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Эндогенный рост в модели с неоднородными агентами и голосованием по поводу общественных благ
на английском языке

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We consider a Barro-type endogenous growth model in which the government's purchases of goods and services enter into the production function. The provision of government services is financed by flat-rate (linear) income or lump-sum taxes. It is assumed that individuals differing in their discount factors vote on the tax rates. We propose a concept of voting equilibrium leading to some versions of the median voter theorem for steady-state equilibria, fully characterize steady-state equilibria and show that if the median voter discount factor is sufficiently low, the long-run rate of growth in the case of flat-rate income taxation is higher than that in the case of lump-sum taxation.
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Издание осуществлено за счет средств проекта создания специализации по природным ресурсам и экономике энергетики «ЭксонМобил».

Endogenous growth in a model with heterogeneous agents and voting on public goods

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A well-known model of endogenous economic growth proposed by Barro (1990) includes public services as a productive input for private producers. In this model, the production with fixed government purchases is subject to diminishing returns to accumulated capital. But if government purchases rise along with the capital stock, the production function specifies constant returns in government purchases and capital. This is the reason for the economy to be capable of endogenous growth.

A crucial assumption of this model is made when describing the consumers behavior. It is assumed that there is one representative infinitely-lived consumer seeking to maximize her overall utility of consumption. The rate of growth, then, depends on the discount factor describing how the representative consumer values her future consumption. This leads to an issue of choosing the discount factor, which somehow aggregates intertemporal preferences of all existing consumers.

To relax the restrictive and unrealistic assumption of a representative consumer, we propose a model with heterogeneous agents differing in their discount factors. A Ramsey-type model with consumer impatience heterogeneity was introduced by Becker (1980), who showed that if an agent's objective function over an infinite horizon is represented by a stationary, additive, discounted-utility function with a constant pure rate of time preference, then the income distribution in the steady state is determined by the lowest discount rate. The consumer with the lowest rate of discount owns all the capital and earns the capital rent as well as the wage income; all other consumers receive the wage income only.

There is a rich literature on models of economic growth with consumers having different rates of impatience surveyed by Becker (2006). However, most existing papers on this topic ignore public sector. Exceptions include papers by Sarte

(1997) and Sorger (2002), who study the impact of a progressive income tax on the long-run distribution of wealth.

In Becker's model, the long-run capital intensity is determined by the discount factor of the most patient consumer. It is reasonable to expect that in an endogenous growth model the long-run rate of growth is determined by the discount factor of the most patient consumer. This would be true in the Barro model with consumers having different discount factors if tax rates were given exogenously.

However the assumption of an exogenously given tax rate embedded into the Barro model, does not seem reasonable. From a positive point of view, taxes should be considered as endogenous. When we consider a model with heterogeneous consumers it is more sensible to assume that when determining tax rates any government is likely to be responsive to the wishes of a majority of population. To capture this responsiveness we adopt a politico-economic approach and assume that tax rates are determined endogenously by a voting procedure, which serves as a metaphor for real political process. Even in a dictatorship, a majority of population influences economic policy decisions.

Our model is akin to a model proposed by Alesina and Rodrik (1994), who study Barro's model under the assumption that public investment is financed by a tax on capital income and heterogeneous consumers vote on the tax rate, but our focus is different. In that model, as in most other politico-economic models of economic growth with infinitely lived consumers , individuals have identical discount factors, but differ in income and/or wealth (see Bassetto and Benhabib (2006); Grossmann (2003); Corbae and Kuruscu (2009); Fiaschi (1999); Azzimonti *et al.* (2008); Krusell and Rios-Rull (1999) among others).

In our model, consumers are differentiated by their fixed rates of time preference. We focus on the effects of consumer heterogeneity in time preference on policy choices and on the long-run rate of economic growth. In particular, we compare the long-run rates of growth under flat-rate and lump sum taxation.

Another distinction between our approach to modeling policy decisions and that of Alesina and Rodrik (1994) is that they assume the applicability of the median voter theorem to steady-state equilibria from the outset. They justify this assumption by the fact that they proceed from steady-state equilibria. By contrast, we treat the problem as truly dynamic and following Borissov *et al.* (2010) first propose a notion of voting equilibrium path (which is not necessarily balanced) and then prove two versions of the median voter theorem for steady-state equilibria.

The application of the median voter theorem to dynamic models of the political economy requires a suitable analytical simplification of the political and economic environment. Models of this kind are much harder to analyze than their static counterparts and than the usual intertemporal models without political components. The performance of majoritarian institutions in dynamic settings has attracted growing interest and attention (see e.g. Baron (1996); Krusell *et al.* (1997); Cooley and Soares (1999); Rangel (2003); Bernheim and Slavov (2009)). However, the development of the theory is still in its infancy and the consensus on how to model dynamic majoritarian voting has not been achieved.

Without going into detail, notice that our approach to voting is different from approaches accepted in the above-mentioned papers. Our definition of voting equilibrium can be considered as a dynamic version of the Bowen equilibrium (Bowen, 1943; Bergstrom, 1979). It is closely related to Kramer-Shepsle's equilibrium (Kramer, 1972; Shepsle, 1979).

In this paper, we describe the structure of steady-state voting equilibria in the cases of flat-rate taxation and lump-sum taxation and compare them and derive explicit formulas the long-run rates of growth. In particular, we show that in both cases the long-run equilibrium rate of growth is increasing in the discount factors of the most patient and the median consumers. In the case of flat-rate income taxation, the long-run equilibrium rate of growth is completely determined by these discount factors. In the case of lump-sum taxation, if the median discount factor is equal to the highest one, the long-run equilibrium rate of growth also is increasing in the relative wealth of the median voter.

We also compare the equilibrium rates of growth in the cases of flat-rate and lump sum taxation and show that if the median voter discount factor is sufficiently low, the rate of growth in the case of flat-rate income taxation is higher than that in the case of lump sum taxation. This result is in some contrast with the conventional wisdom that lump sum taxes are always preferable to distorting taxes.

The paper is organized as follows. In section 1, we introduce our main assumptions. In section 2, we consider the case of flat-rate income taxes and section 3 analyzes the case of lump sum taxes. In section 4, we compare the equilibrium rates of growth in these two cases. Conclusion follows in section 5.

1. The model

1.1. Consumers

There is an odd number L of consumers. We assume that all consumers live for an infinite period of time and are identical in all respect except their discount factors. Each time each consumer supply one unit of labor force in the labor market. Thus the total labor supply at each time is L.

