absolutely unpredictable consequences for consumers. It follows that when solving his utility maximization problem, a consumer can foresee and take into account the consequences of his saving and consumption decisions only until a possible revolution, after which an absolutely new economic order will be established. This is true irrespective of whether this new order will be better or worse than before the revolution. Thus, as the probability of a revolution increases, the effective discount factor of each consumer decreases.

We show that unlike models with exogenously given discount rates, steady-state equilibria in our two models are indeterminate in the sense that the set of all equilibria is a continuum that can be parametrized by an index of inequality. The dependence of economic development (proxied by the national output or by the growth rate) on this index is not unambiguous. However, under some reasonable additional assumption, this dependence has an inverted U-shaped form.

The rest of the paper is organized as follows. In Section 2 we describe our models and in Section 3 we analyze them under the simplifying assumption that the discount factors of consumers does not depend on the probability of a disruptive revolution. We formulate our main result in Section 4 and conclude in Section 5.

2. The models

2.1 Technology

Technology is given by a production function

$Y=F(K,AL)$,

where Y is output, K is the capital stock, L is the input of labor force, which is assumed to be constant over time and be equal to the labor supply normalized to unity (L=1), and A is the state of technology. The production function $F:\Re^2_+ \to \Re_+$ is assumed to be continuous, concave, homogeneous of degree one and continuously differentiable on $\operatorname{int}\Re^2_+$. Technological progress is assumed to be Harrod-neutral and AL is interpreted as effective labor. For simplicity, the capital stock fully depreciates during one time period.

We consider simultaneously two models in discrete time t=0,1,…: an exogenous-growth model (M^{Ex}) and an endogenous-growth model (M^{En}), varying in their assumptions about dynamics of A. Model M^{Ex} is based on assumption

Ex) $A_t=(1+\bar{g})^t A_0$, where $\bar{g}>0$ is an exogenously given rate of growth,

whereas model M^{En} is based on assumption

En) $A_t=K_t/\bar{k}L$, where $\bar{k}>0$ is exogenously given.

Here K_t is the stock of capital and A_t is the state of technology at time t.

Assumption *Ex*) is a traditional textbook one. As for *En*), it is borrowed from Frankel (1962) and Romer (1986). This assumption specifies the class of so called AK-models. All markets are competitive and therefore the relationships between the capital intensity of effective labor, $k=K/AL$, the interest rate, r, and the wage rate earned by one unit of effective labor, w, are the following:

$$r=r(k)\equiv f'(k)-1,\ w=w(k)\equiv f(k)-f'(k)k,$$

where

$$f(k)\equiv F(k,1)$$

is the production function in intensive form. It should be noticed that, given w and A, the real wage rate earned by one unit of labor force is Aw. In model M^{En}, for all t,

$$k_t=\bar{k},\ r_t=\bar{r}\equiv f'(\bar{k})-1,\ w_t=\bar{w}\equiv f(\bar{k})-f'(\bar{k})\bar{k}.$$

For model M^{Ex}, in what follows we assume for simplicity that

$$r(0)=\infty,\ r(\infty)<\bar{g}.$$

2.2 Consumers

Each consumer is endowed at each time with one unit of labor force. Given the initial level of his savings, $s_{-1}\geq 0$, at time $t=0$ the consumer solves the following maximization problem:

$$\max\{\textstyle\sum_{t=0}^{\infty}\beta^t u(c_t)\mid c_t+s_t=(1+r_t)s_{t-1}+w_tA_t,$$

$$s_t\geq 0,\ t=0,1,\ldots,\ s_{-1}\geq 0 \text{ is given}\}, \tag{1}$$

where c_t and s_t are respectively consumption and savings of the consumer in period t, $\beta\in(0,1)$ is his discount factor and

$$u(c)=\log c.$$

It should be emphasized that in our models there are borrowing constraints, $s_t\geq 0$, $t=0,1,\ldots$, so that future income cannot be discounted to the present. Our models may be considered as models of sequential generations with altruism. We leave it for the reader to generalize them to the case of overlapping generations with altruism and/or to the case of felicity functions with constant intertemporal elasticity of substitution, $\nu>0$,

$$u(c)=\begin{cases}(c^{1-\nu}-1)/(1-\nu), & \nu\neq 1\\ \log c, & \nu=1\end{cases}.$$

In models with homogenous consumers with exogenously given discount factors, the relationship between the equilibrium steady-growth rates of interest and growth, r and g, is the following:

$$1+g=\beta(1+r). \tag{2}$$

In this equation, r is an endogenous variable and $g=\bar{g}$ if we consider model M^{Ex}, and g is endogenous and $r=\bar{r}$ if we consider model M^{En}.

