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Models and Games with Adaptation and Mitigation

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Models and Games with Adaptation and Mitigation

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Abstract

The paper discusses and explores several prospective economic-environmental models with separate investments into mitigation and adaptation. The offered model generates essential implications about associated long-term environmental policies such as the optimal adaptation/mitigation ratio. The author focuses on an analytic model of two countries in competitive and collaborative cases.

JEL Classifications: Q52, Q 56, C61, E22

Keywords: environmental adaptation, mitigation, optimal investment, long-term climate policy

1. Introduction

Two major long-term strategies to deal with forthcoming climate change are to mitigate (decrease) greenhouse gas emissions or to adapt to global environmental changes caused by the emissions. The efficient design and implementation of these strategies bring up many relevant questions. What is the priority of the investment into adaptation versus mitigation? At what extent are the investments into adaptation and mitigation substitutable? How does the optimal investment depend on the country's stage of development? Which analytical and/or numerical tools and techniques can be used to investigate and develop rational investment strategies? How to avoid the conflict of interest between a separate country and the entire region? The list of such questions can be continued. A number of research papers address some of these issues. They use different approaches and tools: analytic, empiric, simulation, numeric, etc. As one of such tools, economic-mathematical models cannot give a complete answer to any of these questions, but are nonetheless useful in getting better insight about this complicated and important global problem.

Gradus and Smulders (1993) are among the first who analyze long-term economic growth models with pollution that involve spending on abatement activities. Stokey (1998) analyzes the optimal technology choice in alternative models with pollution, but with no directed spending on abatement or pollution clean-up. The abatement process is modeled in Byrne (1997) and Vellinga (1999) similarly to an environmental clean-up process. Chimeli and Braden (2005) use the central planner framework and steady-state analysis to explore the relationship between a country's per-capita income and environmental quality. In contrast to the aggregate "environmental protection expenditure" of Chimeli and Braden (2005), Brechet et al (2013) distinguish between abatement and adaptation investments and their specific impact on the economy and the environment.

In contrast to abatement and mitigation, the economic modeling of adaptation is scarce. Lecocq and Shalizi (2009) stress the need for an integrated portfolio of policy actions to minimize the climate bill, but only a few studies explicitly consider adaptation and mitigation as complementary policy responses to climate change. Some papers are purely descriptive (*e.g*. Kane and Yohe, 2000; Smit *et al*., 2000; Agrawal and Fankhauser, 2008; EEA, 2007; UNFCCC, 2007). Other papers use game-theoretic frameworks, either static (Shalizi and Lecocq, 2009; Kane and Shogren, 2000) or dynamic (Buob and Stephan, 2011). The analytic economic-environmental models of (Bretschger and Valente 2011) and (Millner and Dietz 2011) analyze the optimal choice of adaptation expenses but do not consider mitigation. Millner and Dietz (2011) provide a thorough analytic and numeric study of a Ramsey-Koopmans optimal growth model with productive and adaptive capital stocks. They conclude that the optimal investment ratio between adaptive and productive capitals for developing countries is large at the beginning of planning horizon and declines later. Bretschger and Valente (2011) analytically study the effects of climate change and climate adaptation on long-run economic development using two models with endogenous capital, stock pollution and adaptation expenditures. A distinguished feature of their model is the assumption that the physical capital depreciates faster under the climate change.

Computational integrated assessment models have been recently used for the analysis of optimal adaptation (Bosello *et al*., 2010; de Bruin *et al*., 2009; Argawala et al. 2011). These papers address two key questions. First, are mitigation and adaptation substitutes or complementary policy instruments? Second, does the country's stage of development affect the optimal policy mix between mitigation and adaptation? To date, the first question still remains open. Buob and Stephan (2010) partially answer the second question by arguing that high-income countries should invest in both mitigation and adaptation, while low-income countries should invest only in mitigation. Brechet et al. (2013) analytically demonstrate that the substitutability between the two instruments depends on the country’s stage of development. The present paper contributes to this discussion. It extends a macroeconomic analytic framework for studying such issues and exploring associated environmental policies at national and international levels.