The utility function of consumer i is of the form $\sum_{t=0}^{\infty} \beta_i^t u(C_{i,t})$, where β_i is the discount function of this consumer and $C_{i,t}$ is his consumption at time t. We assume that

$$u(C) = \ln C.$$

We assume that the consumers are sorted in the ascending order of their discount factors:

$$0 < \beta_1 \le \beta_2 \le \cdots \le \beta_L \equiv \beta_{max} < 1.$$

By I we denote the set of agents with the highest discount factor:

$$I = \{i = 1, \ldots, L \mid \beta_i = \beta_L\}.$$

We consider two types of taxes : a flat-rate income tax and a lump sum tax.

In the case of flat-rate income taxation the budget constraints of consumer i is of the following form

$$C_{i,t} + S_{i,t} \le (1 - \theta_t) \left[(1 + r_t) S_{i,t-1} + W_t \right], \quad S_{i,t} \ge 0, \quad t = 0, 1, \ldots,$$

Here r_t and W_t are interest and wage rates at time t, $S_{i,t}$ are the savings of consumer i at time t and θ_t ($0 \le \theta_t \le 1$), $t = 0, 1, \ldots,$ are the tax rates. Consumers are prohibited to borrow against their future wage earnings. Therefore, their savings must be non-negative.

In the case of lump sum taxation at given tax rates $\tau_t \ge 0$, $t = 0, 1, \ldots,$ the budget constraint of consumer i takes the form

$$C_{i,t} + S_{i,t} \le (1 + r_t) S_{i,t-1} + W_t - \tau_t, \quad S_{i,t} \ge 0, \quad t = 0, 1, \ldots$$

1.2. Production

We assume that output at each time t, Y_t, is given by

$$Y_t = \phi(g_t) F(K_t, L), \tag{1}$$

where K_t is the capital stock and g_t is the per capita quantity of public good at time t. Capital fully depreciates during one time period. We assume that the production function is Cobb-Douglas:

$$F(K, L) = K^{\alpha} L^{1-\alpha}, \qquad 0 < \alpha < 1,$$

and that

$$\phi(g) = g^{1-\alpha}.$$

Thus,

$$Y_t = g_t^{1-\alpha} K_t^{\alpha} L^{1-\alpha}.$$

Hence the production function given by (1) is of the constant returns to scale in K_t and L for any given quantity of per capita public good g_t. At the same time, it is of constant return to scale in the stock of capital, K_t, and the quantity of per capita public good, g_t.

Per capita output $y_t = Y_t/L$ can be written as follows

$$y_t = \phi(g_t) f(k_t) = g_t^{1-\alpha} k_t^{\alpha},$$

where $k_t = K_t/L$ and $f(k_t) = k_t^{\alpha}$.

2. Endogenous growth with flat-rate income tax

In this section we assume that the government collects flat-rate income taxes. Further, in section 3, we will give necessary definitions and formulate corresponding theorems for the case of lump sum taxation.

2.1. Equilibrium paths and steady-state equilibria

Let the sequence $\Theta = (\theta_t)_{t=0}^{\infty}$ of tax rates and an initial state specified by $\hat{g}_0 > 0$ and a tuple $\left\{\hat{S}_{i,-1}\right\}_{i=1,\dots,L}$ such that

$$\hat{S}_{i,-1} \geq 0, \quad i = 1, \dots, L, \quad \sum_{i=1}^{L} \hat{S}_{i,-1} > 0,$$

be given. We define an *equilibrium path* starting from this initial state as a sequence

$$\left\{k_t^*, r_t^*, W_t^*, g_t^*, \left(C_{i,t}^*, S_{i,t}^*\right)_{i=1,\dots,L}\right\}_{t=0,1,\dots}$$

such that $g_0^* = \hat{g}_0$ and

1) for each $i = 1, \dots, L$, the sequence $\left(C_{i,t}^*, S_{i,t}^*\right)_{t=0,1,\dots}$ is a solution to the following problem:

$$\max \sum_{t=0}^{\infty} \beta_i^t u(C_{i,t}), \tag{2}$$

$$C_{i,t} + S_{i,t} \leq (1 - \theta_t)\left[(1 + r_t^*) S_{i,t-1} + W_t^*\right], \quad t = 0, 1, \dots,$$

$$S_{i,t} \geq 0, \quad t = 0, 1, \dots$$

where $S_{i,-1} = \hat{S}_{i,-1}$.

2) total consumer savings are equal to gross investment:

$$Lk_t^* = \sum_{i=1}^{L} S_{i,t-1}^*, \quad t = 0, 1, \ldots,$$

3) capital is paid its marginal product:

$$1 + r_t^* = \phi(g_t^*) f'(k_t^*), \quad t = 0, 1, \ldots,$$

4) labor is paid its marginal product:

$$W_t^* = \phi(g_t^*)(f(k_t^*) - f'(k_t^*)k_t^*), \quad t = 0, 1, \ldots,$$

5) at each time public sector is financed via taxes collected in the previous period:

$$g_{t+1}^* = \theta_t \phi(g_t^*) f(k_t^*), \quad t = 0, 1, \ldots$$

We shall make our emphasis on steady-state equilibria. They are defined as follows.

Let the tax rate be constant over time: $\theta_t = \theta, \quad t = 0, 1, \ldots$. A tuple

$$\left\{\gamma^*, k^*, r^*, W^*, g^*, (C_i^*, S_i^*)_{i=1,\ldots,L}\right\}$$

is called a *steady-state equilibrium* if the sequence

$$\left\{k_t^*, r_t^*, W_t^*, g_t^*, \left(C_{i,t}^*, S_{i,t}^*\right)_{i=1,\ldots,L}\right\}_{t=0,1,\ldots}$$

given for $t = 0, 1, \ldots$ and $i = 1, \ldots, L$ by

$$\begin{aligned} k_t^* &= (1+\gamma^*)^t k^*, & 1 + r_t^* &= 1 + r^*, & W_t^* &= (1+\gamma^*)^t W^*, \\ g_t^* &= (1+\gamma^*)^t g^*, & C_{i,t}^* &= (1+\gamma^*)^t C_i^*, & S_{i,t}^* &= (1+\gamma^*)^t S_i^*, \end{aligned}$$

is an equilibrium path.