Approximately the same is true if we consider models with heterogeneous consumers varying only in their discount factors. The only difference is that, in this case, in (2) β is not the discount factor shared by all consumers, but is the discount factor of the most patient consumers. Moreover, in steady state equilibria, all the capital belongs to the most patient consumers. This was noted by Uzawa (1968) and Becker (1980) for the case of exogenous growth and can easily be proved for the case of an AK technology.

2.3 Main assumption

Our main assumption is that the formation of the effective discount factor of a consumer, β, takes place endogenously. Namely,

$$\beta=(1-p)\varphi,$$

where φ is his subjective discount factor and p is the probability that a revolution leading to the full collapse of the pre-revolutionary economic order will happen during one time period. All consumers are assumed to be identical in their exogenous parameters and risk neutral.

The subjective discount factor of a consumer is an increasing function of his relative income, that is, of the ratio of his personal income to per capita income. By income we mean, as usually, the amount a person could have spent whilst maintaining the value of his wealth intact. To be more precise, suppose that A, k, $w=w(k)$, $r=r(k)$ are given. If the savings of the consumer are s, then his income is $rs+Aw$, and since per capita income is $A(f(k)-k)$, his relative income is $\frac{rs+Aw}{A(f(k)-k)}$. Thus, our assumption is that

$$\varphi=\varphi\left(\frac{rs+Aw}{A(f(k)-k)}\right),$$

where the function $\varphi:[0,\infty)\rightarrow(0,1)$ is assumed to be increasing and continuous. The results of the paper would not change if we assumed that the subjective discount factor is an increasing function of some other measure of relative well-being, for example, of relative consumption or felicity.

As for p, it is assumed to be an increasing (or at least non-decreasing) function of some measure of income inequality that will be specified later. It should be emphasized that if the values of φ are different for consumers with different incomes, p is the same for all consumers.

For any given distribution of income, the objective function of each consumer is a well-defined concave function of the form $\sum_{t=0}^{\infty}\beta^t u(c_t)$. However, if we tried to define an equilibrium path starting from an arbitrary chosen starting point, we would face the problem of time inconsistency. Indeed, the discount factor accepted by a consumer at some time would not be equal to the discount factor of the same consumer accepted previously. Therefore, his objective function would change over time. This implies that the optimal plan chosen at time t_1 need not be optimal as of time t_2. However, we will consider steady states only and the problem will be resolved on its own.

4 Preliminary analysis

In this section we analyze our models under the simplifying assumption that $p\in(0,1)$ is determined exogenously:

$$p=\text{const.}$$

Suppose we are given constant over time values of the wage, interest and growth rates, w, r and g. Suppose also that $A_0=1$. Then, irrespective of whether the rate of growth is formed exogenously or endogenously,

$$A_t=(1+g)^t,\ t=0,1,\ldots,$$

and problem (1) can be rewritten as follows:

$$\max\{\sum_{t=0}^{\infty}\beta^t u(c_t)\ \mid\ c_t+s_t=(1+r)s_{t-1}+w(1+g)^t,$$
$$s_t\geq 0,\ t=0,1,\ldots,\ s_{-1}\geq 0 \text{ is given}\}. \qquad (3)$$

The first-order conditions for this problem are

$$c_{t+1}\geq\beta(1+r)c_t\ (=\text{ if } s_t>0),\ t=0,1,\ldots\ .$$

Given β, w, r and $g\in(-1,r)$, we call the pair (c,s) a *steady-growth consumer equilibrium* if the sequence $(c_t,s_t)_{t=0,1,\ldots}$ given by

$$c_t=(1+g)^t c,\ s_t=(1+g)^{t+1}s,\ t=0,1,\ldots, \qquad (4)$$

is a solution to (3) at $s_{-1}=s$.

The following lemma follows from definitions.

Lemma 1. *Suppose that* β *and* $g\in(-1,r)$ *are given. Steady-growth consumer equilibria exist if and only if* $1+g\geq\beta(1+r)$.

If $1+g=\beta(1+r)$, *then any pair* (c,s) *such that* $s\geq 0$ *and*

$$c=w+(r-g)s \qquad (5)$$

is a steady-growth consumer equilibrium.