The paper is organized as follows. Section 2 describes an economic-environmental model with adaptation and mitigation in a closed one-country world. Section 3 discusses several relevant modifications of this model and the next two sections explore two modifications in details. Section 4 provides the steady-state analysis of the model in order to estimate the impact of capital deterioration on the optimal policies. Section 5 analytically explores the optimal long-term investments into the environmental adaptation and mitigation in the model with two countries.

2. Economic-Environmental Model with Adaptation and Mitigation

The economic-environmental model of (Brechet et al 2013) employs the Solow-Swan one-sector growth framework (Barro and Sala-i-Martin 1995), in which the economy uses a Cobb-Douglas technology with constant returns to produce a single final good Y. The social planner allocates the final good across consumption C, investment I_K in physical capital K, investment I_D in environmental adaptation D, and emission abatement expenditures B in order to maximize the utility of an infinitely lived representative household:

$$\max_{I_K, I_D, C} \int_0^\infty e^{-\rho t} U\left[\ln C - \eta(D) \frac{P^{1+\mu}}{1+\mu} \right] dt \,, \tag{2.1}$$

$$I_K(t) \geq 0, \qquad I_D(t) \geq 0, \qquad C(t) \geq 0, \tag{2.2}$$

subject to the following constraints:

$$Y(t) = AK^{\alpha}(t) = I_K(t) + I_D(t) + B(t) + C(t), \tag{2.3}$$

$$K'(t) = I_K(t) - \delta_K K(t), \qquad K(0) = K_0, \tag{2.4}$$

$$D'(t) = I_D(t) - \delta_D D(t), \qquad D(0) = D_0, \tag{2.5}$$

$$P'(t) = -\delta_P P(t) + \gamma Y(t)/B(t), \qquad P(0) = P_0. \tag{2.6}$$

where $\rho > 0$ is the rate of time preference, $A > 0$ and $0 < \alpha < 1$ are parameters of the Cobb-Douglas production function, $\delta_K \geq 0$, $\delta_D \geq 0$ are deterioration coefficients for physical capital and adaptation capital.

Environmental quality is characterized by the pollution stock P, which is represented by the concentration of greenhouse gases in atmosphere and described by (2.6). The pollution inflow (net emission) is assumed to be proportional to the output Y. The abatement activity B is also a flow (Gradus and Smulders, 1993; Vellinga, 1999). The pollution stock grows with net emissions and declines as the abatement expenditures B increase. Despite its simplicity, our specification captures the major qualitative features of the abatement activity B (see Gradus and Smulders, 1993).[1] In the pollution equation (2.6), the emission factor γ>0 reflects the net flow of pollution, that is, the flow resulting from productive activity net of abatement efforts. The pollution stock P increases with this flow and deteriorates in time at a constant natural decay rate $\delta_P > 0$.

The utility function in (2.1) depends on the consumption C, pollution P, and environmental adaptation capital D, where the *environmental vulnerability* $\eta(D)$ of the economy can be reduced by investing in adaptation. The dependence $\eta(D)$ reflects the *efficiency of adaptation measures* to protect people from climate change adverse impacts.

The model (2.1)-(2.5) allows us to discuss the optimal policy mix between emission abatement and adaptation with respect to the stage of development of the economy. Combining the comparative static analysis with perturbation techniques, Brechet et al (2013) obtain approximate analytical expressions for the steady-state optimal policy mix between emission abatement and environmental adaptation at the macroeconomic level. It is shown that the relevance of adaptation depends on the stage of development of the

[1] Smulders and Gradus (1996) also consider a more general polluting model as the flow $Y^{\varpi}B^{-\lambda}$, $\lambda > \varpi$.

country. Specifically, the optimal policy mix between abatement and adaption investments (the ratio *D/B*) depends on the country's economic potential. One of theoretic findings is the inverted U-shape dependence of the optimal ratio *D/B* on the economy. If the economic efficiency is very weak (lower than a certain critical level), then the optimal policy may be no adaptation at all. The subsequent estimation of applicable parameter ranges and numeric simulation demonstrate that this critical efficiency level is very low and the optimal adaptation/abatement ratio *D/B* is smaller for countries with higher economic efficiency. The optimal investment ratio *D/K* between adaptive and productive capitals possesses a similar qualitative behavior.

