EUROPEAN
UNÍVERSÍTY AT
SAINT-PETERSBURG

Kirill Borissov

Alexander Surkov

Common and private property to exhaustible resources: theoretical implications for economic growth

Working paper Ec-02/10

Department of Economics

St. Petersburg
2010

УДК 330.35
ББК 65.012.2
В78

Европейский университет в Санкт-Петербурге

Кирилл Борисов, Александр Сурков
Общественная и частная собственность на невосполнимые ресурсы в моделях экономического роста
на английском языке

Серия препринтов; Ec-02/10; Факультет экономики
Санкт-Петербург, 2010

Borissov K., Surkov A.
B78 Common and private property to exhaustible resources: theoretical implications for economic growth / Kirill Borissov, Alexander Surkov: Working paper Ec-02/10; Department of Economics. — St. Petersburg: EUSP Press, 2010. — 20 p.

We develop two models of economic growth with exhaustible natural resources and consumers heterogeneous in time preferences. The first model assumes private ownership of natural resources. In the second model, natural resources are commonly owned and the resource extraction rate is chosen by voting. We show that if discount factors are given exogenously, the long-run rate of growth under private property is higher than or equal to that under common property. If the discount factors are formed endogenously, under some circumstances common property can result in a higher rate of growth than private property.
The authors gratefully acknowledge financial support from ExxonMobil. A.S. acknowledges support from Academician Nikolai Fedorenko International Scientific Foundation of Economic Research. The authors thank Prof. Mikhail Klimenko for his useful comments. The authors are grateful to the participants of the Monte Veritá Conference on Sustainable Resource Use and Economic Dynamics — SURED 2010 (Ascona, Switzerland), the 2010 World Conference on Natural Resource Modeling (Helsinki, Finland) and the 11th Annual Conference of the Association for Public Economic Theory (PET10, Istanbul, Turkey) for stimulating discussions.

Издание осуществлено за счет средств проекта создания специализации по природным ресурсам и экономике энергетики «ЭксонМобил».

Common and private property to exhaustible resources: theoretical implications for economic growth

Kirill Borissov and Alexander Surkov

European University at St. Petersburg, St. Petersburg Institute for Economics and Mathematics RAS

3 Gagarinskaya Str., St. Petersburg 191187, Russia, kirill@eu.spb.ru, asurkov@eu.spb.ru

Property rights are central to debates about natural resource policy. Which individual, group, or entity should hold the rights to a resource? Until three decades ago, the accepted answer was generally "the government". A widespread viewpoint today is that with private ownership of resources, owners experience directly the costs and benefits of their decisions and thus, under the logic of the market, will use their resources wisely. The aim of this paper is to look at the controversies over the superiority of private vs. common property from the standpoint of the theory of economic growth.

To do this, we propose two models of economic growth with heterogeneous consumers and exhaustible resources differing in property regimes. These models are modifications to a well-known Ramsey-type model of economic growth with exhaustible resources (see, e.g., Heal *et al.* (1974)). Our goal is to describe possible effects of different property regimes on the resource utilization rate and the rate of growth.

The first model assumes private ownership of resource stocks (mines, oilfields etc.) perceived by consumers as assets which they can buy and sell and in which they can invest their savings. This implies that the resource owners can earn a rent. The second model supposes common property, with resource stocks owned collectively (e.g., Heltberg (2002); Ostrom and Hess (2007)), and the resource rent equally divided among all the consumers.

Following Becker (1980) we assume that consumers are heterogeneous in their intertemporal preferences. Discount factors are higher for more patient consumers and lower for less patient ones.

First we consider the model with private property. In this model, in the long run all capital and resources belong to the most patient consumers. We show that the discount factor of the most patient consumers determines the long-run rate of resource utilization and the long-run rate rate of growth, the rate of resource utilization being decreasing and the rate of growth increasing in the discount factor of the most patient consumers.

Then we pass to the model with common property. In this model, consumer heterogeneity leads to different preferences on the rate of resource extraction, since less patient consumers prefer the resources to be extracted faster than the more patient consumers. The resource extraction rate is chosen by voting. Our approach to voting in a dynamic framework is borrowed from Borissov *et al.* (2010)*. We prove that in the model with common property it is the discount factor of the median voter that determines the long-run resource utilization rate and the long-run rate of growth. It is interesting to notice that the latter does not depend on the discount factor of the most patient consumers.

It follows that the long-run rate of growth in the case of private ownership of natural resources is less than or equal to the long-run rate of growth in the case of common ownership. However, this conclusion is subject to some reservations. More specifically, we show that if discount factors are endogenous, under some circumstances common property can result in a higher rate of growth.

The paper is organized as follows. Section 1 is devoted to the model with private property to resources. We define equilibrium paths, fully characterize steady-state equilibria and derive an explicit formula for the equilibrium rate of growth. In section 2, we develop the model with common resource stocks, describe our approach to voting, characterize steady state voting equilibria and derive a formula the the long-run rate of growth. In section 3, we modify our models by introducing endogenous time preferences.

1. Economic growth model with private ownership of resource stocks

1.1. Consumers

Suppose that there is an odd number L of consumers. The consumers live for an infinite period of time and are identical in all respect except their discount factors. Each time each consumer supply one unit of labor force in the labor market. Thus the total labor supply at each time is L.

The utility function of consumer i is of the form

$$\sum_{t=0}^{\infty} \beta_i^t u(C_{i,t}),$$

where β_i is the discount factor of this consumer and $C_{i,t}$ is his consumption at time t. We assume that $u(C) = \ln C$.