If $1+g>\beta(1+r)$, *then there is a unique steady-growth consumer equilibrium. This equilibrium is given by* $(c,s)=(w,0)$. *It also satisfies* (5).

Given k, $w=w(k)$, $r=r(k)$, $g\in(-1,r)$ and p, we call the pair (c,s) a *consistent steady-growth consumer equilibrium* if the sequence $(c_t,s_t)_{t=0,1,\ldots}$ determined by (4) is a solution to (3) at $s_{-1}=s$ and

$$\beta=(1-p)\,\varphi\left(\frac{rs+w}{f(k)-k}\right).$$

The following lemma follows from Lemma 1 and definitions.

Lemma 2. *Suppose that* k, $w=w(k)$, $r=r(k)$, $g\in(-1,r)$ *and* p *are given. The pair* (c,s) *is a consistent steady-growth consumer equilibrium if and only if*

$$c=w+(r-g)s$$

and either s *is a solution in* x *to the equation*

$$1+g=(1-p)\,\varphi\left(\frac{rx+w}{f(k)-k}\right)(1+r) \tag{6}$$

or

$$1+g>(1-p)\,\varphi\left(\frac{w}{f(k)-k}\right)(1+r) \text{ and } s=0.\square$$

Now we should introduce the notion of balanced-growth equilibrium. To make this definition clear, several points should be made. Suppose we know that the economy is in an equilibrium and that the equilibrium values of capital intensity of effective labor and equilibrium wage, interest and growth rates are respectively k^*, $w^*=w(k^*)$, $r^*=r(k^*)$ and $g^*\in(-1,r^*)$. Then on the corresponding path of balanced growth, for each consumer his consumption c_t and savings s_t satisfy the following relationships:

$$s_{-1}=s,\ c_t=(1+g^*)^t c,\ s_t=(1+g^*)^{t+1}s,\ t=0,1,\ldots,$$

where (c,s) is a consistent balanced growth consumer equilibrium at $w=w^*$, $r=r^*$, $g=g^*$. It follows from Lemma 2 that

$$c=w^*+(r^*-g^*)s,$$

and either $s=0$ or s is a solution to (6) at $k=k^*$, $w=w^*$, $r=r^*$, $g=g^*$.

It follows that in any equilibrium the population splits into (at most) two types of agents in such a way that the agents of the first type have some positive savings and the agents of the other type have no savings and spend all their wages for consumption. Agents of the first type will be denoted by the subscript h and agents of the second type by the subscript l. Since the former are wealthier than the latter, they will be called the rich and the poor respectively (following

Mankiw (2000), they might be called savers and spenders). We denote by σ the fraction of the rich in the population and by $1-\sigma$ the fraction of the poor. It should be emphasized that these fractions are determined endogenously in the following definition.

We define *a balanced growth equilibrium at a given p*, (g^*, k^*, σ^*, c_l^*, c_h^*, s_h^*), by the following conditions

- (c_l^*,0) is a consistent balanced growth consumer equilibrium at $w=w^*=w(k^*)$, $r=r^*=r(k^*)$, $g=g^*$;
- (c_h^*,s_h^*) is a consistent balanced growth consumer equilibrium at $w=w^*=w(k^*)$, $r=r^*=r(k^*)$, $g=g^*$;
- $\sigma^* s_h^*=k^*$;

and either

- $g^*=\bar{g}$ for model M^{Ex}

or

- $k^*=\bar{k}$ for model M^{En}.

It is clear that each balanced growth equilibrium (g^*, k^*, σ^*, c_l^*, c_h^*, s_h^*) determines a path of balanced equilibrium growth $\{K_t^*$, $(s_{j,t}^*)_{j=l,h}$, $(c_{j,t}^*)_{j=l,h}\}_{t=0,1,\ldots}$ by the following relationships:

$$s_{l,t}^*=0,\; s_{h,t}^*=(1+g^*)^{t+1}s_h^*,\; t=-1,0,1,\ldots,$$

$$K_{t+1}^*=(1+g^*)^{t+1}k^*=\sigma^* s_{h,t}^*,\; t=-1,0,1,\ldots,$$

$$c_{l,t}^*=(1+g^*)^t c_l^*,\; c_{h,t}^*=(1+g^*)^t c_h^*,\; t=0,1,\ldots,$$

where K_0^* equals $k^*(=k^*L)$ in the case of model M^{Ex} or is arbitrary chosen in the case of model M^{En}. Indeed, for this path,