To describe steady-state equilibria we need the following definition.

Suppose that for some r, W, γ, θ,

$$r_t = r, \quad W_t = (1+\gamma)^t W, \quad \theta_t = \theta, \quad t = 0, 1, \ldots$$

We call a couple $\left(C_i^*, S_i^*\right)$ a *balanced optimum of consumer i* if the sequence $(C_{i,t}^*, S_{i,t}^*)_{t=0}^{\infty}$ given by

$$C_{i,t}^* = (1+\gamma)^t C_i^*, \qquad S_{i,t}^* = (1+\gamma)^t S_i^*, \quad t = 0, 1, \ldots,$$

is a solution to problem (2) at $\hat{S}_{i,-1} = (1+\gamma)^{-1} S_i^*$.

It is clear that for any constant over time tax rate, θ, a tuple

$$\left\{\gamma^*, k^*, r^*, W^*, g^*, (C_i^*, S_i^*)_{i=1,\ldots,L}\right\}$$

is a steady-state equilibrium if and only if

$$1 + r^* = \phi(g^*) f'(k^*), \tag{3}$$

$$W^* = \phi(g^*)[f(k^*) - f'(k^*)k^*], \tag{4}$$

$$(1+\gamma^*)\, k^* L = \sum_{i=1}^{L} S_i^*, \tag{5}$$

$$(1+\gamma^*)\, g^* = \theta\phi(g^*) f(k^*), \tag{6}$$

and, for each $i = 1, \ldots, L$, the couple $\left(C_i^*, S_i^*\right)$ is a balanced optimum of consumer i at $r = r^*$ and $W = W^*$.

To describe properties of steady-state equilibria we formulate the following simple lemma.

Lemma 1. *Given r, W, γ and θ,*

1) a balanced optimum of consumer i exists if and only if

$$(1-\theta)\beta_i \le \frac{1+\gamma}{1+r};$$

2) if $(1-\theta)\beta_i = (1+\gamma)\,/\,(1+r)$, then any couple $\left(C_i^*, S_i^*\right)$ such that

$$C_i^* + S_i^* = (1-\theta)\left(\frac{1+r}{1+\gamma} S_i^* + W\right), \quad C_i^* \ge 0, \quad S_i^* \ge 0,$$

is a balanced optimum of consumer i;

3) if $(1-\theta)\beta_i < (1+\gamma)\,/\,(1+r)$, then there is a unique balanced optimum of consumer i, $\left(C_i^*, S_i^*\right)$; it is given by $C_i^* = (1-\theta)\,W$, $S_i^* = 0$.

Now we can formulate an important proposition describing the structure of steady-state equilibria in the case of flat-rate income taxation. It follows from Lemma 1.

Proposition 1. *Let a constant over time tax rate, θ, be given. A tuple*

$$\left\{\gamma^*, k^*, r^*, W^*, g^*, (C_i^*, S_i^*)_{i=1,\ldots,L}\right\}$$

is a steady-state equilibrium if and only if it satisfies conditions (3)–(6) and

$$(1-\theta)\beta_{max} = \frac{1+\gamma^*}{1+r^*}, \tag{7}$$

$$C_i^* + S_i^* = (1-\theta)\left(\frac{1+r^*}{1+\gamma^*}S_i^* + W^*\right), \quad C_i^* \geq 0, \quad S_i^* \geq 0, \quad i \in I, \tag{8}$$

$$S_i^* = 0, \quad C_i^* = (1-\theta)W^*, \quad i \notin I. \tag{9}$$

To prove this proposition it is sufficient to repeat a well-known argument by Becker (1980, 2006). The proposition says that only the most patient consumers make positive savings and own all the capital.

2.2. Voting equilibria

Consider an equilibrium path

$$\left\{k_t^*, r_t^*, W_t^*, g_t^*, \left(C_{i,t}^*, S_{i,t}^*\right)_{i=1,\dots,L}\right\}_{t=0,1,\dots}$$

and ask each consumer i, weather she prefers to increase or decrease the tax rate at time t. We assume that when answering this question, consumers take into account the fact that the taxes collected at time t are equal to the supply of public goods at time $t+1$ and that the latter impacts the total factor productivity and hence marginal products of capital and labor.

To be more precise, first note that for each i, $(C_{i,t}^*, S_{i,t}^*)_{t=0,1,\dots}$ is a solution to the following problem:

$$\max \sum_{t=0}^{\infty} \beta_i^t u(C_{i,t}), \tag{10}$$

$$C_{i,t} + S_{i,t} \leq (1-\theta_t)[\phi(g_t^*) f'(k_t^*) S_{i,t-1} + \phi(g_t^*)(f(k_t^*) - f'(k_t^*)k_t^*)], \tag{11}$$

$$S_{i,t} \geq 0, \quad t = 0, 1, \dots,$$

where $S_{i,-1} = \hat{S}_{i,-1}$. Recall that

$$g_0^* = \hat{g}_0, \quad g_t^* = \theta_{t-1}\phi(g_{t-1}^*) f(k_{t-1}^*), \quad t = 0, 1, \dots \tag{12}$$

Let us consider the value of (10), V_i, as a function of $\Theta = (\theta_t)_{t=0}^{\infty}$. We assume that the attitude of consumer i to a possible change in θ_t is determined by the sign of the derivative $\partial V_i/\partial\theta_t$, if it exists. If $\partial V_i/\partial\theta_t > 0$, consumer i *is in favor of increasing* θ_t. If $\partial V_i/\partial\theta_t < 0$, consumer i is *in favor of decreasing* θ_t.

When calculating the derivative $\partial V_i/\partial\theta_t$, consumer i treats k_t^*, $t = 0, 1, \dots$, as independent of θ_t. Thus she takes account of the effect of the supply of public goods and hence of the tax choice on total factor productivity but she is ignorant of the effect of the tax choice on the capital stocks in intertemporal general equilibrium.

Definition 1. *We call a sequence*

$$\left\{\theta_t^*, k_t^*, r_t^*, W_t^*, g_t^*, \left(C_{i,t}^*, S_{i,t}^*\right)_{i=1,\ldots,L}\right\}_{t=0,1,\ldots}$$

a voting equilibrium path if $\left\{k_t^*, r_t^*, W_t^*, g_t^*, \left(C_{i,t}^*, S_{i,t}^*\right)_{i=1,\ldots,L}\right\}_{t=0,1,\ldots}$ *is an equilibrium path at* $\Theta = (\theta_t^*)_{t=0}^{\infty}$ *and for each t, the number of consumers who are in favor of increasing* θ_t *and the number of those who are in favor of decreasing* θ_t *is less than* $L/2$.