*The performance of majoritarian institutions in dynamic settings has attracted growing interest and attention in recent years (see e.g. Baron (1996); Krusell *et al.* (1997); Cooley and Soares (1999); Rangel (2003); Bernheim and Slavov (2009)). Without going into detail, notice that our approach to voting is different from the approaches accepted in these papers.

We suppose the households to be sorted in ascending order of their discount factors:

$$0 < \beta_1 \leq \beta_2 \leq \cdots \leq \beta_L \equiv \beta_{max} < 1.$$

By J we denote the set of agents with the highest discount factor:

$$J = \{i = 1, \ldots, L \mid \beta_i = \beta_L\}.$$

The budget constraints of consumer i are of the following form

$$C_{i,t} + S_{i,t} \leq (1 + r_t)\, S_{i,t-1} + W_t, \quad S_{i,t} \geq 0, \quad t = 0, 1, \ldots, \quad S_{i,-1} = \hat{S}_{i,-1}. \quad (1)$$

Here r_t and W_t are the interest and wage rates at time t, and $S_{i,t}$ are the savings of consumer i at time t. Consumers are prohibited to borrow against their future wage earnings. Therefore, their savings must be non-negative. They can be invested in physical capital as well as in natural resources. According to Hotelling's rule (Hotelling (1931), see also Stiglitz (1974)) the return to the investments into phisical capital and into natural resources must be equal in equilibrium. Hence, the resource price P_t in equilibrium grows at the rate r_t:

$$P_t = (1 + r_t)\, P_{t-1}, \quad t = 0, 1, \ldots.$$

Initially consumer i is supposed to be endowed with some amount of physical capital $\hat{K}_{i,0}$ and some amount of natural resources $\hat{R}_{i,0}$ which are assumed to be given. Therefore at the initial time $t = 0$ the savings of consumer i are

$$S_{i,-1} = P_{-1}\hat{R}_{i,0} + \hat{K}_{i,0} \geq 0$$

where P_{-1} is the price of natural resources at time $t = -1$.

1.2. Production

We assume that output Y_t at each time t is given by the Cobb-Douglas production function:

$$Y_t = A_t K_t^{\alpha_1} L^{\alpha_2} E_t^{\alpha_3}, \quad \alpha_j > 0, \quad j = 1, 2, 3, \quad \sum_{j=1}^{3} \alpha_j = 1, \quad (2)$$

where A_t is the coefficient of total factor productivity, K_t is the physical capital stock at time t, L is the labor supply, and E_t is the volume of extraction of exhaustible resources. Capital fully depreciates during one time period. The total factor productivity grows at an exogenously given rate λ:

$$A_t = (1 + \lambda)^t\,.$$

The resources expended for production decrease their available stock:

$$R_{t+1} = R_t - E_t, \quad t = 0, 1, \ldots.$$

We denote by ρ_t the resource extraction rate: $\rho_t = E_t/R_t$, so that

$$E_t = \rho_t R_t.$$

1.3. Equilibrium paths and steady-state equilibria

Suppose that we are given an initial state represented by an initial distribution of physical capital $\left(\hat{K}_{i,0}\right)_{i=1,\ldots,L}$ and natural resources $\left(\hat{R}_{i,0}\right)_{i=1,\ldots,L}$ among the consumers. We assume that

$$\hat{K}_{i,0} \geq 0, \quad \hat{R}_{i,0} \geq 0, \quad i = 1, \ldots, L,$$

$$\hat{K}_0 \equiv \sum_{i=1}^{L} \hat{K}_{i,0} > 0, \qquad \hat{R}_0 \equiv \sum_{i=1}^{L} \hat{R}_{i,0} > 0.$$

and define an *equilibrium path* starting from the initial state $\left(\hat{K}_{i,0}, \hat{R}_{i,0}\right)_{i=1,\ldots,L}$ as a sequence

$$\left\{K_t^*, R_t^*, 1 + r_t^*, W_t^*, P_t^*, E_t^*, \left(C_{i,t}^*, S_{i,t}^*\right)_{i=1,\ldots,L}\right\}_{t=0,1,\ldots}$$

such that

1) for each $i = 1, \ldots, L$, the sequence $\left(C_{i,t}^*, S_{i,t}^*\right)_{t=0,1,\ldots}$ is a solution to the following problem:

$$\max \sum_{t=0}^{\infty} \beta_i^t u(C_{i,t}), \tag{3}$$

$$C_{i,t} + S_{i,t} \leq (1 + r_t) S_{i,t-1} + I_t, \quad t = 0, 1, \ldots,$$

$$S_{i,t} \geq 0, \quad t = 0, 1, \ldots$$

at $r_t = r_t^*$, $\pi_t = \pi_t^*$, $I_t = W_t^*$, and

$$S_{i,-1} = \frac{P_0^* \hat{R}_{i,0}}{1 + r_0^*} + \hat{K}_{i,0};$$

2) capital is paid its marginal product:

$$1 + r_t^* = \frac{\alpha_1 A_t L^{\alpha_2} E_t^{*\alpha_3}}{K_t^{*\,1-\alpha_1}}, \quad t = 0, 1, \ldots,$$

where $K_0^* = \hat{K}_0$;

3) labor is paid its marginal product:

$$W_t^* = \frac{\alpha_2 A_t K_t^{*\alpha_1} E_t^{*\alpha_3}}{L^{1-\alpha_2}}, \quad t = 0, 1, \ldots;$$