- $(c_{l,t}^*, s_{l,t}^*)_{t=0,1,\ldots}(=(c_{l,t}^*,0)_{t=0,1,\ldots})$ is a solution to (3) at $w=w^*$, $r=r^*$, $g=g^*$, $s_{-1}=0$,

$$\beta=(1-p)\,\varphi\left(\frac{r^* s_{l,t-1}^*+(1+g^*)^t w^*}{(1+g^*)^t(f(k^*)-k^*)}\right)\left(=(1-p)\,\varphi\left(\frac{w^*}{f(k^*)-k^*}\right)\right),\; t=0,1,\ldots;$$

- $(c_{h,t}^*, s_{h,t}^*)_{t=0,1,\ldots}$ is a solution to (3) at $w=w^*$, $r=r^*$, $g=g^*$, $s_{-1}=s_h^*$,

$$\beta=(1-p)\,\varphi\left(\frac{r^* s_{h,t-1}^*+(1+g^*)^t w^*}{(1+g^*)^t(f(k^*)-k^*)}\right)\left(=(1-p)\,\varphi\left(\frac{r^* s_h^*+w^*}{f(k^*)-k^*}\right)\right),\; t=0,1,\ldots;$$

- $K_{t+1}{}^{*}=\sigma^{*} s^{*}_{h,t}+(1-\sigma^{*})\, s^{*}_{l,t}$, $t=-1,0,1,\ldots$;
- $K_{t+1}{}^{*}+\sigma^{*} c^{*}_{h,t}+(1-\sigma^{*})\, c^{*}_{l,t}=F(K_t{}^{*}, A_t L)$, where $A_t=(1+g^{*})^t$, $t=0,1,\ldots$.

Suppose that we are given a $p\in(0,1)$ and an $\eta\geq 1$. Consider the following equation:

$$1+g=(1-p)\,\varphi(\eta)(1+r).$$

When we consider model M^{Ex}, we will denote by $\rho(\eta)$ a solution to this equation in r at $g=\bar{g}$:

$$\rho(\eta)=\frac{1+\bar{g}}{(1-p)\varphi(\eta)}-1,$$

and when we consider model M^{En}, we will denote by $\gamma(\eta)$ a solution to this equation in g at $r=\bar{r}$:

$$\gamma(\eta)=(1-p)\,\varphi(\eta)(1+\bar{r})-1.$$

The following lemma says, in particular, that at a given p, the set of balanced growth equilibria, is a continuum (cf. Borissov (2002)). It follows from Lemma 2.

Lemma 3. *Suppose that p is given exogenously. Then for any* $\eta\geq 1$ *there exists a balanced growth equilibrium* (g^{*}, k^{*}, σ^{*}, c_l^{*}, c_h^{*}, s_h^{*}) *such that*

$$\frac{r(k^{*})s_h^{*}+w(k^{*})}{f(k^{*})-k^{*}}=\eta$$

and either

$$r(k^{*})=\rho(\eta)\ \ (\textit{in model } \mathrm{M}^{\mathrm{Ex}})$$

or

$$g^{*}=\gamma(\eta)\ \ (\textit{in model } \mathrm{M}^{\mathrm{En}}).\square$$

Corollary 1. *In model* M^{Ex}*, given p, the set of equilibrium values of the capital intensity of effective labor is the interval* $[k',k'')$*, where* k' *and* k'' *are determined by*

$$r(k')=\rho(1),\ r(k'')=\rho(\infty).\square$$

Corollary 2. *In model* M^{En}*, given p, the set of equilibrium values of the growth rate is the interval* $[g',g'')$*, where*

$$g'=\gamma(1),\ g''=\gamma(\infty).\square$$

To clarify the above lemma, note that, like in models with a representative consumer, any balanced growth equilibrium satisfies the modified golden rule:

$$1+g^{*}=\beta(1+r^{*}),\ 1+r^{*}=f'(k^{*}).$$

Unlike most models with a representative consumer, β is not given exogenously, but is formed endogenously. Moreover, this β is not the discount factor of the representative consumer, but the effective discount factor of only one group of consumers, the rich. It depends on relative income of this group and hence on the proportion between the number of the rich and the number of the poor.