Here we certainly do not mean that the individuals votes for any change in a tax rate. This consideration merely reflects the idea that the government responds somehow to the wishes of a majority when choosing tax rates.

We will not discuss the existence and properties of voting equilibria in general form, but we will describe voting steady-state equilibria.

Definition 2. *If the sequence* $\left\{\theta_t^*, k_t^*, r_t^*, W_t^*, g_t^*, \left(C_{i,t}^*, S_{i,t}^*\right)_{i=1,\ldots,L}\right\}_{t=0,1,\ldots}$ *given by*

$$\begin{gathered}\theta_t^* = \theta^*, \qquad k_t^* = k^*\left(1+\gamma^*\right)^t, \qquad g_t^* = g^*\left(1+\gamma^*\right)^t, \\ C_{i,t}^* = C_i^*\left(1+\gamma^*\right)^t, \qquad S_{i,t}^* = S_i^*\left(1+\gamma^*\right)^t, \\ i = 1,\ldots,L, \quad t = 0,1,\ldots\end{gathered}$$

forms a voting equilibrium path, then the tuple

$$\left\{\theta^*, \gamma^*, k^*, r^*, W^*, g^*, (C_i^*, S_i^*)_{i=1,\ldots,L}\right\}$$

is called a voting steady-state equilibrium.

The following theorem describes voting steady-state equilibria in the case of flat-rate income taxation. It reads that an important role in formating steady-state equilibria is played by the median consumer $m = (L+1)/2$.

Theorem 1. *A tuple*

$$\left\{\theta^*, \gamma^*, k^*, r^*, W^*, g^*, (C_i^*, S_i^*)_{i=1,\ldots,L}\right\}$$

represents a voting steady-state equilibrium, if and only if it satisfies conditions (3)–(9) at $\theta = \theta^*$ *and*

$$1+\gamma^* = (1-\theta^*)\beta_m\phi'(g^*)f(k^*). \tag{13}$$

Proof. Let the tuple

$$\left\{\gamma^*, k^*, r^*, W^*, g^*, (C_i^*, S_i^*)_{i=1,\dots,L}\right\}$$

be a steady-state equilibrium constructed at $\theta = \theta^*$ and

$$\left\{k_t^*, r_t^*, W_t^*, g_t^*, \left(C_{i,t}^*, S_{i,t}^*\right)_{i=1,\dots,L}\right\}_{t=0,1,\dots}$$

be an equilibrium path corresponding to this steady-state equilibrium.

By the envelope theorem, taking into account (12), for all $i = 1, \dots, L$ and all $t = 0, 1, \dots,$ we have

$$\begin{aligned}
\frac{\partial V_i}{\partial \theta_t} &= -\beta_i^t u'\left(C_{i,t}^*\right)\left[(1 + r_t^*) S_{i,t-1}^* + W_t^*\right] \\
&\quad + \beta_i^{t+1} u'\left(C_{i,t+1}^*\right)(1 - \theta^*)\left[\phi'\left(g_{t+1}^*\right) f'\left(k_{t+1}^*\right) S_{i,t}^*\right. \\
&\qquad \left. + \ \phi'\left(g_{t+1}^*\right)\left(f\left(k_{t+1}^*\right) - f'\left(k_{t+1}^*\right) k_t^*\right)\right] \phi\left(g_t^*\right) f\left(k_t^*\right) \\
&= -\beta_i^t u'\left(C_{i,t}^*\right)\left[(1 + r^*) S_{i,t-1}^* + W_t^*\right] \\
&\quad + \beta_i^{t+1} u'\left(C_{i,t+1}^*\right)(1 - \theta^*) \frac{\phi'\left(g_{t+1}^*\right)}{\phi\left(g_{t+1}^*\right)} \\
&\qquad \times \frac{f\left(k_{t+1}^*\right)}{f\left(k_{t+1}^*\right)}\left[(1 + r^*) S_{i,t}^* + W_{t+1}^*\right] \phi\left(g_t^*\right) f\left(k_t^*\right).
\end{aligned}$$

Also we have

$$\begin{gathered}
g_{t+1}^* = (1 + \gamma^*) g_t^*, \quad k_{t+1}^* = (1 + \gamma^*) k_t^*, \\
C_{i,t+1}^* = (1 + \gamma^*) C_{i,t}^*, \quad u'\left(C_{i,t+1}^*\right) = \frac{u'\left(C_{i,t}^*\right)}{1 + \gamma^*}, \\
\left[(1 + r^*) S_{i,t}^* + W_{t+1}^*\right] = (1 + \gamma^*)\left[(1 + r^*) S_{i,t-1}^* + W_t^*\right], \\
\phi\left(g_{t+1}^*\right) f\left(k_{t+1}^*\right) = (1 + \gamma^*) \phi\left(g_t^*\right) f\left(k_t^*\right), \\
\phi'\left(g_{t+1}^*\right) f\left(k_{t+1}^*\right) = \phi'\left(g^*\right) f\left(k^*\right).
\end{gathered}$$

Therefore,

$$\begin{aligned}
\frac{\partial V_i}{\partial \theta_t} &= -\beta_i^t u'\left(C_{i,t}^*\right)\left[(1 + r^*) S_{i,t-1}^* + W_t^*\right] \\
&\qquad \times \left[\frac{\beta_i (1 - \theta^*)}{1 + \gamma^*} \phi'(g_{t+1}^*) f(k_{t+1}^*) - 1\right]
\end{aligned}$$

and hence

$$\operatorname{sign}\frac{\partial V_i}{\partial \theta_t} = \operatorname{sign}\left\{\frac{\beta_i\,(1-\theta^*)}{1+\gamma^*}\,\phi'\left(g^*_{t+1}\right)f\left(k^*_{t+1}\right)-1\right\}$$
$$= \operatorname{sign}\left\{\frac{\beta_i\,(1-\theta^*)}{1+\gamma^*}\,\phi'\left(g^*\right)f\left(k^*\right)-1\right\}.$$

To complete the proof it is sufficient to notice that

$$\frac{\partial V_i}{\partial \theta_t} \gtreqless 0 \quad \Leftrightarrow \quad \beta_i(1-\theta^*)\phi'(g^*)f(k^*) \gtreqless 1+\gamma^*.$$