4) the price of exhaustible resources is equal to their marginal product:

$$P_t^* = \frac{\alpha_3 A_t K_t^{*\alpha_1} L^{\alpha_2}}{E_t^{*\,1-\alpha_3}}, \quad t = 0, 1, \ldots;$$

5) Hotelling's rule holds true:

$$P_{t+1}^* = \left(1 + r_{t+1}^*\right) P_t^*, \quad t = 0, 1, \ldots;$$

6) total consumer savings are equal to the investments into physical capital and exhaustible resources

$$\sum_{i=1}^{L} S_{i,t}^* = P_t^* R_{t+1}^* + K_{t+1}^*, \quad t = 0, 1, \ldots;$$

7) the natural balance of exhaustible resources is fulfilled

$$R_{t+1}^* = R_t^* - E_t^*, \quad t = 0, 1, \ldots,$$

where $R_0^* = \hat{R}_0$.

We make our emphasis on steady-state equilibria. They are defined as follows. A tuple

$$\left\{\gamma^*, \rho^*, K^*, R^*, 1 + r^*, W^*, P^*, E^*, (C_i^*, S_i^*)_{i=1,\ldots,L}\right\}$$

is called a *steady-state equilibrium* if the sequence

$$\left\{K_t^*, R_t^*, 1 + r_t^*, W_t^*, P_t^*, E_t^*, \left(C_{i,t}^*, S_{i,t}^*\right)_{i=1,\ldots,L}\right\}_{t=0,1,\ldots}$$

given for $t = 0, 1, \ldots$ and $i = 1, \ldots, L$ by

$$\begin{aligned} K_t^* &= (1+\gamma^*)^t K^*, & 1 + r_t^* &= 1 + r^*, \\ W_t^* &= (1+\gamma^*)^t W^*, & P_t^* &= (1+r^*)^t P^*, \\ R_t^* &= (1-\rho^*)^t R^*, & E_t^* &= (1-\rho^*)^t E^*, \\ C_{i,t}^* &= (1+\gamma^*)^t C_i^*, & S_{i,t}^* &= (1+\gamma^*)^t S_i^*, \end{aligned}$$

is an equilibrium path for some initial state.

Suppose that for some r, I, γ,

$$r_t = r, \quad I_t = (1+\gamma)^t I, \quad t = 0, 1, \ldots$$

We call a couple $\left(C_i^*, S_i^*\right)$ a *balanced optimum of consumer i* if the sequence $\left(C_{i,t}^*, S_{i,t}^*\right)_{t=0}^{\infty}$ given by

$$C_{i,t}^* = (1+\gamma)^t\, C_i^*, \qquad S_{i,t}^* = (1+\gamma)^t\, S_i^*, \quad t = 0, 1, \ldots,$$

is a solution to problem (3) at $\hat{S}_{i,-1} = (1+\gamma)^{-1}\, S_i^*$.

It is clear that a tuple

$$\left\{\gamma^*, \rho^*, K^*, R^*, 1+r^*, W^*, P^*, E^*, (C_i^*, S_i^*)_{i=1,\ldots,L}\right\}$$

is a steady-state equilibrium if and only if

$$1 + r^* = \frac{\alpha_1 L^{\alpha_2} E^{*\alpha_3}}{K^{*1-\alpha_1}}, \qquad W^* = \frac{\alpha_2 K^{*\alpha_1} E^{*\alpha_3}}{L^{1-\alpha_2}}, \qquad P^* = \frac{\alpha_3 K^{*\alpha_1} L^{\alpha_2}}{E^{*1-\alpha_3}}, \tag{4}$$

$$(1+\gamma^*)\left(\frac{P^* R^*}{1+r^*} + K^*\right) = \sum_{i=1}^{L} S_i^*, \tag{5}$$

$$E^* = \rho^* R^*, \tag{6}$$

$$(1+\gamma^*)^{1-\alpha_1} = (1+\lambda)\,(1-\rho^*)^{\alpha_3}, \tag{7}$$

$$1 + r^* = \frac{1+\gamma^*}{1-\rho^*}, \tag{8}$$

and, for each $i = 1, \ldots, L$, the couple $\left(C_i^*, S_i^*\right)$ is a balanced optimum of consumer i at $r = r^*$, $\gamma = \gamma^*$, and $I = W^*$.

To describe properties of steady-state equilibria we formulate the following simple lemma.

Lemma 1. *Given r, I, and γ,*

1) a balanced optimum of consumer i exists if and only if

$$\beta_i \le \frac{1+\gamma}{1+r};$$

2) if

$$\beta_i = \frac{1+\gamma}{1+r},$$

then any couple $\left(C_i^*, S_i^*\right)$ *such that*

$$C_i^* + S_i^* + = \frac{1+r}{1+\gamma} S_i^* + I, \quad C_i^* \geq 0, \quad S_i^* \geq 0$$

is a balanced optimum of consumer i;

3) *if*

$$\beta_i < \frac{1+\gamma}{1+r},$$

then there is a unique balanced optimum of consumer i, $\left(C_i^*, S_i^*\right)$*; it is given by* $C_i^* = I$*,* $S_i^* = 0$*.*

Now we can formulate an important proposition describing the structure of steady-state equilibria. It follows from Lemma 1.