The outcomes of Brechet et al (2013) are non-trivial and coincide with some other studies but contradict others. Naturally, all such results are driven by model assumptions and model parameters and are not generic. For example, Millner and Dietz (2011) argue that the optimal *D/K* decreases in time, which cannot be captured in our steady state analysis. Buob and Stephan (2011) study the optimal mix of adaptation and mitigation in a two-stage dynamic game of several identical regions with cooperative and non-cooperative behavior. Their economic model is different from (2.1)-(2.6) and completely ignores production (the income is exogenously given to regions), its "environmental quality level" linearly depends on perfect substitutable adaptation and mitigation, and the cost of adaptation is a priori assumed to depend inversely on the mitigation effort. They arrive to the same result as Brechet et al (2013) that, while high income countries invest in both mitigation and adaptation, the low income countries should invest only in mitigation. In contrast, recent adaption-related IAMs (Agrawala et al 2011) conclude that there would be a greater emphasis on adaptation for developing world in earlier decades in response to the impacts of climatic changes. So, the modeling outcomes depend on modeling choice and further research coupled with fair scientific discussion is required to address this global issue.

3. Possible Extensions of Basic Model

In this section, we identify several relevant problems that require extending the results of (Brechet et al., 2013) for model (2.1)-(2.6).

3.1. Impact of Capital Deterioration on Optimal Policy

In order to override essential technical challenges in a qualitative optimal analysis, Brechet et al (2013) considered the model (2.1)-(2.6) in the case with no capital deterioration. They proved the existence of a unique steady–state trajectory and found approximate analytic expressions for the steady state, which lead to essential economic implications. Although the assumption about zero capital depreciation is common in economic literature, it might be too restrictive for long-term policies that use steady–state analysis. The corresponding economic disturbances can be essential. Section 4 of the present paper considers the model (2.1)-(2.6) in the general case with capital deterioration and analyzes effects of capital deterioration on the optimal dynamics of the modeled economy.

3.2. Model with Several Countries

An imperative avenue for further research of adaptation and mitigation is the multiple-country setting. The model (2.1)-(2.6) explores the optimal mix of adaptation and mitigation investments in a closed (world) economy. By ‘closed’ we not only mean the absence of external trade but also a closed interaction between the economy and the environment. In model (2.1)-(2.6), the environment is not a public good because all costs and benefits of environmental degradation accrue to the country. It is well recognized that climate change has the nature of a global public good and that its international dimension constitutes one of its cornerstones. Extending our model to an *n*-country model with strategic behaviors is highly desirable. An essential step in this direction is done in Section

5 below. The objective is to analyze the optimal environmental policy of a separate country in two-country world and how the international cooperation affects this policy.

3.3. Alternative Models of Pollution Dynamics and Mitigation

The choice of the law of motion (2.5) for the environmental pollution P is critical. The industrial pollution emission and abatement in equation (2.5) can take different forms. An alternative specification of emissions in equation (2.5) could be

$$\Phi(K,B) = \gamma K^{\varpi} B^{-\lambda}, \qquad \Phi(Y,B) = \gamma Y^{\varpi} B^{-\lambda}, \qquad \Phi(Y,B) = \gamma Y^{\varpi}/(B_0+B)^{\lambda}, \tag{3.1}$$

where γ, $\lambda \geq \varpi$, and B_0 are positive constants. However, a preliminary analysis shows that these specifications make the model analysis more challenging. On the other hand, the current choice of Y/B is reasonable from a practical viewpoint. Under specifications of model (2.1)-(2.6), the optimal steady state B is always strictly positive because the value of the objective function is getting worst as B approaches 0. Nevertheless, the use of various pollution laws in modified model (2.1)-(2.6) is highly desirable.

3.4 Modeling of Environmental Impact on Economy

Despite a relatively young age of climate change economics, several aggregate analytic approaches have been developed to model a negative impact of undesirable environmental changes on the economy and human welfare.

Approach 1: Decreased Productive Value. A simple economically-oriented approach emphasizes that the temperature increase causes direct global losses of the economic output. It introduces a *damage function* $G(T)$, $0 < G(T) \leq 1$, that translates the temperature increase T into global losses of the product output Q. Next, a multiplicative utility function $U(G(T)C)$ is employed instead of the standard utility $U(C)$ in related optimization models.