□

This theorem allows us to calculate the equilibrium growth and tax rates in the case of flat-rate income taxation. Recalling that the production function is Cobb-Douglas,

$$\phi(g_t) = g_t^{1-\alpha}, \qquad f(k_t) = k_t^{\alpha},$$

one obtains from (6)

$$1+\gamma^* = \theta^*\frac{\phi\left(g^*\right)f\left(k^*\right)}{g^*} = \theta^*\left(\frac{k^*}{g^*}\right)^{\alpha}.$$

At the same time (3) and (7) give

$$1+\gamma^* = \alpha\beta_{max}(1-\theta^*)\left(\frac{g^*}{k^*}\right)^{1-\alpha},$$

and (13) yields

$$1+\gamma^* = (1-\alpha)\beta_m(1-\theta^*)\left(\frac{k^*}{g^*}\right)^{\alpha}.$$

It follows that

$$\theta^* = \frac{(1-\alpha)\beta_m}{1+(1-\alpha)\beta_m}, \qquad \frac{k^*}{g^*} = \frac{\alpha\beta_{max}}{(1-\alpha)\beta_m},$$

and hence

$$1+\gamma^* = \frac{(\alpha\beta_{max})^{\alpha}\left[(1-\alpha)\beta_m\right]^{1-\alpha}}{1+(1-\alpha)\beta_m}. \tag{14}$$

Thus, the equilibrium tax rate grows as the patience of the median consumer represented by β_m increases. As for the long-run rate of growth, it increases with the discount factors of the median and the most patient consumers.

3. Endogenous growth under lump sum taxation

In this section we consider the case of lump sum taxation. In this case the definition of equilibrium path is slightly different from that in the case of flat-rate income taxation.

3.1. Equilibrium paths and steady-state equilibria

Suppose that tax rates τ_t, $t = 0, 1, \ldots$ and an initial state $\hat{g}_0$, $\{\hat{S}_{i,-1}\}_{i=1,\ldots,L}$ are given. We define an *equilibrium path* starting from this initial state as a sequence

$$\left\{k_t^\star, r_t^\star, W_t^\star, g_t^\star, \left(C_{i,t}^\star, S_{i,t}^\star\right)_{i=1,\ldots,L}\right\}_{t=0,1,\ldots}$$

such that

1) $1 + r_t^\star = \phi(g_t^\star) f'(k_t^\star)$;
2) $W_t^\star = \phi(g_t^\star)\left[f(k_t^\star) - k_t^\star f'(k_t^\star)\right]$;
3) for each $i = 1, \ldots, L$, the sequence $\left(C_{i,t}^\star, S_{i,t}^\star\right)_{t=0,1,\ldots}$ is a solution to the following problem:

$$\max \sum_{t=0}^{\infty} \beta_i^t u(C_{i,t}), \tag{15}$$

$$C_{i,t} + S_{i,t} \leq \left(1 + r_t^\star\right) S_{i,t-1} + W_t^\star - \tau_t, \quad t = 0, 1, \ldots,$$

$$S_{i,t} \geq 0, \quad t = 0, 1, \ldots,$$

   where $S_{i,-1} = \hat{S}_{i,-1}$;
4) $Lk_t^\star = \sum_{i=1}^{L} S_{i,t-1}^\star, \quad t = 0, 1, \ldots;$
5) $g_{t+1}^\star = \tau_t, \quad t = 0, 1, \ldots.$

When defining equilibrium paths we take the tax rates τ_t, $t = 0, 1, \ldots$, as given and assume that the taxes are not too burdensome: $\tau_t < W_t^\star$, $t = 0, 1, \ldots$. To define a steady-state equilibrium it is convenient to take as given another magnitude because on a balanced equilibrium path τ_t must grow at the same rate as the economy as a whole. But this growth rate is endogenously determined and is not known a priori.

For any balanced equilibrium path

$$\left\{k_t^\star, r_t^\star, W_t^\star, g_t^\star, \left(C_{i,t}^\star, S_{i,t}^\star\right)_{i=1,\ldots,L}\right\}_{t=0,1,\ldots}$$

the proportion $\tau_t / [\phi(g_t) f(k_t)]$ is constant over time. To define steady-state equilibria it is sufficient to take this proportion as given. Denote it by μ and notice that $\tau < W_t^\star$ is equivalent to $\mu < 1 - \alpha$.

Let $\mu < 1 - \alpha$ be given. We define a *steady-state equilibrium* as a tuple

$$\left\{\gamma^\star, k^\star, r^\star, W^\star, g^\star, \left(C_i^\star, S_i^\star\right)_{i=1,\dots,L}\right\},$$

such that the sequence $\left\{k_t^\star, r_t^\star, W_t^\star, g_t^\star, \left(C_{i,t}^\star, S_{i,t}^\star\right)_{i=1,\dots,L}\right\}_{t=0,1,\dots}$ given for $t = 0, 1, \dots$ and $i = 1, \dots, L$ by

$$\begin{aligned} k_t^\star &= (1+\gamma^\star)^t k^\star, & 1 + r_t^\star &= 1 + r^\star, & W_t^\star &= (1+\gamma^\star)^t W^\star \\ g_t^\star &= (1+\gamma^\star)^t g^\star, & C_{i,t}^\star &= (1+\gamma^\star)^t C_i^\star, & S_{i,t}^\star &= (1+\gamma^\star)^t S_i^\star, \end{aligned}$$

is an equilibrium path at $(\tau_t)_{t=0}^\infty$ determined by

$$\tau_t = \mu\phi(g_t^\star) f(k_t^\star), \quad t = 0, 1, \dots.$$

The definition of a balanced consumer optimum can be given as follows.

For given r, W, γ, $\tau < W$, we call a couple $(C_i^\star, S_i^\star)$ a *balanced consumer optimum* if the sequence $(C_{i,t}^\star, S_{i,t}^\star)_{t=0}^\infty$ given by

$$C_{i,t}^\star = C_i^\star (1+\gamma)^t, \qquad S_{i,t}^\star = S_i^\star (1+\gamma)^t, \quad t = 0, 1, \dots,$$

is a solution to the following problem:

$$\max \sum_{t=0}^{\infty} \beta_i^t u(C_{i,t}),$$

$$C_{i,t} + S_{i,t} \le (1 + r_t) S_{i,t-1} + (1+\gamma)^t W - (1+\gamma)^t \tau, \quad t = 0, 1, \dots,$$

$$S_{i,t} \ge 0, \quad t = 0, 1, \dots,$$

at $S_{i,-1} = (1+\gamma)^{-1} S_i^\star$.

