EUROPEAN
UNIVERSITY AT
ST. PETERSBURG

Sergey Ivashchenko

Estimating nonlinear DSGE models with moments based methods

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Европейский университет в Санкт-Петербурге
Сергей Иващенко

Оценка нелинейной модели ДСОЭР при помощи методов основанных на вычислении моментов
На английском языке

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Keywords: DSGE; DSGE-VAR, GMM, nonlinear estimation.
JEL Classification: C13; C32; E32

Сергей Иващенко Санкт-Петербургский экономико-математический институт РАН, ул. Чайковского 1, Санкт-Петербург, 191187, Россия, e-mail: glucke_ru@pisem.net

Estimating nonlinear DSGE models with moments based methods

By Ivashchenko Sergey[1]

Abstract

This article suggests new approach to approximation of nonlinear DSGE models moments. These approximations are fast and accurate enough to use them for estimation of nonlinear DSGE models. The small financial DSGE model is repeatedly estimated by several modifications of suggested approach. Approximations of moments are close to the large sample Monte Carlo estimation of them. The quality of estimation with suggested approach is close to the Central Difference Kalman Filter (the CDKF) based. At the same time suggested approach is much faster.

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[1] St. Petersburg Institute for Economics and Mathematics (Russian Academy of Sciences); 1 Tchaikovsky Str., St. Petersburg, 191187 RUSSIA e-mail: glucke_ru@pisem.net; tel: +7-921-746-32-12

Introduction

Modern macroeconomics seeks to explain the aggregate economy using theories based on strong microeconomic foundations. The advantage of such an approach is description of models in terms of "deep structural" parameters which are not influenced by economic policy [Wickens (2008)]. Never less, this parameters should be estimated for usage of DSGE models. There are different econometric techniques for models estimation but empirical literature has concentrated its attention on the estimation of first-order linearized DSGE models [Tovar (2008)].

Non-linear approximations of DSGE models have some important properties, in particular, they allow uncertainty to effect economic choices [Ruge-Murcia (2012)]. Linear approximation of DSGE models behavior could differ from the higher order (more accurate) approximations significantly [Collard and Juillard (2001)]. Second order approximation makes difference between models and approximation behavior much smaller. That is why important to estimate DSGE models on base of non-linear approximation.

There are two main approaches for estimation of DSGE models: moments based and likelihood based [DeJong and Dave (2007), Canova (2007)]. Likelihood based approaches use nonlinear filters for construction of the likelihood function. The first of them is the particle filter. This tool could produce all advantages of nonlinear approximation, including sharper the likelihood function and smaller variance of parameters estimator [An and Schorfheide (2006)]. But the particle filters have some disadvantages: the likelihood evaluation with the particle filter is random variable (each evaluation of the likelihood for the same model and observations produce different value). Thus usual maximization algorithms can't be used and Markov Chain Monte Carlo inefficiency greatly increases (Pitt, Silva et al. (2012) show that number of draws should be 10 (from 5 to 400 depending on the number of particles) times higher for the same accuracy of MCMC).

There are alternative filters that could be used for the likelihood calculation. For example, Andreasen (2008) has shown the advantage of the Central Difference Kalman Filter over few versions of particle filter (in terms of the quality and computing time (100 times faster)). The Quadratic Kalman Filter has advantage in the quality with some loss in computing time over the Unscented Kalman filter (Julier and Uhlmann (1997)) and the Central Difference Kalman Filter (Ivashchenko (2013)).

The moments based approaches for estimation of DSGE models are more robust [Ruge-Murcia (2007), Creel and Kristensen (2011)]. The first of them is the instrumental variables approach which is the special case of the generalized method of moments [Canova (2007)]. All variables of the DSGE model should be observed for usage of this approach. It could be true only for small-scale DSGE models. Another version of the GMM is used for a linearized model because wide range of empirical targets can be calculated analytically [DeJong and Dave (2007)]. The simulated method of moments have almost the same statistical efficient as the GMM, but it is more computational demanding [Ruge-Murcia (2007)]. The SMM could be implemented for non-linear DSGE models [Ruge-Murcia (2012), Kim and Ruge-Murcia (2009)]. But it's too slow for the estimation of medium-scale or large-scale models which are used for policymaking. Another moments based approach is the indirect inference. Theoretically it's more efficient then the GMM [Creel and Kristensen (2011)]. The usage of the indirect inference is more complicated than the GMM, because it requires knowledge of moments distribution function. It could be calculated easily only for a narrow range of moments (usually it is parameters estimation of some econometric model, example is DSGE-VAR model). So, usually this approach is described differently [DeJong and Dave (2007)].

This article suggests an approach for fast calculation of non-linear DSGE model's moments. This moments calculations are compared with alternative approaches. A small non-linear

DSGE model is estimated with moments based approaches. The quality of such estimations is compared with linear maximum likelihood estimation and the CDKF based quasi-maximum likelihood estimation.

The approach for moments calculation

The equation (1) describes the data generating process for state variables (X_t) which is approximation of a rational expectation model with perturbation method. Exogenous shocks (ε_t) have normal distribution with covariance matrix Ω_ε and mean equal zero. The measurement equation (2) describes the dependence of observed variables (Y_t) on state variables and measurement errors (u_t) which have normal distribution with zero mean and covariance matrix Ω_u.

$$X_t = \begin{bmatrix} B_X & B_\varepsilon \end{bmatrix} \begin{bmatrix} X_{t-1} \\ \varepsilon_t \end{bmatrix} + C + \\ + \begin{bmatrix} A_{xx} & A_{x\varepsilon} & 0 & A_{\varepsilon\varepsilon} \end{bmatrix} \begin{bmatrix} X_{t-1} \otimes X_{t-1} \\ X_{t-1} \otimes \varepsilon_t \\ \varepsilon_t \otimes X_{t-1} \\ \varepsilon_t \otimes \varepsilon_t \end{bmatrix} \qquad (1)$$

$$Y_t = S + DX_t + u_t \qquad (2)$$

The measurement equation (2) is linear, therefore dependence between moments of observed variables (Y_t) and state variables (X_t) is standard. Equations for the first and the second moments of state variables (3)-(5) are more complicated.