It should be noted that in model M^{En} this dependence is such that the smaller the fraction of the rich in the population, the larger the equilibrium rate of growth. Indeed, we have

$$1+g^*=(1-p)\varphi\left(\frac{\bar{r}s_h^*+\bar{w}}{f(\bar{k})-\bar{k}}\right)(1+\bar{r}).$$

At the same time, $s_h^*=\bar{k}/\sigma^*$ and hence

$$\frac{\bar{r}s_h^*+\bar{w}}{f(\bar{k})-\bar{k}}=\frac{\alpha}{\sigma^*}+1-\alpha, \tag{7}$$

where

$$\alpha=\frac{\bar{r}\bar{k}}{f(\bar{k})-\bar{k}}. \tag{8}$$

It follows that

$$1+g^*=(1-p)\varphi\left(\frac{\alpha}{\sigma^*}+1-\alpha\right)(1+\bar{r}).$$

Therefore, g^* increases as σ^* decreases.

As for model M^{Ex}, the conclusion that the smaller the fraction of the rich in the population, the larger the equilibrium value of capital intensity of effective labor and hence of quantity of output per unit of effective labor is true under some additional conditions on the production function. For example, this conclusion is true if the production function is (modified) Cobb-Douglas:

$$f(k)=ak^{\alpha}+k,\ a>0,\ 0<\alpha<1. \tag{9}$$

In this case, we have:

$$1+\bar{g}=(1-p)\varphi\left(\frac{r(k^*)s_h^*+w(k^*)}{f(k^*)-k^*}\right)(1+r(k^*)).$$

At the same time, $s_h^*=k^*/\sigma^*$ and hence

$$\frac{r(k^*)s_h^*+w(k^*)}{f(k^*)-k^*}=\frac{\alpha}{\sigma^*}+1-\alpha. \tag{10}$$

It follows that

$$1+\bar{g}=(1-p)\varphi\left(\frac{\alpha}{\sigma^*}+1-\alpha\right)(1+r(k^*)).$$

Therefore, as σ^* goes up, $(1-p)\varphi\left(\frac{\alpha}{\sigma^*}+1-\alpha\right)$ goes down and hence $r(k^*)$ goes up. Thus, k^* and $f(k^*)$ increase as σ^* decreases.

5 Main result

To relax our previous assumption that p is given exogenously, we need to specify how to measure inequality and how does p depend on this measure. There are many ways of measuring inequality, all of which have some intuitive or mathematical appeal. Cowell (1995) contains details of at least 12 summary measures of inequality. For example, we could take the Gini coefficient as a measure of inequality. However, since we are interested only in balanced growth equilibria, our task becomes simpler since we know that, given p, at any balanced growth equilibrium the population is divided into at most two classes, the rich and the poor. In such a context we can take as an index of inequality the relative income of a rich consumer, $\frac{r^* s_h^* + w^*}{f(k^*)-k^*}$.

It should be noted that for the case of model M^{Ex} where the production function is given by (9), and for model M^{En}, this index is equivalent to the Gini coefficient as a measure of inequality in equilibria in the sense that they represent the same inequality ordering over the set of distributions of income between the two classes. Indeed, let (g^*, k^*, σ^*, c_l^*, c_h^*, s_h^*) be a balanced growth equilibrium. The Lorenz curve characterizing the income distribution in this equilibrium is presented on Fig. 1. It is not difficult to check that the Gini coefficient corresponding to this Lorenz curve is $\alpha(1-\sigma^*)$. Here α is the parameter that appears in (9) (in the case of model M^{Ex}) or is determined by (8) (in the case of model M^{En}). At the same time, taking into account (7) or (10), σ^* decreases as $\frac{r^* s_h^* + w^*}{f(k^*)-k^*}$ increases. Therefore, the Gini coefficient increases as $\frac{r^* s_h^* + w^*}{f(k^*)-k^*}$ increases.

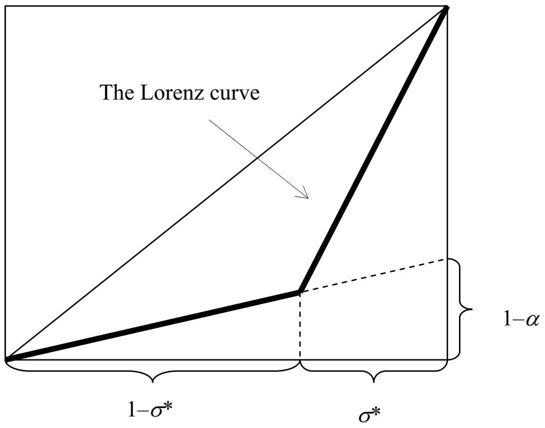

Figure 1. Lorenz curve

To give a general definition of balanced growth equilibrium, let us assume that

$$p = p\left(\frac{rs_H + Aw}{A(f(k)-k)}\right),$$

where p:[1,∞)→[0,1) is a continuous non-decreasing function and s_H is the savings of the richest consumer.