It is evident that, for a given $\mu < 1 - \alpha$, a tuple

$$\left\{\gamma^\star, k^\star, r^\star, W^\star, g^\star, \left(C_i^\star, S_i^\star\right)_{i=1,\dots,L}\right\}$$

is a steady-state equilibrium, if and only if it satisfies conditions (3)-(5),

$$\left(1+\gamma^\star\right) g^\star = \mu\phi(g^\star) f(k^\star), \tag{16}$$

and, for all $i = 1, \dots, L$, the couple $\left(C_i^\star, S_i^\star\right)$ is a balanced consumer optimum at $r = r^\star$, $W = W^\star$, $\gamma = \gamma^\star$, $\tau = \mu\phi(g^\star) f(k^\star)$.

The following lemma is useful for describing the structure of steady-state equilibria.

Lemma 2 (cf. Lemma 1)**.** *Let r, W, γ, $\tau < W$ be given.*

1) A balanced consumer optimum for consumer i exists, if, and only if

$$\beta_i \leq \frac{1+\gamma}{1+r}.$$

2) If $\beta_i = (1+\gamma)/(1+r)$, then any couple $\left(C_i^\star, S_i^\star\right)$, such that

$$C_i^\star + S_i^\star = \frac{1+r}{1+\gamma} S_i^\star + W - \tau,$$

is a balanced consumer optimum.

3) If $\beta_i < (1+\gamma)/(1+r)$, then the only balanced consumer optimum is the couple $\left(C_i^\star, S_i^\star\right)$ given by $C_i^\star = W - \tau$, $S_i^\star = 0$.

Proposition 2 (cf. Proposition 1)**.** *A tuple*

$$\left\{\gamma^\star, k^\star, r^\star, W^\star, g^\star, \left(C_i^\star, S_i^\star\right)_{i=1,\ldots,L}\right\}$$

forms a steady-state equilibrium, if and only if it satisfies conditions (3)-(5), (16), and for $\tau = \mu\phi(g^\star) f(k^\star)$, the following conditions hold:

$$1+\gamma^\star = \beta_{max}(1+r^\star), \tag{17}$$

$$C_i^\star + S_i^\star = \frac{1+r^\star}{1+\gamma^\star} S_i^\star + W^\star - \tau, \quad S_i \geq 0, \quad i \in I, \tag{18}$$

$$S_i^\star = 0, \qquad C_i^\star = W^\star - \tau, \quad i \notin I. \tag{19}$$

3.2. Voting equilibria

Consider the value of problem (15) as a function of tax rates, $\mathrm{T} = (\tau_t)_{t=0}^{\infty}$, and denote this value as $V_i(\mathrm{T})$.

As in the case of flat-rate taxes, to define voting equilibrium paths in case of lump sum taxation we assume that when voting, consumer i looks at the value of the derivative $\partial V_i/\partial\tau_t$, if one exists. If $\partial V_i/\partial\tau_t > 0$, this consumer is *in favor of increasing* of τ_t. If $\partial V_i/\partial\tau_t < 0$, the consumer is *in favor of decreasing* of τ_t. If $\partial V_i/\partial\tau_t = 0$, this consumer is indifferent in this respect.

We call a sequence

$$\left\{\tau_t^\star, k_t^\star, r_t^\star, W_t^\star, g_t^\star, \left(C_{i,t}^\star, S_{i,t}^\star\right)_{i=1,\ldots,L}\right\}_{t=0,1,\ldots}$$

a voting equilibrium path if $\left\{k_t^\star, r_t^\star, W_t^\star, g_t^\star, \left(C_{i,t}^\star, S_{i,t}^\star\right)_{i=1,\dots,L}\right\}_{t=0,1,\dots}$ is an equilibrium path at $\mathrm{T}^\star = (\tau_t^\star)_{t=0}^\infty$, $\tau_t^\star < W_t^\star$, $t = 0, 1, \dots$, and for each t, the number of consumers who are in favor of increasing τ_t and the number of those who are in favor of decreasing τ_t is less than $L/2$.

We will not discuss the existence and properties of voting equilibrium paths, but will describe voting steady-state equilibria.

A tuple

$$\left\{\mu^\star, k^\star, r^\star, W^\star, g^\star, \gamma^\star, \left(C_i^\star, S_i^\star\right)_{i=1,\dots,L}\right\}$$

is called a *voting steady-state equilibrium* if the tuple

$$\left\{\tau_t^\star, k_t^\star, r_t^\star, W_t^\star, g_t^\star, \left(C_{i,t}^\star, S_{i,t}^\star\right)_{i=1,\dots,L}\right\}_{t=0,1,\dots}$$

given for $i = 1, \dots, L$ and $t = 0, 1, \dots$ by

$$k_t^\star = k^\star\left(1+\gamma^\star\right)^t, \qquad g_t^\star = g^\star\left(1+\gamma^\star\right)^t, \qquad \tau_t^\star = \mu^\star\phi(g_t^\star)f(k_t^\star),$$
$$C_{i,t}^\star = C_i^\star\left(1+\gamma^\star\right)^t, \qquad S_{i,t}^\star = S_i^\star\left(1+\gamma^\star\right)^t,$$

forms a voting equilibrium path starting from the initial state $\hat{g}_0$, $\left\{\hat{S}_{i,-1}\right\}_{i=1,\dots,L}$ given by

$$\hat{g}_0 = g^\star, \quad \hat{S}_{i,-1} = (1+\gamma^\star)^{-1}S_i^\star, \quad i = 1, \dots, L.$$

Theorem 2 (cf. Theorem 1)**.** *In the case of lump sum taxation, a tuple*

$$\left\{\mu^\star, k^\star, r^\star, W^\star, g^\star, \gamma^\star, \left(C_i^\star, S_i^\star\right)_{i=1,\dots,L}\right\}$$

is a voting steady-state equilibrium, if and only if, for $\tau^\star = \mu^\star\phi(g^\star)f(k^\star)$, it satisfies conditions (3)-(5), (16)-(19), $\mu^\star < 1 - \alpha$ and

$$1+\gamma^\star = \beta_m\phi'(g^\star)\left[\frac{f'(k^\star)}{1+\gamma^\star}S_m^\star + f(k^\star) - k^\star f'(k^\star)\right], \tag{20}$$

where index m denotes the consumer being the median among consumers sorted ascending in their equilibrium savings $S_i^\star$ and their discount factors β_i.