$$E(X_t) = B_X E(X_t) + C + A_{xx} E(X_t \otimes X_t) + A_{\varepsilon\varepsilon}\Omega_\varepsilon \qquad (3)$$

$$\begin{aligned}
&E(X_t \otimes X_t) = (\mathrm{B_X} \otimes \mathrm{B_X})E(X_t \otimes X_t) + (\mathrm{B_X} \otimes \mathrm{C})E(X_t) + \\
&(\mathrm{B_X} \otimes \mathrm{A_{XX}})E(X_t \otimes X_t \otimes X_t) + (\mathrm{B_X} \otimes \mathrm{A}_{\varepsilon\varepsilon})(E(X_t) \otimes \Omega_\varepsilon) + \\
&(\mathrm{B}_\varepsilon \otimes \mathrm{B}_\varepsilon)\Omega_\varepsilon + (\mathrm{B}_\varepsilon \otimes \mathrm{A}_{X\varepsilon})\mathrm{U_{EXE_XEE}}(E(X_t) \otimes \Omega_\varepsilon) + \\
&(\mathrm{C} \otimes \mathrm{B_X})E(X_t) + \mathrm{C} \otimes \mathrm{C} + (\mathrm{C} \otimes \mathrm{A_{XX}})E(X_t \otimes X_t) + \\
&(\mathrm{C} \otimes \mathrm{A}_{\varepsilon\varepsilon})\Omega_\varepsilon + (\mathrm{A_{XX}} \otimes \mathrm{B_X})E(X_t \otimes X_t \otimes X_t) + \\
&(\mathrm{A_{XX}} \otimes \mathrm{C})E(X_t \otimes X_t) + (\mathrm{A_{XX}} \otimes \mathrm{A}_{\varepsilon\varepsilon})(E(X_t \otimes X_t) \otimes \Omega_\varepsilon) + \\
&(\mathrm{A}_{X\varepsilon} \otimes \mathrm{B}_\varepsilon)(\mathrm{X} \otimes \Omega_\varepsilon) + (\mathrm{A}_{\varepsilon\varepsilon} \otimes \mathrm{B_X})(\Omega_\varepsilon \otimes E(X_t)) + \\
&(\mathrm{A}_{X\varepsilon} \otimes \mathrm{A}_{X\varepsilon})\mathrm{U_{XEXE_EEXX}}(\Omega_\varepsilon \otimes E(X_t \otimes X_t)) + \\
&(\mathrm{A}_{\varepsilon\varepsilon} \otimes \mathrm{C})\Omega_\varepsilon + (\mathrm{A}_{\varepsilon\varepsilon} \otimes \mathrm{A_{XX}})(\Omega_\varepsilon \otimes E(X_t \otimes X_t)) + \\
&(\mathrm{A}_{\varepsilon\varepsilon} \otimes \mathrm{A}_{\varepsilon\varepsilon})\Omega_{\varepsilon\varepsilon} + (\mathrm{A_{XX}} \otimes \mathrm{A_{XX}})E(X_t \otimes X_t \otimes X_t \otimes X_t)
\end{aligned}$$

(4)

$$\begin{aligned}
&E(X_t \otimes X_{t-s}) = (B_X \otimes I_X)E(X_t \otimes X_{t-s+1}) + (C \otimes I_X)E(X_t) + \\
&(A_{xx} \otimes I_X)E(X_t \otimes X_t \otimes X_{t-s+1}) + (A_{\varepsilon\varepsilon} \otimes I_X)(\Omega_\varepsilon \otimes E(X_t))
\end{aligned}$$

(5)

where $\Omega_{\varepsilon\varepsilon}$ is the fourth moment of vector ε_t, I_X is identity matrix of the same size as state variables vector (X_t), U_{EXE_XEE} and U_{XEXE_EEXX} are the permutation matrixes which describe dependence between vectors indicated in the subscript (equation (6) is example):

$$\varepsilon_t \otimes X_t \otimes \varepsilon_t = U_{EXE\_XEE}(X_t \otimes \varepsilon_t \otimes \varepsilon_t) \qquad (6)$$

The equation (7) shows matrix formula for calculation of the fourth moment (for ε_t):

$$vec(\Omega_{\varepsilon\varepsilon};n_\varepsilon^4;1)=vec(vec(\Omega_\varepsilon;n_\varepsilon;n_\varepsilon)\otimes vec(\Omega_\varepsilon;n_\varepsilon^2;1)+$$
$$vec(\Omega_\varepsilon;n_\varepsilon;n_\varepsilon)\otimes vec(\Omega_\varepsilon;n_\varepsilon;n_\varepsilon)+ \qquad (7)$$
$$vec(\Omega_\varepsilon;n_\varepsilon;n_\varepsilon)\otimes vec(\Omega_\varepsilon;1;n_\varepsilon^2);n_\varepsilon^4;1)$$

where n_ε is the number of elements in the vector ε_t, *vec(M;n;m)* is the vectorization function which transform a matrix M to the matrix with n rows and m columns.

The solution of equation (3) require knowledge of vector's X_t second moment. The equation (4) for the second moment requires knowledge of the third and the fourth moments. So, it is impossible to solve these equations directly. There are some approaches for approximation of equation (3)-(5) solution. The key question is how to be with the higher order moments. The first way is to think that the higher order moments are zero. The second way is to approximate the higher order moments (normal distribution would be used for this approximation).

Equations (8)-(9) are formulas for the third and the fourth moments of normal distribution:

$$E(X_t\otimes X_t\otimes X_t)=\mu_X\otimes\mu_X\otimes\mu_X+\Omega_X\otimes\mu_X+$$
$$\mu_X\otimes\Omega_X+U_{X\mu X\_\mu XX}\left(\mu_X\otimes\Omega_X\right) \qquad (8)$$

$$
\begin{aligned}
&E(X_t \otimes X_t \otimes X_t \otimes X_t) = \mu_X \otimes \mu_X \otimes \mu_X \otimes \mu_X + \\
&U_{X\mu X\mu\_\mu\mu XX}\left(\mu_X \otimes \mu_X \otimes \Omega_X\right) + U_{X\mu\mu X\_\mu\mu XX}\left(\mu_X \otimes \mu_X \otimes \Omega_X\right) + \\
&U_{\mu XX\mu\_\mu\mu XX}\left(\mu_X \otimes \mu_X \otimes \Omega_X\right) + U_{\mu X\mu X\_\mu\mu XX}\left(\mu_X \otimes \mu_X \otimes \Omega_X\right) + \\
&\Omega_X \otimes \mu_X \otimes \mu_X + \mu_X \otimes \mu_X \otimes \Omega_X + \Omega_{XX}
\end{aligned}
$$

(9)

where μ_X is the first moment of a vector X_t, Ω_X is the second central moment of a vector X_t, Ω_{XX} is the fourth central moment of a vector X_t.

Equations (3)-(4) with normal approximation of the higher order moments are nonlinear (in moments). Analytical solution of large non-linear system is problematic. Thus numeric solution is required. The simple numeric method is used:

$$
\mu_{X,k+1} = B_X \mu_{X,k} + C + A_{xx}\mu_{XX,k} + A_{\varepsilon\varepsilon}\Omega_\varepsilon \qquad (10)
$$