Now we are ready to give a general formal definition.

The tuple (p^*, g^*, k^* σ^*, c_l^*, c_h^*, s_h^*) is called a *balanced growth equilibrium* if it satisfies the following properties:

- (c_l^*,0) is a consistent balanced growth consumer equilibrium at $w=w^*=w(k^*)$, $r=r^*=r(k^*)$, $g=g^*$, $p=p^*$;
- (c_h^*,s_h^*) is a consistent balanced growth consumer equilibrium at $w=w^*=w(k^*)$, $r=r^*=r(k^*)$, $g=g^*$, $p=p^*$;
- $p^* = p\left(\frac{r^* s_h^* + w^*}{f(k^*)-k^*}\right)$;
- $\sigma^* s_h^* = k^*$;

and either

- $g^*=\bar{g}$ for model M^{Ex}

or

- $k^*=\bar{k}$ for model M^{En}.

Define the function ψ:[1,∞)→(0,1) by

$\psi(\eta)\equiv(1-p(\eta))\varphi(\eta)$.

Suppose that we are given an $\eta\geq1$ and consider the following equation:

$$1+g=\psi(\eta)(1+r).$$

When we consider model M^{Ex}, we will denote by $\rho^*(\eta)$ a solution to this equation in r at $g=\bar{g}$:

$$\rho^*(\eta)=\frac{1+\bar{g}}{\psi(\eta)}-1,$$

and when we consider model M^{En}, we will denote by $\gamma^*(\eta)$ a solution to this equation in g at $r=\bar{r}$:

$$\gamma^*(\eta)=\psi(\eta)(1+\bar{r})-1.$$

The following theorem follows from definitions and Lemma 3.

Theorem. *For any* $\eta\geq1$ *there exists a balanced growth equilibrium* (p^*, g^*, k^*, σ^*, c_l^*, c_h^*, s_h^*) *such that*

$$\frac{r(k^*)s_h^* + w(k^*)}{f(k^*)-k^*}=\eta$$

and either

$$r(k^*)=\rho^*(\eta) \quad (\textit{in model } M^{Ex})$$

or

$$g^*=\gamma^*(\eta) \quad (\textit{in model } M^{En}).\square$$

This theorem says that in our models, like in the model proposed by Borissov (2002), the set of equilibria is a continuum. Moreover, this set can be parameterized by a simple index of inequality, η. The dependence of the national output (in model M^{Ex}) or the growth rate (in model M^{En}) on η is in general ambiguous. It is completely determined by the shape of the function $\psi(\eta)$, which in its turn depends on the shapes of $p(\eta)$ and $\varphi(\eta)$. By the moment we have assumed no more than that the functions p:[1,∞)→(0,1) and φ:[1,∞)→(0,1) are monotonically increasing and continuous. At the same time it is not unreasonable to assume in addition that as η increases, $\psi(\eta)$ first increases, peaks, and then decreases. Indeed, one might expect that $\varphi(\eta)$ is a concave function having a noticeable positive slope when η is sufficiently small

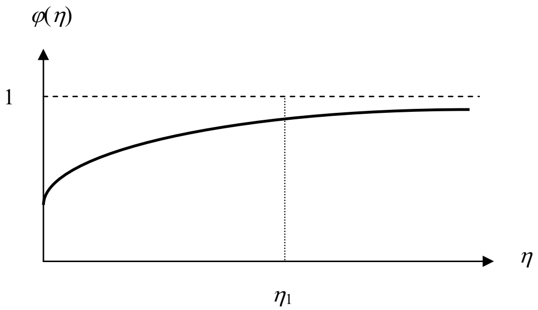

Figure 2. The shape of $\varphi(\eta)$

and getting more and more flat as η as goes up. As for $p(\eta)$, one is inclined to think that it is zero or close to zero when η is sufficiently small, then there is an interval on which it goes up sufficiently steeply and then $p(\eta)$ asymptotically converges to some positive number (≤ 1). If on the interval where $p(\eta)$ goes up fast, $\varphi(\eta)$ is sufficiently flat, then $\psi(\eta)$ and therefore the dependence of economic development on inequality has an inverted U-shaped form.