Prior to proving this theorem, notice that ascending sorting in $S_i^\star$ does not contradict to ascending sorting in β_i. Indeed, in any steady-state equilibrium only the most patient consumers $i \in I$ can make positive savings. They have the same discount factors $\beta_i = \beta_{max}$ and they are sorted ascending in their savings $S_i^\star$ in the set of most patient consumers. Less patient consumers $i \notin I$ have zero equilibrium savings $S_i^\star = 0$ and they are sorted ascending in their discount factors β_i in the set of less patient consumers.

Proof. Let a tuple

$$\left\{\mu^\star, k^\star, r^\star, W^\star, g^\star, \gamma^\star, \left(C_i^\star, S_i^\star\right)_{i=1,\dots,L}\right\}$$

be a steady-state voting equilibrium at $\mu = \mu^\star$ and

$$\left\{k_t^\star, r_t^\star, W_t^\star, g_t^\star, \left(C_{i,t}^\star, S_{i,t}^\star\right)_{i=1,\dots,L}\right\}_{t=0,1,\dots}$$

be a corresponding equilibrium path.

For each $i = 1, \dots, L$, the sequence $(C_{i,t}^\star, S_{i,t}^\star)_{t=0}^\infty$ is a solution to the following problem

$$\max \sum_{t=0}^{\infty} \beta_i^t u(C_{i,t}),$$

$$C_{i,t} + S_{i,t} \le \phi(g_t^\star) f'(k_t^\star) S_{i,t-1} + \phi(g_t^\star)[f(k_t^\star) - f'(k_t^\star)k_t^\star] - \tau_t,$$
$$S_{i,t} \ge 0, \quad t = 0, 1, \dots,$$

where $S_{i,-1} = (1+\gamma^\star)^{-1} S_i^\star$, $\hat{g}_0 = g^\star$, $\tau_t = (1+\gamma^\star)^t \mu^\star \phi(g_0^\star) f(k_0^\star)$, $g_{t+1}^\star = \tau_t$, $t = 0, 1, \dots$

By the envelope theorem, for all $i = 1, \dots, L$ and all $t = 0, 1, \dots,$ we have

$$\begin{aligned}\frac{\partial V_i}{\partial \tau_t} = &-\beta_i^t u'\left(C_{i,t}^\star\right) + \beta_i^{t+1} u'\left(C_{i,t+1}^\star\right) \\ &\times \phi'\left(g_{t+1}^\star\right)\left[f'\left(k_{t+1}^\star\right) S_{i,t}^\star + f(k_{t+1}^\star) - f'\left(k_{t+1}^\star\right) k_t^\star\right].\end{aligned}$$

Also we have

$$g_t^\star = (1+\gamma^\star)^t g^\star, \quad k_t^\star = (1+\gamma^\star)^t k^\star,$$
$$C_{i,t}^\star = (1+\gamma^\star)^t C_i^\star, \quad S_{i,t}^\star = (1+\gamma^\star)^t S_i^\star,$$
$$u'\left(C_{i,t+1}^\star\right) = \frac{u'(C_{i,t}^\star)}{1+\gamma^\star},$$
$$\phi'(g_{t+1}^\star) f(k_{t+1}^\star) = \phi(g^\star) f(k^\star),$$
$$\frac{f(k_{t+1}^\star) - f'(k_{t+1}^\star)k_{t+1}^\star}{f(k_{t+1}^\star)} = 1 - \alpha = \frac{f(k^\star) - f'(k^\star)k^\star}{f(k^\star)},$$
$$\frac{f'(k_{t+1}^\star)}{f(k_{t+1}^\star)} = \frac{\alpha}{k_{t+1}^\star} = \frac{\alpha}{(1+\gamma^\star)^{t+1} k^\star} = \frac{f'(k^\star)}{(1+\gamma^\star)^{t+1} f(k^\star)}.$$

Therefore,

$$\frac{\partial V_i}{\partial \tau_t} = \beta_i^t u'\left(C_{i,t}^\star\right)\left\{\frac{\beta_i}{1+\gamma^\star}\frac{\phi'\left(g_{t+1}^\star\right) f\left(k_{t+1}^\star\right)}{f\left(k_{t+1}^\star\right)}\right.$$
$$\left.\times\left[f'\left(k_{t+1}^\star\right) S_{i,t}^\star + f\left(k_{t+1}^\star\right) - f'\left(k_{t+1}^\star\right) k_{t+1}^\star\right] - 1\right\} = \beta_i^t u'\left(C_{i,t}^\star\right)$$
$$\times\left\{\frac{\beta_i}{1+\gamma^\star}\phi'\left(g^\star\right)\left[\frac{f'\left(k^\star\right)}{1+\gamma^\star} S_i^\star + f\left(k^\star\right) - f'\left(k^\star\right) k^\star\right] - 1\right\}$$

and hence

$$\operatorname{sign}\frac{\partial V_i}{\partial \tau_t} = \operatorname{sign}\left\{\frac{\beta_i}{1+\gamma^\star}\phi'\left(g^\star\right)\left[\frac{f'\left(k^\star\right)}{1+\gamma^\star} S_i^\star + f\left(k^\star\right) - f'\left(k^\star\right) k^\star\right] - 1\right\}.$$

To complete the proof it is sufficient to notice that

$$\frac{\partial V_i}{\partial \tau_t} \gtreqless 0 \quad \Leftrightarrow \quad \beta_i \phi'(g^\star)\left[\frac{f'\left(k^\star\right)}{1+\gamma^\star} S_i^\star + f\left(k^\star\right) - f'\left(k^\star\right) k^\star\right] \gtreqless 1+\gamma^\star.$$

This completes the proof of the theorem. □

It should be noticed that if $\beta_m < \beta_{max}$, then the savings of the median voter are zero and hence (20) takes the form

$$1+\gamma^\star = \beta_m \phi'\left(g^\star\right)\left[f\left(k^\star\right) - k^\star f'\left(k^\star\right)\right]. \tag{21}$$

If $\beta_m = \beta_{max}$, then the median voter can have positive savings in a steady-state equilibrium, and the set of possible equilibria is a continuum. This set can be parameterized by the relative wealth of the median voter, defined as

$$\xi_m^\star = \frac{f'(k_t^\star) S_{m,t-1}^\star + f(k_t^\star) - k_t^\star f'(k_t^\star)}{f(k_t^\star)} = 1 - \left(\frac{S_m^\star}{1+\gamma^\star} - k^\star\right)\frac{f'(k^\star)}{f(k^\star)}, \tag{22}$$

and (20) can be rewritten as

$$1+\gamma^\star = \beta_m \xi_m^\star \phi'(g^\star) f(k^\star). \tag{23}$$

(if $\beta_m < \beta_{max}$ it reduces to (21)).