$$
\begin{aligned}
&\mu_{XX,k+1} = (\mathrm{B_X} \otimes \mathrm{B_X})\mu_{XX,k} + (\mathrm{B_X} \otimes \mathrm{C})\mu_{X,k} + \\
&(\mathrm{B_X} \otimes \mathrm{A_{XX}})\mu_{XXX,k} + (\mathrm{B_X} \otimes \mathrm{A}_{\varepsilon\varepsilon})(\mu_{X,k} \otimes \Omega_\varepsilon) + \\
&(\mathrm{B}_\varepsilon \otimes \mathrm{B}_\varepsilon)\Omega_\varepsilon + (\mathrm{B}_\varepsilon \otimes \mathrm{A}_{X\varepsilon})\mathrm{U_{EXE\_XEE}}(\mu_{X,k} \otimes \Omega_\varepsilon) + \\
&(\mathrm{C} \otimes \mathrm{B_X})\mu_{X,k} + \mathrm{C} \otimes \mathrm{C} + (\mathrm{C} \otimes \mathrm{A_{XX}})\mu_{XX,k} + \\
&(\mathrm{C} \otimes \mathrm{A}_{\varepsilon\varepsilon})\Omega_\varepsilon + (\mathrm{A_{XX}} \otimes \mathrm{B_X})\mu_{XXX,k} + (\mathrm{A_{XX}} \otimes \mathrm{C})\mu_{XX,k} + \\
&(\mathrm{A_{XX}} \otimes \mathrm{A}_{\varepsilon\varepsilon})(\mu_{XX,k} \otimes \Omega_\varepsilon) + (\mathrm{A}_{X\varepsilon} \otimes \mathrm{B}_\varepsilon)(\mu_{X,k} \otimes \Omega_\varepsilon) + \qquad (11) \\
&(\mathrm{A}_{X\varepsilon} \otimes \mathrm{A}_{X\varepsilon})\mathrm{U_{XEXE\_EEXX}}(\Omega_\varepsilon \otimes \mu_{XX,k}) + \\
&(\mathrm{A}_{\varepsilon\varepsilon} \otimes \mathrm{B_X})(\Omega_\varepsilon \otimes \mu_{X,k}) + (\mathrm{A}_{\varepsilon\varepsilon} \otimes \mathrm{C})\Omega_\varepsilon + \\
&(\mathrm{A}_{\varepsilon\varepsilon} \otimes \mathrm{A_{XX}})(\Omega_\varepsilon \otimes \mu_{XX,k}) + \\
&(\mathrm{A}_{\varepsilon\varepsilon} \otimes \mathrm{A}_{\varepsilon\varepsilon})\Omega_{\varepsilon\varepsilon} + (\mathrm{A_{XX}} \otimes \mathrm{A_{XX}})\mu_{XXXX,k}
\end{aligned}
$$

where $\mu_{X,k}$, $\mu_{XX,k}$, $\mu_{XXX,k}$, $\mu_{XXXX,k}$ are the first, the second, the third and the fourth moments of a vector X_t after iteration k.

It should be noted that only a few iteration could be calculated due to computational costs. It means that general properties of convergence are less important. The numerical comparison of moments and estimation based on them are made instead of it.

The equation (5) could be solved without numerical approximation (just with normal approximation of third moment of vector $[X_t;X_{t-s}]$). This approach would be called the normal approximation of the higher then 2 moments (the NAHM2).

Alternative approach (where the higher order moments are zeros) is also used at this article. Due to lower computational costs the third moment of X_t could be calculated (large formula (12)).

$$\begin{aligned}
&\mu_{\mathrm{XXX,k+1}} = (\mathrm{B_X} \otimes \mathrm{B_X} \otimes \mathrm{B_X})\mu_{\mathrm{XXX,k}} + (\mathrm{B_X} \otimes \mathrm{B_X} \otimes \mathrm{C})\mu_{\mathrm{XX,k}} + \\
&(\mathrm{B_X} \otimes \mathrm{B_X} \otimes \mathrm{A}_{\varepsilon\varepsilon})(\mu_{\mathrm{XX,k}} \otimes \Omega_{\varepsilon}) + (\mathrm{B_X} \otimes \mathrm{B}_{\varepsilon} \otimes \mathrm{B}_{\varepsilon})(\mu_{\mathrm{X,k}} \otimes \Omega_{\varepsilon}) + \\
&(\mathrm{B_X} \otimes \mathrm{B}_{\varepsilon} \otimes \mathrm{A}_{X\varepsilon})\mathrm{U}_{\mathrm{XEXE_EEXX}}(\Omega_{\varepsilon} \otimes \mu_{\mathrm{XX,k}}) + (\mathrm{B_X} \otimes \mathrm{C} \otimes \mathrm{B_X})\mu_{\mathrm{XX,k}} + \\
&(\mathrm{B_X} \otimes \mathrm{C} \otimes \mathrm{C})\mu_{\mathrm{X,k}} + (\mathrm{B_X} \otimes \mathrm{C} \otimes \mathrm{A_{XX}})\mu_{\mathrm{XXX,k}} + \\
&(\mathrm{B_X} \otimes \mathrm{C} \otimes \mathrm{A}_{\varepsilon\varepsilon})(\mu_{\mathrm{X,k}} \otimes \Omega_{\varepsilon}) + (\mathrm{B_X} \otimes \mathrm{A_{XX}} \otimes \mathrm{C})\mu_{\mathrm{XXX,k}} + \\
&(\mathrm{B_X} \otimes \mathrm{A_{XX}} \otimes \mathrm{A}_{\varepsilon\varepsilon})(\mu_{\mathrm{XXX,k}} \otimes \Omega_{\varepsilon}) + (\mathrm{B_X} \otimes \mathrm{A}_{X\varepsilon} \otimes \mathrm{B}_{\varepsilon})(\mu_{\mathrm{XX,k}} \otimes \Omega_{\varepsilon}) + \\
&(\mathrm{B_X} \otimes \mathrm{A}_{X\varepsilon} \otimes \mathrm{A}_{X\varepsilon})\mathrm{U}_{\mathrm{XXEXE_EEXXX}}(\Omega_{\varepsilon} \otimes \mu_{\mathrm{XXX,k}}) +
\end{aligned}$$

(12)

$$+(B_X \otimes A_{\varepsilon\varepsilon} \otimes B_X)U_{XEEX\_EEXX}(\Omega_\varepsilon \otimes \mu_{XX,k}) +$$

$$(B_X \otimes A_{\varepsilon\varepsilon} \otimes C)(\mu_{X,k} \otimes \Omega_\varepsilon) + (B_X \otimes A_{\varepsilon\varepsilon} \otimes A_{\varepsilon\varepsilon})(\mu_{X,k} \otimes \Omega_{\varepsilon\varepsilon}) +$$

$$(B_X \otimes A_{\varepsilon\varepsilon} \otimes A_{XX})U_{XEEXX\_EEXXX}(\Omega_\varepsilon \otimes \mu_{XXX,k}) +$$

$$(B_\varepsilon \otimes B_X \otimes B_\varepsilon)U_{EXE\_XEE}(\mu_{X,k} \otimes \Omega_\varepsilon) + (B_\varepsilon \otimes C \otimes B_\varepsilon)\Omega_\varepsilon +$$

$$(B_\varepsilon \otimes B_X \otimes A_{X\varepsilon})U_{EXXE\_EEXX}(\Omega_\varepsilon \otimes \mu_{XX,k}) +$$

$$(B_\varepsilon \otimes B_\varepsilon \otimes B_X)(\Omega_\varepsilon \otimes \mu_{X,k}) + (B_\varepsilon \otimes B_\varepsilon \otimes C)\Omega_\varepsilon +$$

$$(B_\varepsilon \otimes B_\varepsilon \otimes A_{XX})(\Omega_\varepsilon \otimes \mu_{XX,k}) + (B_\varepsilon \otimes B_\varepsilon \otimes A_{\varepsilon\varepsilon})\Omega_{\varepsilon\varepsilon} +$$