Proposition 1. *A tuple*

$$\left\{\gamma^*, \rho^*, K^*, R^*, 1+r^*, W^*, P^*, E^*, (C_i^*, S_i^*)_{i=1,\dots,L}\right\}$$

is a steady-state equilibrium if and only if it satisfies conditions (4)–(8) and

$$\beta_{max} = \frac{1+\gamma^*}{1+r^*},$$

$$C_i^* + S_i^* = \frac{1+r}{1+\gamma} S_i^* + W^*, \quad C_i^* \geq 0, \quad S_i^* \geq 0, \quad i \in J,$$

$$C_i^* = W, \quad C_i^* \geq 0, \quad S_i^* = 0, \quad i \notin J.$$

To prove this proposition it is sufficient to repeat a well-known argument by Becker (1980, 2006). It allows us to note the following:

- The equilibrium resource extraction rate is determined by the patience of the most patient consumers:

  $$\rho^* = 1 - \beta_{max}.$$

  The more patient are these consumers, the lower is the resource extraction rate.

- The growth rate is determined by the rate of technological change and the discount factor of the most patient consumer

  $$1 + \gamma^* = \left[(1+\lambda)\beta_{max}^{\alpha_3}\right]^{\frac{1}{1-\alpha_1}}.$$

  The higher is patience of the most patient consumers, the higher is the growth rate.

2. Economic growth model with common ownership of resources

In this section we propose a model of economic growth with heterogeneous agents and common resource stocks. The income obtained from the sale of the exhaustible resource is equally distributed among the consumers. Consumers choose resource extraction rate by voting. We describe equilibrium paths in the basic Ramsey-type model and after that introduce voting procedure and define voting equilibrium.

We maintain our earlier assumptions and rewrite the budget constraints (1) of consumer i as follows

$$C_{i,t} + S_{i,t} \leq (1 + r_t) S_{i,t-1} + W_t + \Omega_t, \quad S_{i,t} \geq 0, \quad t = 0, 1, \ldots, \quad S_{i,-1} = \hat{S}_{i,-1}$$

Here r_t is the interest time t, and $S_{i,t}$ are the savings of consumer i at time t invested in physical capital. Consumer's income includes the wage W_t and resource income Ω_t, which is the per capita income from the sale of the extracted resource equally distributed among the consumers.

2.1. Equilibrium paths and steady-state equilibria

As in section 1.3, first we define equilibrium paths. Suppose that a sequence $\mathcal{R} = (\rho_t)_{t=0}^{\infty}$ of resource extraction rates is given. The initial state is given by a tuple of initial savings $\left\{\hat{S}_{i,-1}\right\}_{i=1,\ldots,L}$ such that

$$\hat{S}_{i,-1} \geq 0, \quad i = 1, \ldots, L, \qquad \sum_{i=1}^{L} \hat{S}_{i,-1} > 0,$$

and the initial stock of exhaustible resource $\hat{R}_0 > 0$. We define an *equilibrium path* starting from the initial state $\left\{\left(\hat{S}_{i,-1}\right)_{i=1,\ldots,L}, \hat{R}_0\right\}$ as a sequence

$$\left\{K_t^{**}, R_t^{**}, 1 + r_t^{**}, W_t^{**}, \Omega_t^{**}, P_t^{**}, E_t^{**}, \left(C_{i,t}^{**}, S_{i,t}^{**}\right)_{i=1,\ldots,L}\right\}_{t=0,1,\ldots}$$

such that

1) for each $i = 1, \ldots, L$, the sequence $\left(C_{i,t}^{**}, S_{i,t}^{**}\right)_{t=0,1,\ldots}$ is a solution to problem (3) at $r_t = r_t^{**}$, $I_t = W_t^{**} + \Omega_t^{**}$, $S_{i,-1} = \hat{S}_{i,-1}$;

2) aggregate savings are equal to the capital stock

$$K_t^{**} = \sum_{i=1}^{L} S_{i,t-1}^{**}, \quad t = 0, 1, \ldots;$$

3) capital is paid its marginal product:

$$1 + r_t^{**} = \frac{\alpha_1 A_t L^{\alpha_2} E_t^{**\alpha_3}}{K_t^{**1-\alpha_1}}, \quad t = 0, 1, \ldots;$$

4) labor is paid its marginal product:

$$W_t^{**} = \frac{\alpha_2 A_t K_t^{**\alpha_1} E_t^{**\alpha_3}}{L^{1-\alpha_2}}, \qquad t = 0, 1, \ldots$$

5) the resource income is given by

$$\Omega_t^{**} = \frac{P_t^{**} E_t^{**}}{L}, \qquad t = 0, 1, \ldots;$$

6) the price of the exhaustible resources is equal to their marginal product

$$P_t^{**} = \frac{\alpha_3 A_t K_t^{**\alpha_1} L^{\alpha_2}}{E_t^{**1-\alpha_3}}, \quad t = 0, 1, \ldots;$$

7) the extraction of resource is determined by

$$E_t^{**} = \rho_t R_t^{**}, \quad t = 0, 1, \ldots;$$

8) the natural balance of exhaustible resource is fulfilled

$$R_{t+1}^{**} = R_t^{**} - E_t^{**}, \quad t = 0, 1, \ldots,$$

where $R_0^{**} = \hat{R}_0$.