Following the politico-economic literature about income inequality (see, e.g., Meltzer and Richard (1981)), an income distribution is called more equal, the higher the median income is relative to the mean (this is only reasonable in the case where the median income does not exceed the mean, which is considered as a typical situation). For our model this implies that if $\beta_m = \beta_{max}$, lower inequality has a positive effect on the equilibrium rate of growth.

Recalling that the production function is Cobb-Douglas,

$$\phi(g_t) = g_t^{1-\alpha}, \qquad f(k_t) = k_t^{\alpha},$$

by (17) and (23), we have

$$1 + \gamma^{\star} = \alpha\beta_{max}\left(\frac{g^{\star}}{k^{\star}}\right)^{1-\alpha},$$

$$1 + \gamma^{\star} = (1-\alpha)\beta_m\xi_m\left(\frac{k^{\star}}{g^{\star}}\right)^{\alpha}.$$

Therefore,

$$\mu^{\star} = (1-\alpha)\,\beta_m\xi_m^{\star}, \qquad \frac{k^{\star}}{g^{\star}} = \frac{\alpha\beta_{max}}{(1-\alpha)\beta_m\xi_m^{\star}},$$

and hence

$$1 + \gamma^{\star} = (\alpha\beta_{max})^{\alpha}\left[(1-\alpha)\beta_m\xi_m^{\star}\right]^{1-\alpha}. \tag{24}$$

Thus, as in the case of flat-rate income taxation, the long-run rate growth increases monotonically with the discount factor of the median consumer, β_m, and the discount factor of the most patient consumer, β_{max}, but here it also depends on the median voter relative wealth, $\xi_m^{\star}$.

4. Lump-sum and flat-rate taxes compared

Let a tuple

$$\left\{\gamma^{\star}, k^{\star}, r^{\star}, W^{\star}, g^{\star}, \left(C_i^{\star}, S_i^{\star}\right)_{i=1,\dots,L}\right\}$$

be a steady-state equilibrium in the case of lump sum taxation. Let further

$$\bar{\beta}\left(\xi_m^{\star}\right) = \frac{1}{1-\alpha}\left[\frac{1}{(\xi_m^{\star})^{1-\alpha}} - 1\right],$$

where where $\xi_m^{\star}$ is given by (22). It should be noticed that if $|I| < L/2$, i.e. $\beta_m < \beta_{max}$, then $\bar{\beta}\,(\xi_m^{\star})$ does not depend on $\xi_m^{\star}$:

$$\bar{\beta}\left(\xi_m^{\star}\right) = \frac{1}{1-\alpha}\left[\frac{1}{(1-\alpha)^{1-\alpha}} - 1\right].$$

Now we are ready to formulate our main result implying that there is a domain of parameters where the equilibrium growth rate under flat-rate income taxation, γ^{*}, is larger then that under lump sum taxation, $\gamma^{\star}$. This domain of parameters is described by the following theorem.

Theorem 3. $\gamma^* \gtreqless \gamma^\star \quad \Leftrightarrow \quad \bar{\beta}(\xi_m^\star) \gtreqless \beta_m.$

Proof. The proof of the theorem is readily follows from the comparison of the growth rates given by (14) and (24). □

In particular, this theorem says that if $\beta_m < \bar{\beta}(\xi_m^\star)$, than flat-rate taxation provides higher growth rate then lumps-sum taxation: $\gamma^* > \gamma^\star$. The intuition is simple. Impatient and poor individuals consider lump sum taxation as more burdensome then flat-rate taxation providing the same growth rate, because in the case of flat-rate taxation it is rich and patient individuals who carry a larger share of the tax burden. If the median consumer is sufficiently impatient and poor, in the case of lump sum taxation she will vote in favor of a less generous program of public spending, than in the case of flat rate taxation. Therefore, in a steady-state voting equilibrium, underprovision of public services under lump sum taxation can be more severe than under distorting flat-rate taxation.

It should be noted that this effect can not be observed in models with homogenous consumers or representative household because in that cases $\beta_m = \beta_{max}$ and $\xi_m^\star = 1$, so that $\bar{\beta}(\xi_m^\star) = 0$ and $\beta_m < \bar{\beta}(\xi_m^\star)$ is impossible.

5. Conclusion

In this paper, we have considered a Barro-type endogenous growth model. The key feature of our model is that individuals differ in their discount factors allowing them to compare utility from present and future consumption. The aggregate production function is taken to be linearly homogeneous in capital and productive government services taken together, and this can lead to endogenous growth driven by expansion of the capital stock and government services. The provision of government services is financed by a flat-rate (linear) income tax or, in another variant of the model, by a lump sum tax.

Taxation decreases the individuals consumption and savings at present but leads to growth in government services, output, wages and interest rate in the future. So, each individual has her own idea on the preferable tax rate determined by her intertemporal preferences. Larger discount factor means that future consumption is more valuable for an individual leading to larger preferable tax rate.

To associate economic policy and individual preferences we have introduced a notion of voting equilibrium leading to some form of the median voter theorem, according to which the tax rate selected by the government is the one preferred by the median voter. We have fully characterized the structure of steady-state equilibria in the two versions of our model and have derived explicit formulas for the long-run rate of growth.

These formulas say that, irrespective of the tax system, the larger is the discount factor of the median voter, the larger is the tax rate determined by the voting procedure, and consequently the higher is the rate of growth. They also show that in the case of lump sum taxation the rate of growth is increasing in the relative wealth of the median voter and that in the both cases it is increasing in the discount factor of the most patient of consumer.

Finally, it follows from the derived formulas that under some condition distorting flat-rate income taxation can lead to higher growth rate then undistorting lump sum taxation.

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