$$(B_\varepsilon \otimes C \otimes A_{X\varepsilon})U_{EXE\_XEE}(\mu_{X,k} \otimes \Omega_\varepsilon) + (A_{XX} \otimes B_X \otimes C)\mu_{XXX,k} +$$

$$(B_\varepsilon \otimes A_{XX} \otimes B_\varepsilon)U_{EXXE\_EEXX}(\Omega_\varepsilon \otimes \mu_{XX,k}) +$$

$$(B_\varepsilon \otimes A_{XX} \otimes A_{X\varepsilon})U_{EXXXE\_EEXXX}(\Omega_\varepsilon \otimes \mu_{XXX,k}) +$$

$$(B_\varepsilon \otimes A_{X\varepsilon} \otimes B_X)U_{EXEX\_EEXX}(\Omega_\varepsilon \otimes \mu_{XX,k}) +$$

$$(B_\varepsilon \otimes A_{X\varepsilon} \otimes C)U_{EXE\_XEE}(\mu_{X,k} \otimes \Omega_\varepsilon) + (B_\varepsilon \otimes A_{\varepsilon\varepsilon} \otimes B_\varepsilon)\Omega_{\varepsilon\varepsilon} +$$

$$(B_\varepsilon \otimes A_{X\varepsilon} \otimes A_{XX})U_{EXEXX\_EEXXX}(\Omega_\varepsilon \otimes \mu_{XXX,k}) +$$

$$(B_\varepsilon \otimes A_{X\varepsilon} \otimes A_{\varepsilon\varepsilon})U_{EXEEE\_XEEEE}(\mu_{X,k} \otimes \Omega_{\varepsilon\varepsilon}) +$$

$$(B_\varepsilon \otimes A_{\varepsilon\varepsilon} \otimes A_{X\varepsilon})U_{EEEXE\_XEEEE}(\mu_{X,k} \otimes \Omega_{\varepsilon\varepsilon}) + (C \otimes B_X \otimes B_X)\mu_{XX,k} +$$

$$(C \otimes B_X \otimes C)\mu_{X,k} + (C \otimes B_X \otimes A_{XX})\mu_{XXX,k} +$$

$$(C \otimes B_X \otimes A_{\varepsilon\varepsilon})(\mu_{X,k} \otimes \Omega_\varepsilon) + (C \otimes B_\varepsilon \otimes B_\varepsilon)\Omega_\varepsilon +$$

$$(C \otimes B_\varepsilon \otimes A_{X\varepsilon})U_{EXE\_XEE}(\mu_{X,k} \otimes \Omega_\varepsilon) + (C \otimes C \otimes B_X)\mu_{X,k} +$$

$$C \otimes C \otimes C + (C \otimes C \otimes A_{XX})\mu_{XX,k} + (C \otimes C \otimes A_{\varepsilon\varepsilon})\Omega_\varepsilon +$$

$$(C \otimes A_{XX} \otimes B_X)\mu_{XXX,k} + (C \otimes A_{XX} \otimes C)\mu_{XX,k} +$$

$$(C \otimes A_{XX} \otimes A_{\varepsilon\varepsilon})(\mu_{XX,k} \otimes \Omega_\varepsilon) + (C \otimes A_{X\varepsilon} \otimes B_\varepsilon)(\mu_{X,k} \otimes \Omega_\varepsilon) +$$

$$(C \otimes A_{X\varepsilon} \otimes A_{X\varepsilon})U_{XEXE\_EEXX}(\Omega_\varepsilon \otimes \mu_{XX,k}) +$$

$$(C \otimes A_{\varepsilon\varepsilon} \otimes B_X)(\Omega_\varepsilon \otimes \mu_{X,k}) + (C \otimes A_{\varepsilon\varepsilon} \otimes C)\Omega_\varepsilon +$$

$$(C \otimes A_{\varepsilon\varepsilon} \otimes A_{XX})(\Omega_\varepsilon \otimes \mu_{XX,k}) + (C \otimes A_{\varepsilon\varepsilon} \otimes A_{\varepsilon\varepsilon})\Omega_{\varepsilon\varepsilon} +$$

(12)

$$
\begin{aligned}
&+(A_{XX}\otimes B_{\varepsilon}\otimes B_{\varepsilon})(\mu_{XX,k}\otimes\Omega_{\varepsilon})+(A_{XX}\otimes C\otimes B_{X})\mu_{XXX,k}+\\
&(A_{XX}\otimes B_{X}\otimes A_{\varepsilon\varepsilon})(\mu_{XXX,k}\otimes\Omega_{\varepsilon})+\\
&(A_{XX}\otimes A_{\varepsilon\varepsilon}\otimes C)(\mu_{XX,k}\otimes\Omega_{\varepsilon})+(A_{X\varepsilon}\otimes B_{\varepsilon}\otimes C)(\mu_{X,k}\otimes\Omega_{\varepsilon})+\\
&(A_{XX}\otimes B_{\varepsilon}\otimes A_{X\varepsilon})U_{XXEXE\_EEXXX}(\Omega_{\varepsilon}\otimes\mu_{XXX,k})+\\
&(A_{XX}\otimes C\otimes C)\mu_{XX,k}+(A_{XX}\otimes C\otimes A_{\varepsilon\varepsilon})(\mu_{XX,k}\otimes\Omega_{\varepsilon})+\\
&(A_{XX}\otimes A_{X\varepsilon}\otimes B_{\varepsilon})(\mu_{XXX,k}\otimes\Omega_{\varepsilon})+\\
&(A_{XX}\otimes A_{\varepsilon\varepsilon}\otimes B_{X})U_{XXEEX\_EEXXX}(\Omega_{\varepsilon}\otimes\mu_{XXX,k})+\\
&(A_{XX}\otimes A_{\varepsilon\varepsilon}\otimes A_{\varepsilon\varepsilon})(\mu_{XX,k}\otimes\Omega_{\varepsilon\varepsilon})+\\
&(A_{X\varepsilon}\otimes B_{X}\otimes B_{\varepsilon})U_{XEXE\_EEXX}(\Omega_{\varepsilon}\otimes\mu_{XX,k})+\\
&(A_{X\varepsilon}\otimes B_{X}\otimes A_{X\varepsilon})U_{XEXXE\_EEXXX}(\Omega_{\varepsilon}\otimes\mu_{XXX,k})+\\
&(A_{X\varepsilon}\otimes B_{\varepsilon}\otimes B_{X})U_{XEEX\_EEXX}(\Omega_{\varepsilon}\otimes\mu_{XX,k})+\\
&(A_{X\varepsilon}\otimes B_{\varepsilon}\otimes A_{XX})U_{XEEXX\_EEXXX}(\Omega_{\varepsilon}\otimes\mu_{XXX,k})+\\
&(A_{X\varepsilon}\otimes B_{\varepsilon}\otimes A_{\varepsilon\varepsilon})(\mu_{X,k}\otimes\Omega_{\varepsilon\varepsilon})+(A_{X\varepsilon}\otimes C\otimes B_{\varepsilon})(\mu_{X,k}\otimes\Omega_{\varepsilon})+\\
&(A_{X\varepsilon}\otimes C\otimes A_{X\varepsilon})U_{XEXE\_EEXX}(\Omega_{\varepsilon}\otimes\mu_{XX,k})+\\
&(A_{X\varepsilon}\otimes A_{XX}\otimes B_{\varepsilon})U_{XEXXE\_EEXXX}(\Omega_{\varepsilon}\otimes\mu_{XXX,k})+\\
&(A_{X\varepsilon}\otimes A_{X\varepsilon}\otimes B_{X})U_{XEXEX\_EEXXX}(\Omega_{\varepsilon}\otimes\mu_{XXX,k})+\\
&(A_{X\varepsilon}\otimes A_{X\varepsilon}\otimes C)U_{XEXE\_EEXX}(\Omega_{\varepsilon}\otimes\mu_{XX,k})+\\
&(A_{X\varepsilon}\otimes A_{X\varepsilon}\otimes A_{\varepsilon\varepsilon})U_{XEXEEE\_EEEEXX}(\Omega_{\varepsilon\varepsilon}\otimes\mu_{XX,k})+\\
&(A_{X\varepsilon}\otimes A_{\varepsilon\varepsilon}\otimes B_{\varepsilon})(\mu_{X,k}\otimes\Omega_{\varepsilon\varepsilon})+\\
&(A_{X\varepsilon}\otimes A_{\varepsilon\varepsilon}\otimes A_{X\varepsilon})U_{XEEEXE\_EEEEXX}(\Omega_{\varepsilon\varepsilon}\otimes\mu_{XX,k})+\\
&(A_{\varepsilon\varepsilon}\otimes B_{X}\otimes B_{X})(\Omega_{\varepsilon}\otimes\mu_{XX,k})+\\
&(A_{\varepsilon\varepsilon}\otimes B_{X}\otimes C)(\Omega_{\varepsilon}\otimes\mu_{X,k})+(A_{\varepsilon\varepsilon}\otimes B_{X}\otimes A_{XX})(\Omega_{\varepsilon}\otimes\mu_{XXX,k})+\\
&(A_{\varepsilon\varepsilon}\otimes B_{X}\otimes A_{\varepsilon\varepsilon})U_{EEXEE\_XEEEE}(\mu_{X,k}\otimes\Omega_{\varepsilon\varepsilon})+\\
&(A_{\varepsilon\varepsilon}\otimes B_{\varepsilon}\otimes B_{\varepsilon})\Omega_{\varepsilon\varepsilon}+(A_{\varepsilon\varepsilon}\otimes A_{XX}\otimes B_{X})(\Omega_{\varepsilon}\otimes\mu_{XXX,k})+
\end{aligned}
$$