To describe steady-state equilibria suppose that the resource extraction rate is constant over time: $\rho_t = \rho$, $t = 0, 1, \ldots$

A tuple

$$\left\{\gamma^{**}, K^{**}, R^{**}, 1 + r^{**}, W^{**}, \Omega^{**}, P^{**}, \pi^{**}, E^{**}, (C_i^{**}, S_i^{**})_{i=1,\ldots,L}\right\}$$

is called a *steady-state equilibrium* if the sequence

$$\left\{K_t^{**}, R_t^{**}, 1 + r_t^{**}, W_t^{**}, \Omega_t^{**}, P_t^{**}, E_t^{**}, \left(C_{i,t}^{**}, S_{i,t}^{**}\right)_{i=1,\ldots,L}\right\}_{t=0,1,\ldots}$$

given for $t = 0, 1, \ldots$ and $i = 1, \ldots, L$ by

$$K_t^{**} = (1 + \gamma^{**})^t K^{**}, \qquad 1 + r_t^{**} = 1 + r^{**},$$
$$W_t^{**} = (1 + \gamma^{**})^t W^{**}, \qquad \Omega_t^{**} = (1 + \gamma^{**})^t \Omega^{**},$$
$$P_t^{**} = (1 + \pi^{**})^t P^{**}, \qquad R_t^{**} = (1 - \rho)^t R^{**}, \qquad E_t^{**} = (1 - \rho)^t E^{**},$$
$$C_{i,t}^{**} = (1 + \gamma^{**})^t C_i^{**}, \qquad S_{i,t}^{**} = (1 + \gamma^{**})^t S_i^{**},$$

is an equilibrium path. Here we do not suppose that Hotelling's rule holds true. Under some circumstances it may be that $\pi^{**} \neq r^{**}$.

It is clear that for any constant over time resource extraction rate, ρ, a tuple

$$\left\{\gamma^{**}, K^{**}, R^{**}, 1+r^{**}, W^{**}, \Omega^{**}, P^{**}, \pi^{**}, E^{**}, (C_i^{**}, S_i^{**})_{i=1,\dots,L}\right\}$$

is a steady-state equilibrium if and only if

$$1+r^{**} = \frac{\alpha_1 L^{\alpha_2} E^{**\alpha_3}}{K^{**\,1-\alpha_1}}, \qquad W^{**} = \frac{\alpha_2 K^{**\alpha_1} E^{**\alpha_3}}{L^{1-\alpha_2}}, \tag{9}$$

$$\Omega^{**} = \frac{P^{**} E^{**}}{L}, \qquad P^{**} = \frac{\alpha_3 K^{**\alpha_1} L^{\alpha_2}}{E^{**\,1-\alpha_3}}, \tag{10}$$

$$(1+\gamma^{**})\, K^{**} = \sum_{i=1}^{L} S_i^{**}, \tag{11}$$

$$E^{**} = \rho R^{**}, \tag{12}$$

$$(1+\gamma^{**})^{1-\alpha_1} = (1+\lambda)(1-\rho)^{\alpha_3}, \tag{13}$$

$$1+\gamma^{**} = (1+\pi^{**})(1-\rho), \tag{14}$$

and, for each $i = 1, \dots, L$, the couple $\left(C_i^{**}, S_i^{**}\right)$ is a balanced optimum of consumer i at $r = r^{**}$, $\gamma = \gamma^{**}$, and $I = W^{**} + \Omega^{**}$.

Lemma 1 can be readily applied to the model under consideration. The following proposition describing the structure of steady-state equilibrium follows from Lemma 1.

Proposition 2. *Let a constant over time resource extraction rate, ρ, be given. A tuple*

$$\left\{\gamma^{**}, K^{**}, R^{**}, 1+r^{**}, W^{**}, \Omega^{**}, P^{**}, \pi^{**}, E^{**}, (C_i^{**}, S_i^{**})_{i=1,\dots,L}\right\}$$

is a steady-state equilibrium if and only if it satisfies conditions (9)–(14) and

$$\beta_{max} = \frac{1+\gamma^{**}}{1+r^{**}}, \tag{15}$$

$$C_i^{**} + S_i^{**} = \frac{1+r^{**}}{1+\gamma^{**}} S_i^{**} + W^{**} + \Omega^{**}, \quad C_i^{**} \geq 0, \quad S_i^{**} \geq 0, \quad i \in J, \tag{16}$$

$$C_i^{**} = W^{**} + \Omega^{**}, \quad C_i^{**} \geq 0, \quad S_i^{**} = 0, \quad i \notin J. \tag{17}$$

This proposition can be proved in the same way as Proposition 2. It says that only the most patient consumers make positive savings and own all the capital.

2.2. Voting equilibria

Here we introduce a voting procedure into our model. Consider an equilibrium path

$$\left\{K_t^{**}, R_t^{**}, 1+r_t^{**}, W_t^{**}, \Omega_t^{**}, P_t^{**}, E_t^{**}, \left(C_{i,t}^{**}, S_{i,t}^{**}\right)_{i=1,\dots,L}\right\}_{t=0,1,\dots}$$

and ask each consumer i, weather he prefers to increase or decrease the resource extraction rate ρ_t at time t. We assume that when answering this question consumers take into account the fact that additional resource extracted at time t can be sold, used for production and possibly increase their consumption via corresponding part of the resource rent and via increase of factors income. On the other hand, the resource utilized at time t decrease the resource stock available in the future.