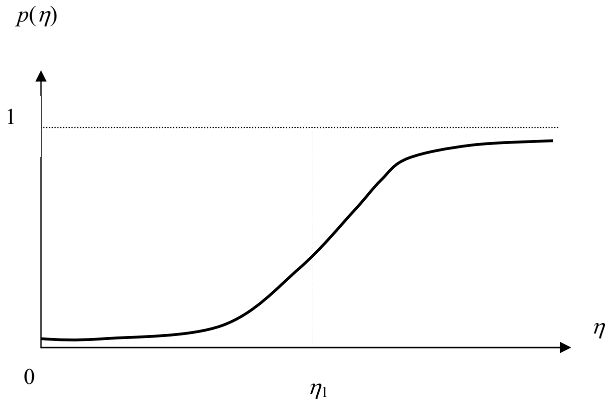

Figure 3. The shape of $p(\eta)$

To be more precise, suppose that the functions $\varphi(\eta)$ and $p(\eta)$ are twice continuously differentiable, that $\varphi(\eta)$ is concave ($\varphi''(\eta)<0$, $\eta>0$, see Fig. 2) and that there is an $\eta_1>0$ such that $p(\eta)$ is convex on the interval $[0,\eta_1]$ and concave on $[\eta_1,\infty)$ ($p''(\eta)>0$ for $0<\eta<\eta_1$, $p''(\eta)<0$ for $\eta>\eta_1$, see Fig. 3). Then the function $\psi(\eta)$ is concave ($\psi''(\eta)<0$) on the interval $[0,\eta_1]$. If, in addition, we suppose that

$$\psi'(\eta)<0, \ \eta>\eta_1, \tag{11}$$

then the function $\psi(\eta)$ reaches its maximum at some $\eta_{\max}<\eta_1$ and, moreover, $\psi(\eta)$ is increasing on $(0,\eta_{\max})$ and decreasing on $(\eta_{\max},\infty)$ (see Fig. 4). It remains to note that the following conditions are sufficient for (11) to hold:

$$1-p(\eta_1)<\varphi(\eta_1),$$

$$\varphi'(\eta)<p'(\eta), \ \eta>\eta_1.$$

Casting a glance on Fig. 2 and Fig. 3, the reader would agree that the first of these inequalities looks quite reasonable. As for the second one, though it looks somewhat peculiar, it does not seem to be extremely unreasonable.

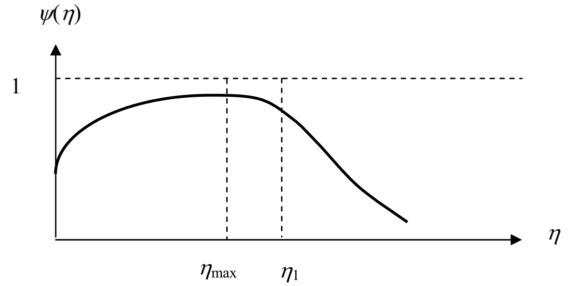

Figure 4. The shape of $\psi(\eta)$

6. Conclusion

In this paper, two one-sector models of (exogenous and endogenous) economic growth with endogenous discounting have been constructed. All consumers are identical in their exogenous parameters. Our main assumption is that when maximizing his discounted utility $\sum_t\beta^t u(c_t)$, where $u(c)=\log c$, a consumers form his effective discount factor β endogenously by $\beta=(1-p)\varphi$, where φ is his subjective discount factor and p is the probability of full collapse of the economy during one time period. The subjective discount factor is a monotonically increasing function of the relative well-being of the consumer and p is a monotonically increasing function of inequality. We have introduced p to reflect the empirically supported observation that inequality can lead to less political stability and hence more uncertainty the consumers bear.

We have shown that, unlike models with exogenously given discount rates, there is a continuum of steady-state equilibria. This is because the population is divided in equilibria into the rich and the poor and, what is important, the division cannot be considered as exogenous since all consumers are identical in their exogenous parameters. The set of equilibria can be parameterized by a simple index of income inequality. In general, the dependence of economic development on this index is ambiguous. However, under some reasonable assumption it has an inverted U-shaped form.

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