(12)

$$
\begin{aligned}
&+(A_{\varepsilon\varepsilon}\otimes B_{\varepsilon}\otimes A_{X\varepsilon})U_{\mathrm{EEEXE\_XEEEE}}(\mu_{\mathrm{X,k}}\otimes\Omega_{\varepsilon\varepsilon})+\\
&(A_{\varepsilon\varepsilon}\otimes C\otimes B_{\mathrm{X}})(\Omega_{\varepsilon}\otimes\mu_{\mathrm{X,k}})+(A_{\varepsilon\varepsilon}\otimes C\otimes C)\Omega_{\varepsilon}+\\
&(A_{\varepsilon\varepsilon}\otimes C\otimes A_{\mathrm{XX}})(\Omega_{\varepsilon}\otimes\mu_{\mathrm{XX,k}})+(A_{\varepsilon\varepsilon}\otimes C\otimes A_{\varepsilon\varepsilon})\Omega_{\varepsilon\varepsilon}+\\
&(A_{\varepsilon\varepsilon}\otimes A_{\mathrm{XX}}\otimes C)(\Omega_{\varepsilon}\otimes\mu_{\mathrm{XX,k}})+\\
&(A_{\varepsilon\varepsilon}\otimes A_{\mathrm{XX}}\otimes A_{\varepsilon\varepsilon})U_{\mathrm{EEXXEE\_EEEEXX}}(\Omega_{\varepsilon\varepsilon}\otimes\mu_{\mathrm{XX,k}})+\\
&(A_{\varepsilon\varepsilon}\otimes A_{X\varepsilon}\otimes B_{\varepsilon})U_{\mathrm{EEXEE\_XEEEE}}(\mu_{\mathrm{X,k}}\otimes\Omega_{\varepsilon\varepsilon})+\\
&(A_{\varepsilon\varepsilon}\otimes A_{X\varepsilon}\otimes A_{X\varepsilon})U_{\mathrm{EEXEXE\_EEEEXX}}(\Omega_{\varepsilon\varepsilon}\otimes\mu_{\mathrm{XX,k}})+\\
&(A_{\varepsilon\varepsilon}\otimes A_{\varepsilon\varepsilon}\otimes B_{X})(\Omega_{\varepsilon\varepsilon}\otimes\mu_{\mathrm{X,k}})+(A_{\varepsilon\varepsilon}\otimes A_{\varepsilon\varepsilon}\otimes C)\Omega_{\varepsilon\varepsilon}+\\
&(A_{\varepsilon\varepsilon}\otimes A_{\varepsilon\varepsilon}\otimes A_{XX})(\Omega_{\varepsilon\varepsilon}\otimes\mu_{\mathrm{XX,k}})+(A_{\varepsilon\varepsilon}\otimes A_{\varepsilon\varepsilon}\otimes A_{\varepsilon\varepsilon})\Omega_{\varepsilon\varepsilon\varepsilon}
\end{aligned}
\qquad (12)
$$

The equation for the third moment (with zero higher moments) is linear in moments of state variables X_t, but it is large scale equation. Equation for the second moments is Lyapunov equation and there are special algorithms for solving of it. The equation for the third moments is large and there is not special algorithms (which use specific structure of matrixes) for solving of it. That is why the same iteration algorithm is used for moments calculation. This approach would be called the zero approximation of the higher then 3 moments (the ZAHM3). The case with zero higher then the second moment is calculated too (the ZAHM2).

The DSGE model

Finance is one of the areas where linear approximation of DSGE models is unsuitable. Therefore a finance model is used here for comparing different estimation approaches. The same model as [Ivashchenko (2013)] is used (but with additional observed variables). Households maximize the expected utility function (13) with budget constraint (14). There are 3 types of expenditure: consumption (Ct) with exogenous price ($Z_{P,t}$), one period bonds (B_t), and stocks (X_t), the price of which is S_t. There are 3 sources of income: exogenous income ($S_t Z_{I,t}$), bonds with interest which were bought one period ago ($R_{t-1} B_{t-1}$), and stocks with dividend which were bought one period ago ($X_{t-1}(S_t+D_t)$).