To describe consumers' decision-making, first note that for each i, $(C_{i,t}^{**}, S_{i,t}^{**})_{t=0,1,\dots}$ is a solution to the following problem:

$$\max \sum_{t=0}^{\infty} \beta_i^t u(C_{i,t}), \tag{18}$$

$$C_{i,t} + S_{i,t} \le A_t K_t^{**\alpha_1} L^{\alpha_2} E_t^{**\alpha_3} \left(\frac{\alpha_1 S_{t-1}}{K_t^{**}} + \frac{\alpha_2+\alpha_3}{L}\right),$$

$$S_{i,t} \ge 0, \quad t = 0, 1, \dots,$$

where $S_{i,-1} = \hat{S}_{i,-1}$. Recall that

$$E_t^{**} = \rho_t R_t^{**} \text{ and hence } R_t^{**} = \hat{R}_0 \prod_{j=0}^{t-1} \left(1-\rho_j\right), \quad t = 0, 1, \dots. \tag{19}$$

Let us consider the value of (18), V_i, as a function of $\mathcal{R} = (\rho_t)_{t=0}^{\infty}$. Defining consumers' decision-making procedure we assume that the attitude of consumer i to a possible change in ρ_t is determined by sign of the derivative $\partial V_i / \partial \rho_t$, if it exists. Namely, if $\partial V_i / \partial \rho_t > 0$, consumer i *is in favor of increasing* ρ_t. If $\partial V_i / \partial \rho_t < 0$, consumer i is *in favor of decreasing* ρ_t.

Definition 1. *We call a sequence*

$$\left\{\rho_t^{**}, K_t^{**}, R_t^{**}, 1+r_t^{**}, W_t^{**}, \Omega_t^{**}, P_t^{**}, E_t^{**}, \left(C_{i,t}^{**}, S_{i,t}^{**}\right)_{i=1,\dots,L}\right\}_{t=0,1,\dots}$$

a voting equilibrium path if

$$\left\{K_t^{**}, R_t^{**}, 1+r_t^{**}, W_t^{**}, \Omega_t^{**}, P_t^{**}, E_t^{**}, \left(C_{i,t}^{**}, S_{i,t}^{**}\right)_{i=1,\dots,L}\right\}_{t=0,1,\dots}$$

is an equilibrium path at $\mathcal{R} = (\rho_t^{**})_{t=0}^{\infty}$ and for each t the number of consumers who are in favor of increasing ρ_t and the number of those who are in favor of decreasing ρ_t is less than $L/2$.

Here we certainly do not mean that the individuals votes for any change in the resource extraction rate. The idea is that the government controlling the resource stock responds somehow to the wishes of the majority when choosing the resource extraction rate.

We will not discuss an existence and properties of voting equilibria in a general form, but will describe voting steady-state equilibria.

Definition 2. *If the sequence*

$$\left\{\rho_t^{**}, K_t^{**}, R_t^{**}, 1 + r_t^{**}, W_t^{**}, \Omega_t^{**}, P_t^{**}, E_t^{**}, \left(C_{i,t}^{**}, S_{i,t}^{**}\right)_{i=1,\ldots,L}\right\}_{t=0,1,\ldots}$$

given by

$$\begin{gathered}\rho_t^{**} = \rho^{**}, \qquad K_t^{**} = (1+\gamma^{**})^t K^{**}, \qquad 1 + r_t^{**} = 1 + r^{**},\\ W_t^{**} = (1+\gamma^{**})^t W^{**}, \qquad \Omega_t^{**} = (1+\gamma^{**})^t \Omega^{**},\\ P_t^{**} = (1+\pi^{**})^t P^{**}, \qquad R_t^{**} = (1-\rho^{**})^t R^{**}, \quad E_t^{**} = (1-\rho^{**})^t E^{**},\\ C_{i,t}^{**} = (1+\gamma^{**})^t C_i^{**}, \qquad S_{i,t}^{**} = (1+\gamma^{**})^t S_i^{**},\\ i = 1, \ldots, L, \quad t = 0, 1, \ldots\end{gathered}$$

forms a voting equilibrium path, then the tuple

$$\left\{\rho^{**}, \gamma^{**}, K^{**}, R^{**}, 1 + r^{**}, W^{**}, \Omega^{**}, P^{**}, \pi^{**}, E^{**}, (C_i^{**}, S_i^{**})_{i=1,\ldots,L}\right\}$$

is called a voting steady-state equilibrium.

The following theorem describes voting steady-state equilibria. It reads that an important role in determining a steady-state equilibrium is played by the median consumer $m = (L + 1)/2$.

Theorem 1. *A tuple*

$$\left\{\rho^{**}, \gamma^{**}, K^{**}, R^{**}, 1 + r^{**}, W^{**}, \Omega^{**}, P^{**}, \pi^{**}, E^{**}, (C_i^{**}, S_i^{**})_{i=1,\ldots,L}\right\}$$

represents a voting steady-state equilibrium, if and only if it satisfies conditions (9)–(14) at $\rho = \rho^{**}$, where $\rho^{**} = 1 - \beta_m$.

Proof. Let

$$\left\{\gamma^{**}, K^{**}, R^{**}, 1+r^{**}, W^{**}, \Omega^{**}, P^{**}, \pi^{**}, E^{**}, (C_i^{**}, S_i^{**})_{i=1,\dots,L}\right\}$$

be a steady-state equilibrium constructed at $\rho = \rho^{**}$ and

$$\left\{K_t^{**}, R_t^{**}, 1+r_t^{**}, W_t^{**}, \Omega_t^{**}, P_t^{**}, E_t^{**}, \left(C_{i,t}^{**}, S_{i,t}^{**}\right)_{i=1,\dots,L}\right\}_{t=0,1,\dots}$$

be an equilibrium path corresponding to this steady-state equilibrium.