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t)^{\gamma}}{\gamma} \to \max_{C;B;X} \qquad (13)$$

$$Z_{P,t} C_t + B_t + X_t S_t = R_{t-1} B_{t-1} + X_{t-1}(S_t + D_t) + S_t Z_{I,t} \qquad (14)$$

The model suggests that dividend growth is exogenous (15), the bond amount is set by the government (16), and the amount of stocks is equal to 1 (17).

$$\frac{D_t}{D_{t-1}} = Z_{D,t} \qquad (15)$$

$$B_t = Z_{B,t} S_t \qquad (16)$$

$$X_t = 1 \qquad (17)$$

The model (13)-(17) is transformed to (18)-(22) where stationarised variables are used. Table 1 shows the relation between the original and stationarised variables.

TABLE 1. The DSGE model variables

Variable	Description	Stationary variable
B_t	Value of bonds bought by households at period t	$b_t = B_t / S_t$
C_t	Consumption at time t	$c_t = \ln(Z_{P,t} C_t / S_t)$
D_t	Dividends at time t	$d_t = \ln(D_t / S_t)$
R_t	Interest rate at time t	$r_t = \ln(R_t)$
S_t	Price of stocks at time t	$s_t = \ln(S_t / S_{t-1})$
X_t	Amount of stocks bought by households at period t	$x_t = X_t$
Λ_t	Lagrange multiplier corresponding to budget restriction of households at period t	$\lambda_t = \Lambda_t$
$Z_{A,B,t}$	Exogenous process corresponding to near-rationality of households with its bond position	$z_{A,B,t} = Z_{A,B,t}$
$Z_{A,C,t}$	Exogenous process corresponding to near-rationality of households with its consumption	$z_{A,C,t} = Z_{A,C,t}$
$Z_{A,S,t}$	Exogenous process corresponding to near-rationality of households with its stocks position	$z_{A,C,t} = Z_{A,C,t}$
$Z_{B,t}$	Exogenous process corresponding to bond amount sold by the government	$z_{B,t} = Z_{B,t}$
$Z_{D,t}$	Exogenous process corresponding to dividends growth	$z_{D,t} = Z_{D,t}$
$Z_{I,t}$	Exogenous process corresponding to households income	$z_{T,t} = Z_{T,t}$
$Z_{P,t}$	Exogenous process corresponding to price level	$z_{P,t} = \ln(Z_{P,t} / Z_{P,t-1})$

$$E_0 \sum_{t=0}^{\infty} \beta^t e^{\gamma \sum_{i=0}^{t} (s_i - z_{P,i})} \frac{\left(e^{c_t}\right)^{\gamma}}{\gamma} \to \max_{c;b;x} \quad (18)$$

$$e^{c_t} + b_t + x_t = e^{r_{t-1} - s_t} b_{t-1} + x_{t-1}\left(1 + e^{d_t}\right) + z_{I,t} \quad (19)$$

$$d_t - d_{t-1} + s_t = z_{D,t} \quad (20)$$

$$b_t = z_{B,t} \quad (21)$$

$$x_t = 1 \quad (22)$$

The optimal conditions of (18)-(19) problems with additional exogenous processes ($z_{A,S,t}$, $z_{A,B,t}$, $z_{A,C,t}$) are the following:

$$e^{\lambda_t + z_{A,S,t}} = E_t e^{\lambda_{t+1} + \ln(\beta) + \gamma\left(s_{t+1} - z_{P,t+1}\right)}\left(1 + e^{d_{t+1}}\right) \quad (23)$$

$$e^{\lambda_t + z_{A,B,t}} = E_t e^{\lambda_{t+1} + r_t - s_{t+1} + \ln(\beta) + \gamma\left(s_{t+1} - z_{P,t+1}\right)} \quad (24)$$

$$\gamma c_t = \lambda_t + c_t + z_{A,C,t} \quad (25)$$

An additional exogenous process ($z_{A,S,t}$, $z_{A,B,t}$, $z_{A,C,t}$) could be interpreted as near-rational households (these processes have zero mean). Another interpretation is the compensation of approximation errors (these processes allows the use of a linear approximation for parameter estimation). All the exogenous processes are AR(1) with the following parameterization:

$$z_{*,t} = \eta_{0,*,t}(1 - \eta_{1,*,t}) + \eta_{1,*,t} z_{*,t-1} + \varepsilon_{*,t} \quad (26)$$

The model parameters are estimated with indirect inference approach (DSGE-VAR with 4 lags and 5 iterations with zero higher then second moments). Monthly data (Average rate on 1-month certificates of deposit; MSCI USA price return; MSCI USA gross return; personal consumption expenditures; compensation of employees) from January 1975 till December 2012 are used. The estimated values are used for generating observations.

The comparison techniques

The two approaches for comparison of the moments calculation techniques are used. The first of them is the comparison of moments approximations with more accurate estimation of them. The second approach is the comparison of estimation error (generation of the observed variables by the DSGE model and the estimation of models with different moments based approaches).

The DSGE model is simulated for 100 000 observations. The moments are calculated. It is repeated 10 times. Mean and standard deviation of the moments are reported. Deviations from this moment’s mean (errors) are reported for the each approach.

The following procedure is used for comparing estimate results:

1. Generation of 400 observations (600 and drop of the first 200 observations) from the second-order approximation of model
2. Parameters estimation by different approaches (linear maximum likelihood; the CDKF based maximum likelihood; the indirect inference maximum likelihood and the GMM with different moment's calculation approaches). The true values of parameters are used as the initial values.
3. Steps 1-2 are repeated 100 times.

The indirect inference maximum likelihood is DSGE-VAR (with infinite weight of prior parameter) maximum likelihood estimation (Del Negro and Schorfheide (2004)). The Newey-West estimator (with window equal $7 \approx 4(T/100)^{2/9}$) is used for calculation of moments variance for the GMM [Hamilton (1994)].

The results

Tables 2-3 show the results of moment's estimation. All approaches have very high errors after the first iteration, but these errors greatly decrease after the second iteration (to the same level for each approach). The errors after 5 and 10 iteration are very close what indicate fast convergence. It should be noted that the errors of all approaches are close to errors of mean over 100 000 sample what indicate very high quality of such simple moment approximations. The quality of the NAHM2 and the ZAHM3 are very similar. The quality of the ZAHM2 is 3-4 times worse after 5-10 iteration.