By the envelope theorem, taking into account (19), for all $i = 1, \dots, L$ and all $t = 0, 1, \dots$, we have

$$\frac{\partial V_i}{\partial \rho_t} = \beta_i^t u'\left(C_{i,t}^{**}\right) \frac{\alpha_3 \Phi_t^{**}}{\rho_t} - \sum_{k=t+1}^{\infty} \beta_i^k u'\left(C_{i,k}^{**}\right) \frac{\alpha_3 \Phi_k^{**}}{1-\rho_t},$$

where

$$\Phi_t^{**} = A_t K_t^{**\alpha_1} L^{\alpha_2} E_t^{**\alpha_3} \left(\frac{\alpha_1 S_{t-1}^{**}}{K_t^{**}} + \frac{\alpha_2 + \alpha_3}{L}\right).$$

Also we have

$$\Phi_{t+1}^{**} = (1+\gamma^{**})\Phi_t^{**}, \qquad C_{i,t+1}^{**} = (1+\gamma^{**})C_{i,t}^{**}, \qquad u'\left(C_{i,t+1}^{**}\right) = \frac{u'(C_{i,t}^{**})}{1+\gamma^{**}}.$$

Therefore,

$$\frac{\partial V_i}{\partial \rho_t} = \alpha_3 \beta_i^t \Phi_t^{**} u'\left(C_{i,t}^{**}\right)\left[\frac{1}{\rho_t} - \frac{\beta_i}{(1-\beta_i)(1-\rho_t)}\right]$$

and hence

$$\operatorname{sign}\frac{\partial V_i}{\partial \rho_t} = \operatorname{sign}\{1-\beta_i-\rho_t\}$$

To complete the proof it is sufficient to notice that

$$\frac{\partial V_i}{\partial \rho_t} \gtreqless 0 \quad \Leftrightarrow \quad \beta_i \lesseqgtr 1-\rho_t.$$

□

It follows that

- The long-run equilibrium resource extraction rate is determined by the patience of the median consumer:

$$\rho^{**} = 1-\beta_m.$$

- Hotelling's rule Hotelling (1931), implying that the resource prices grow with the rate equal to the interest rate $\pi^{**} = r^{**}$ may be violated:

$$\frac{1 + r^{**}}{1 + \pi^{**}} = \frac{\beta_m}{\beta_{max}}.$$

  Indeed, if $\beta_m < \beta_{max}$, then π^{**} is larger than the interest rate r^{**}. See Chermak and Patrick (2002) for a discussion of Hotelling's rule applicability to observable price dynamics.

- The long-run rate of growth is determined by the rate of technological change and the patience of the median consumer

$$1 + \gamma^{**} = [(1 + \lambda)\beta_m^{\alpha_3}]^{\frac{1}{1-\alpha_1}}.$$

  It is interesting to note that the discount factor of the most patient consumers β_{max} does not influence the steady state rate of growth though it impacts the interest rate.

3. Endogenous time preferences

It seems reasonable to make the conclusion from our results that in all conditions private ownership of exhaustible resources is more favorable for economic growth than common ownership. However, this conclusion is somewhat hasty. The point is that private ownership, other thing being equal, leads to higher level of income inequality than common ownership. At the same time, it is generally recognized (see e.g. Alesina and Perotti (1996)) that income inequality, by fuelling social discontent, increases sociopolitical instability and insecurity of property rights and the latter, by creating uncertainty in the politico-economic environment, can reduce investment and economic growth. Thus, if private ownership of natural resources leads to a high level of income inequality, it can result in sociopolitical instability and hence in a higher rate of resource extraction[†] and a lower rate of growth than common ownership.

To model this possibility, let us slightly modify our model and, following Borissov and Lambrecht (2009), make the assumption that the discount factors are formed endogenously. More precisely, let us assume that the objective function of consumer i is of the following form:

$$\sum_{t=0}^{\infty}[(1 - p)\beta_i]^t u(C_{i,t}),$$

[†]Similar reasoning on the impact of the insecurity of property rights on the extraction rate can be found Gaddy and Ickes (2005).

where p is a social objective magnitude reflecting a detrimental effect of social tension, political instability and insecurity of property rights on the economy. We assume $p = \psi(\eta)$, where η is a measure of income inequality and $\psi : [0,1] \to [0,1]$ is a continuous function equal to 0 at η smaller than some $\hat{\eta} > 0$ and increasing for $\eta > \hat{\eta}$. It should be noticed that on a non-balanced equilibrium path the income inequality and p change over time. However, on a balanced-growth path they are constant over time. On a balanced-growth path the income distribution depends on the fraction $\sigma = |J|/N$ of the most patient consumers in the population and the distribution of savings across the set of most patient consumers. To simplify our argument, let us restrict our consideration to the simple case where the savings across the most patient consumers are distributed evenly. In this case it is natural to take the following measure of income inequality:

$$\eta = \alpha(1 - \sigma),$$

where $\alpha = \alpha_1$ in the case of common ownership of natural resources and $\alpha = \alpha_1 + \alpha_3$ in the case of private ownership of natural resources. It is not difficult to check that $\alpha(1 - \sigma)$ is a good approximation to the Gini coefficient of income inequality and is equal to the latter if the set of consumers is a continuum.