TABLE 2. The DSGE model moments estimation

		Mean	Std	NAHM2				ZAHM2	
iteration number				1	2	5	10	1	2
First	obs_pp	9.32E-03	1.76E-04	-1.92E-01	8.18E-04	1.99E-05	1.89E-05	-1.92E-01	8.18E-04
	obs_pg	1.57E-02	1.70E-04	-1.92E-01	8.71E-04	7.53E-05	7.42E-05	-1.92E-01	8.71E-04
	obs_r	1.69E-02	6.56E-05	7.51E-04	-3.99E-05	8.72E-06	4.99E-06	7.51E-04	-3.99E-05
	obs_c	9.32E-03	1.76E-04	-5.88E-02	-1.46E-01	8.04E-04	1.66E-05	-5.88E-02	-1.46E-01
	obs_i	9.32E-03	1.76E-04	-1.92E-01	8.19E-04	2.02E-05	1.92E-05	-1.92E-01	8.18E-04
Second	obs_pp	3.09E-03	1.98E-05	-4.05E-02	1.55E-05	5.96E-06	5.98E-06	-4.05E-02	4.76E-05
	obs_pg	3.22E-03	2.11E-05	-4.30E-02	3.66E-05	7.74E-06	7.75E-06	-4.30E-02	6.77E-05
	obs_r	2.90E-04	2.21E-06	2.48E-05	-1.51E-06	2.69E-07	1.42E-07	2.48E-05	-1.36E-06
	obs_c	2.15E-03	1.45E-05	-5.30E-03	-2.85E-02	-8.35E-04	-8.07E-04	-4.84E-03	-2.41E-02
	obs_i	6.11E-03	2.39E-05	-4.05E-02	-9.33E-06	1.25E-05	1.25E-05	-4.05E-02	2.29E-05
Second lagged	obs_pp	8.87E-05	7.13E-06	-3.42E-03	1.90E-05	4.11E-08	3.29E-08	-3.44E-03	1.23E-06
	obs_pg	2.35E-04	8.35E-06	-6.15E-03	3.20E-05	1.92E-06	1.89E-06	-6.17E-03	1.43E-05
	obs_r	2.90E-04	2.21E-06	2.56E-05	-1.37E-06	2.80E-07	1.53E-07	2.55E-05	-1.39E-06
	obs_c	5.89E-04	1.00E-05	-9.48E-03	4.50E-03	4.15E-04	3.86E-04	-8.57E-03	-1.35E-03
	obs_i	6.41E-05	1.06E-05	-3.42E-03	3.16E-05	-5.22E-06	-5.20E-06	-3.44E-03	1.14E-05
RMSE of first moments				1.51E-01	6.54E-02	3.62E-04	3.61E-05	1.51E-01	6.54E-02
RMSE of second moments				2.58E-02	5.80E-03	1.79E-04	1.75E-04	2.58E-02	5.18E-03
RMSE of second lagged moments (1 lag)				1.13E-02	1.77E-03	9.15E-05	8.46E-05	1.03E-02	1.04E-03
RMSE of all second moments (4 lags)				1.29E-02	2.87E-03	9.02E-05	8.71E-05	1.27E-02	2.55E-03
RMSE of all moments				3.22E-02	1.31E-02	1.13E-04	8.57E-05	3.22E-02	1.31E-02

TABLE 3. The DSGE model moments estimation

		Mean	Std	ZAHM2		ZAHM3			
iteration number				5	10	1	2	5	10
First	obs_pp	9.32E-03	1.76E-04	2.01E-05	1.89E-05	-1.92E-01	8.18E-04	2.02E-05	1.89E-05
	obs_pg	1.57E-02	1.70E-04	7.52E-05	7.40E-05	-1.92E-01	8.71E-04	7.56E-05	7.42E-05
	obs_r	1.69E-02	6.56E-05	8.60E-07	5.88E-07	7.51E-04	-3.99E-05	8.87E-06	6.73E-06
	obs_c	9.32E-03	1.76E-04	-1.97E-04	1.90E-05	-5.88E-02	-1.46E-01	-1.41E-03	1.84E-05
	obs_i	9.32E-03	1.76E-04	2.04E-05	1.92E-05	-1.92E-01	8.18E-04	2.05E-05	1.92E-05
Second	obs_pp	3.09E-03	1.98E-05	1.82E-05	1.83E-05	-4.05E-02	1.64E-05	6.02E-06	6.00E-06
	obs_pg	3.22E-03	2.11E-05	1.98E-05	1.98E-05	-4.30E-02	3.75E-05	7.81E-06	7.77E-06
	obs_r	2.90E-04	2.21E-06	6.01E-08	5.14E-08	2.48E-05	-1.46E-06	2.79E-07	2.07E-07
	obs_c	2.15E-03	1.45E-05	3.06E-04	3.80E-04	-4.84E-03	-2.67E-02	-5.15E-04	-6.29E-04
	obs_i	6.11E-03	2.39E-05	2.47E-05	2.48E-05	-4.05E-02	-8.41E-06	1.25E-05	1.25E-05
Second lagged	obs_pp	8.87E-05	7.13E-06	-6.11E-06	-6.18E-06	-3.42E-03	1.61E-05	-7.80E-07	-8.86E-07
	obs_pg	2.35E-04	8.35E-06	-4.18E-06	-4.27E-06	-6.15E-03	2.94E-05	1.13E-06	1.01E-06
	obs_r	2.90E-04	2.21E-06	5.83E-09	-3.43E-09	2.56E-05	-1.46E-06	2.84E-07	2.12E-07
	obs_c	5.89E-04	1.00E-05	-1.95E-04	-1.96E-04	-9.77E-03	5.80E-04	3.75E-04	4.62E-04
	obs_i	6.41E-05	1.06E-05	-1.17E-05	-1.18E-05	-3.42E-03	2.87E-05	-5.98E-06	-6.12E-06
RMSE of first moments				9.50E-05	3.63E-05	1.51E-01	6.54E-02	6.31E-04	3.64E-05
RMSE of second moments				6.70E-04	6.75E-04	2.58E-02	5.45E-03	1.27E-04	1.43E-04
RMSE of second lagged moments (1 lag)				4.64E-04	4.68E-04	1.13E-02	1.11E-03	1.13E-04	1.30E-04
RMSE of all second moments (4 lags)				3.64E-04	3.67E-04	1.29E-02	2.70E-03	8.02E-05	8.82E-05
RMSE of all moments				3.58E-04	3.60E-04	3.22E-02	1.31E-02	1.47E-04	8.68E-05

RMSE of parameters estimation by the indirect inference (DSGE-VAR with 4 lags) with different moment's calculation techniques presented at the table 4. RMSE for the GMM approach presented at the table 5. It should be noted that the indirect inference with moments calculated by the NAHM2 or the ZAHM2 with 2 iterations produce errors covariance matrix which is not positive-definite. Thus, RMSE of the indirect inference presented only for 5 and 10 iterations. The results for the ZAHM3 are not presented due to computational expense of this approach (the ZAHM3 with 5 iterations requires about 21 second; it requires about 11 seconds for 2 iterations what is much higher than the CDKF).