It is not difficult to check that if we assume that $\eta = \alpha(1 - \sigma)$ and hence $p = \psi[\alpha(1 - \sigma)]$, then, in the case of private ownership of exhaustible resources the equilibrium long-run rate of growth, γ^*, is determined by

$$1 + \gamma^* = 1 + \{(1 + \lambda)\{[1 - \psi[(\alpha_1 + \alpha_3)(1 - \sigma)]]\beta_{max}\}^{\alpha_3}\}^{\frac{1}{1-\alpha_1}}$$

and in the case of common ownership the equilibrium long-run rate of growth, γ^{**}, is given by

$$1 + \gamma^{**} = 1 + \{(1 + \lambda)\{[1 - \psi[\alpha_1(1 - \sigma)]]\beta_m\}^{\alpha_3}\}^{\frac{1}{1-\alpha_1}}$$

It follows that

$$\gamma^* \gtreqless \gamma^{**} \Leftrightarrow \{1 - \psi[(\alpha_1 + \alpha_3)(1 - \sigma)]\}\beta_{max} \gtreqless \{1 - \psi[\alpha_1(1 - \sigma)]\})\beta_m.$$

One can readily see that if the median voter is patient, $\beta_m = \beta_{max}$, and inequality under the private property regime is sufficiently high, $(\alpha_1 + \alpha_3)(1 - \sigma) > \hat{\eta}$, then common ownership leads to a higher equilibrium rate of growth than private ownership. If $\beta_m < \beta_{max}$, the possibility that private ownership results in a higher rate of growth is more likely for lower values of the ratio between the discount factors of the median and the most patient consumers, β_{max}/β_m, and for lower elasticity of output in the input of natural resources, α_3.

4. Conclusion

In this paper, we have developed two Ramsey-type models of economic growth with exhaustible natural resources and consumers heterogeneous in their intertemporal preferences, which leads to different preferences on the rate of resource extraction.

The first model assumes private ownership of the resource stocks. As one would expect, under the private property regime in the long run all capital and natural resources are owned by the most patient consumers, who determine the long-run rate of extraction and the long-run rate of growth.

The second model assumes the resource stocks to be collectively owned and the resource rent to be equally divided among all the consumers. The consumers choose the resource extraction rate by voting. For this model we have introduced a notion of voting equilibrium and fully characterized steady-state voting equilibria. In particular, we have shown that the long-run extraction rate and the long-run rate of growth are determined by the discount factor of the median consumer. Somewhat unexpectedly, the long-run rate of growth does not depend on the discount factor of the most patient consumers.

It follows that if the median consumer belongs to the set of the most patient consumers, the rates of growth under the two regimes are equal, and if the median discount factor is lower than that of the most patient consumers, the long-run rate of growth under the private property regime is higher than under the common property regime. However, this conclusion is no longer true if the discount factors are formed endogenously because, as we have shown, in that case under some circumstances common property can result in a higher rate of growth than private property.

References

Alesina, A. and Perotti, R., 1996. Income distribution, political instability, and investment, *European Economic Review*, 40, 1203–1228.

Baron, D.P., 1996. A dynamic theory of collective goods programs, *American Political Science Review*, 90, 316–330.

Becker, R.A., 1980. On the long–run steady state in a simple dynamic model of equilibrium with heterogeneous households, *The Quarterly Journal of Economics*, 95, 375–382.

Becker, R.A., 2006. Equilibrium dynamics with many agents, *in:* T.M. Rose-Anne Dana, Cuong Le Van and K. Nishimura, eds., *Handbook on Optimal Growth 1. Discrete Time*, Springer Berlin Heidelberg, 385–442.

Bernheim, B. and Slavov, S., 2009. A solution concept for majority rule in dynamic settings, *Review of Economic Studies*, 76 (1), 33–62.

Borissov, K., Brechet, T., and Lambrecht, S., 2010. Median voter environmental maintenance, *CORE Discussion Paper*.

Borissov, K. and Lambrecht, S., 2009. Growth and distribution in an ak–model with endogenous impatience, *Economic Theory*, 39 (1), 93–112.

Chermak, J.M. and Patrick, R.H., 2002. Comparing tests of the theory of exhaustible resources, *Resource and energy economics*, 24 (4), 301–325.

Cooley, T.F. and Soares, J., 1999. A positive theory of social security based on reputation, *Journal of Political Economy*, 107, 135–160.

Gaddy, C.G. and Ickes, B.W., 2005. Resource rents and the russian economy, *Eurasian Geography and Economics*, 46 (8), 559–583.

Heal, J. *et al.*, eds., 1974. *Symposium on the Economics of Exhaustible Resources. Special issue of The Review of economic studies*, vol. 41.

Heltberg, R., 2002. Property rights and natural resource management in developing countries, *Journal of Economic Surveys*, 16 (2), 189–214.

Hotelling, H., 1931. The economics of exhaustible resources, *The Journal of Political Economy*, 39, 137–175.

Krusell, P., Quadrini, V., and Rios-Rull, J.V., 1997. Politico-economic equilibrium and economic growth, *Journal of Economic Dynamics and Control*, 21, 243–272.

Ostrom, E. and Hess, C., 2007. Private and common property rights, SSRN eLibrary, available at http://ssrn.com/paper=1304699.

Rangel, A., 2003. Forward and backward intergenerational goods: Why is social security good for the environment?, *American Economic Review*, 93, 813–834.

Stiglitz, J., 1974. Growth with exhaustible natural resources: efficient and optimal growth paths, *The Review of Economic Studies*, 41, 123–137.

Борисов К.Ю., Сурков А.В. Общественная и частная собственность на невосполнимые ресурсы в моделях экономического роста. — СПб. : Издательство Европейского университета в Санкт-Петербурге, 2010. — 20 с. — (Серия препринтов; факультет экономики, Ec-02/10). — На английском языке.

Отпечатано с оригинал-макета, предоставленного авторами

Подписано в печать 29.09.10
Формат 60x88 1/16. Тираж 50 экз.

Издательство Европейского университета в Санкт-Петербурге
191187, Санкт-Петербург, ул. Гагаринская, 3
books@eu.spb.ru