TABLE 4. The RMSE of parameters estimation

	likelihood – the CDKF	DSGE-VAR(4) the NAHM2, 10 iter.	DSGE-VAR(4) the NAHM2, 5 iter.	DSGE-VAR(4) the ZAHM2, 10 iter.	DSGE-VAR(4) the ZAHM2, 5 iter.
Std of $\varepsilon_{A,B}$	8.93E-05	3.83E-05	5.22E-05	1.13E-02	1.06E-03
Std of $\varepsilon_{A,C}$	1.51E-02	8.01E-03	1.21E-02	1.60E-02	1.43E-02
Std of $\varepsilon_{A,S}$	5.41E-03	2.76E-03	1.98E-03	9.51E-03	6.58E-03
Std of ε_{B}	4.41E-02	6.96E-03	6.98E-03	1.38E-01	6.64E-02
Std of ε_{D}	4.50E-03	7.15E-03	8.93E-03	1.09E-02	1.03E-02
Std of ε_{I}	3.96E-02	1.08E-02	4.51E-03	4.11E-03	3.49E-03
Std of ε_{P}	3.73E-03	7.16E-03	5.30E-03	1.84E-02	2.08E-02
$ln(\beta)$	5.45E-03	3.16E-03	2.58E-03	3.65E-03	4.48E-03
γ	1.58E-03	1.50E-03	1.54E-03	2.12E-02	3.47E-02
$\eta_{0,B}$	3.50E-01	1.26E-01	3.10E-01	3.85E-01	3.73E-01
$\eta_{0,D}$	1.66E-03	1.80E-03	2.35E-03	3.42E-03	3.17E-03
$\eta_{0,I}$	1.35E+00	2.62E-01	4.78E-01	9.31E-01	3.12E-01
$\eta_{0,P}$	3.99E-03	3.85E-03	2.39E-03	1.75E-03	9.07E-04
$\eta_{1,AB}$	1.98E-01	9.39E-03	8.72E-03	9.92E-02	2.16E-01
$\eta_{1,AC}$	1.05E-01	6.36E-04	2.03E-01	1.19E-01	2.03E-01
$\eta_{1,AS}$	1.71E-01	9.06E-02	6.92E-02	3.92E-01	3.75E-01
$\eta_{1,B}$	2.78E-01	1.33E+00	1.47E+00	5.92E-01	4.68E-01
$\eta_{1,D}$	1.35E-01	1.55E-01	1.28E-01	1.61E-01	1.68E-01
$\eta_{1,I}$	1.91E-02	9.80E-03	6.46E-03	1.34E-01	8.07E-02
$\eta_{1,P}$	5.07E-01	7.16E-01	8.11E-01	7.34E-01	5.10E-01
Sum of RMSE	3.23E+00	2.75E+00	3.53E+00	3.79E+00	2.87E+00
Sum of MSE	2.37E+00	2.39E+00	3.20E+00	2.15E+00	9.86E-01
Time for likelihood calculation(sec)*	3.95	0.44	0.25	0.125	0.078

*PC used: Intel core 2 Duo E8400 3 GHz, 1 Gb RAM, Windows XP.

It should be noted that a few parameters have much higher RMSE for each approach which mean that it’s influence is critical for a such measure as sum of MSE. The indirect inference with the ZAHM2 (5 iterations) produces extremely high quality of estimation. The reason is a high amount of local extremums (many parameters values produce errors covariance matrix which is not positive-definite). About a half of cases produce low value of the log-likelihood function (the local extremums which are close to the true initial values produce low RMSE).

The NAHM2 with 10 iterations produce the best quality according to sum of RMSE. However, the ZAHM2 with 10 iterations produce the best quality according to the sum of MSE. The CDKF is the second best for the both measures of quality. It should be noted that the NAHM2 with 10 iterations have almost the same sum of MSE as the CDKF.

Unexpected result is that the ZAHM2 is better then the NAHM2 for the estimation purpose (GMM with 2 iterations) despite worse quality of moments calculation. The NAHM2 is better then the ZAHM2 (GMM with 5 or 10 iterations) according to the both measures of quality. The advantage of the indirect inference over the GMM is expectable [Creel and Kristensen (2011)].

The speed of likelihood functions calculations for normal approximations of higher moments (the NAHM2) is between linear-likelihood and the CDKF-likelihood. For the ZAHM2 the speed is almost the same as for linear-likelihood.

TABLE 5. The RMSE of parameters estimation

	GMM the NAHM2 10 iter	GMM the NAHM2 5 iter	GMM the NAHM2 2 iter	GMM the ZAHM2 10 iter	GMM the ZAHM2 5 iter	GMM the ZAHM2 2 iter
Std of $\varepsilon_{A,B}$	8.61E-03	9.50E-03	6.96E-03	3.42E-02	2.72E-02	1.79E-02
Std of $\varepsilon_{A,C}$	1.20E-02	1.96E-02	1.09E-02	1.49E-02	1.85E-02	1.64E-02
Std of $\varepsilon_{A,S}$	6.16E-04	4.98E-04	1.20E-03	2.66E-03	3.36E-03	1.06E-03
Std of ε_{B}	6.54E-03	1.52E-02	4.27E-02	4.79E-01	4.60E-01	6.12E-02
Std of ε_{D}	3.79E-02	3.31E-02	5.04E-02	2.79E-02	3.64E-02	5.06E-02
Std of ε_{I}	1.79E-02	2.65E-02	2.43E-02	5.96E-02	6.09E-02	1.44E-02
Std of ε_{P}	1.40E-02	5.43E-03	3.59E-04	1.71E-02	2.28E-02	3.66E-04
$ln(\beta)$	1.22E-03	1.06E-03	1.47E-03	3.72E-03	5.58E-03	2.67E-03
γ	4.39E-02	6.23E-02	4.06E-01	1.88E-01	1.45E-01	3.04E-01
$\eta_{0,B}$	2.58E-01	5.38E-01	1.01E+00	9.91E-01	8.27E-01	5.89E-01
$\eta_{0,D}$	2.74E-03	3.43E-03	3.78E-03	2.12E-03	2.68E-03	4.52E-03
$\eta_{0,I}$	8.91E-01	2.75E+00	3.28E+00	5.41E+00	4.79E+00	2.58E+00
$\eta_{0,P}$	3.70E-04	6.07E-04	7.38E-04	1.98E-03	2.54E-03	9.89E-04
$\eta_{1,AB}$	4.07E-01	5.61E-01	6.13E-01	1.20E+00	1.28E+00	1.27E+00
$\eta_{1,AC}$	6.42E-01	5.79E-01	4.56E-01	8.87E-01	7.75E-01	1.08E-01
$\eta_{1,AS}$	4.64E-02	4.26E-03	4.22E-01	3.12E-01	4.30E-01	2.98E-01
$\eta_{1,B}$	1.09E+00	6.98E-01	8.81E-01	3.66E-01	4.20E-01	3.94E-01
$\eta_{1,D}$	7.12E-01	8.47E-01	1.09E+00	5.29E-01	7.12E-01	9.94E-01
$\eta_{1,I}$	1.90E-01	8.56E-01	6.32E-01	2.22E-01	3.92E-02	2.93E-01
$\eta_{1,P}$	3.94E-01	4.46E-01	6.74E-01	5.60E-01	6.36E-01	7.40E-01
Sum RMSE	4.78E+00	7.46E+00	9.60E+00	1.13E+01	1.07E+01	7.75E+00
Sum MSE	3.34E+00	1.06E+01	1.55E+01	3.36E+01	2.74E+01	1.06E+01
Time for likelihood calculation (sec)*	0.44	0.25	0.125	0.125	0.078	0.047

*PC used: Intel core 2 Duo E8400 3 GHz, 1 Gb RAM, Windows XP.

Conclusions

This article suggests the new approach to approximation of nonlinear DSGE models moments. These approximations are fast and accurate enough to use them for estimation of parameters of nonlinear DSGE models. The suggested approaches are 9 or 31 times faster (depending on version) then the CDKF. One of the quality measures is 17.5% worse or 15.0% better (depending on version) then the CDKF. Another measure is 0.7% worse or 9.4% better (depending on version) then the CDKF.

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