For instance, the simulation-based models DICE, RICE, and WITCH (Agrawala et al. 2011) use the *gross damage functions* of the form

$$GD(T) = \alpha_0 T + \alpha_1 T^{\alpha}. \tag{3.2}$$

Alternatively, the output Q can be assumed to negatively depend on the pollution P, for instance, as $Q(P) = AK^{\alpha}P^{-\beta}$, $\beta>0$, with subsequent impact on the consumption C and utility U.

Approach 2: Decreased Amenity Value. The other popular approach emphasizes that the global warming causes direct welfare losses. Then, the utility function U depends on the global temperature T or the greenhouse gases concentration P. The majority of such models use the concentration P, then the utility function is $U = U(C,P)$. To specify the negative impact of the environment, it is convenient to choose the utility $U(C,P)$ to be additively separable:

$$U(C, P) = U_1(C) - U_2(P). \tag{3.3}$$

where $U_1(C)$ is a standard utility function, say, the isoelastic utility $U_1(C) = \dfrac{C^{1-\gamma}}{1-\gamma}$ or the logarithmic utility $U_1(C) = \ln C$. The function $U_2(P)$ increases in P, so the utility function $U(C,P)$ decreases in P. This property is known as the *disutility of environmental pollution*. If $-U_2''(P) < 0$, then the function (3.3) describes an *increasing marginal disutility* of pollution. For example, U can be taken in the form

$$U(C, P) = \ln C - \eta \frac{P^{1+\theta}}{1+\theta}, \tag{3.4}$$

where $\eta > 0$ in represents the *environmental vulnerability* of the economy, while $\theta > 0$ reflects the increasing marginal disutility of pollution. The assumption of increasing marginal disutility of pollution is common in environmental economics, although some simpler models, say, in the game theory, assume a linear disutility $U_2(P) = \eta P$ of the environment damage.

The construction of modifications of the model (2.1)-(2.6) that implement both approaches 1 and 2 and their comparative analysis of their dynamics is imperative for producing robust policy recommendations.

3.5 Modeling of Environmental Adaptation

The concept of environmental adaptation means that the environmental damage can be reduced by investing in adaptation measures. The adaptation control D is usually the amount of *environmental adaptation capital*, which can be modeled as a stock or flow. The specific description of adaptation process depends on the chosen Approach 1 or 2 from Subsection 3.4. Namely, the *efficiency of adaptation measures* in protecting people from climate change adverse impacts can be described by introducing a dependence of the gross damage function $G(T)$ in (3.2) or the environment damage disutility $U_2(P)$ in (3.3) on the adaptation control D.

In Approach 1, the *damage function* $0<G(T)\leq 1$ is decreased by a *adaptation efficiency function* $\eta(D)>0$, so the final utility becomes $U(\eta(D)G(T)C)$. In particular, the integrated-assessment models with adaptation empirically estimate and use an adaptation efficiency function of the form $\eta(D) = 1/(1+D)$.

In Approach 2, the final utility $U(C,P,D)$ depends on the consumption C, pollution P, and adaptation D. In the case of the additively-separable utility (3.3), the *adaptation efficiency function* $\eta(D)$ appears in (3.3) as

$$U(C, P, D) = U_1(C) - \eta(D)U_2(P). \tag{3.5}$$

In particular, the model (2.1)-(2.6) employs the utility function

$$U(C, P, D) = \ln C - \eta(D)\frac{P^{1+\mu}}{1+\mu}, \quad \mu > 0. \tag{3.6}$$

that satisfies the following realistic *properties of adaptation investments*:

(i) no damages are reduced without adaptation;

(ii) the infinite adaptation can reduce almost all or all damages;

(iii) the more adaptation is used, the less effective it will be.

The recent adaptation-related integrated-assessment models empirically estimate and use the *adaptation efficiency function*

$$\eta(D) = 1/(1+D). \tag{3.7}$$

that also possesses the properties (i)-(iii). It would be of obvious interest to provide a comparative analysis of both approaches and derive robust economic policies that do not depend on the specific choice of adaptation venue. More accurate models with environmental adaptation should possibly combine both approaches, but it requires more accurate modeling data. The authors are working in this direction.

4. Analysis of Model with Deterioration

Following Section 2, we investigate the following nonlinear optimal control problem: find the unknown functions $C(t)$, $D(t)$, $P(t)$, $Y(t)$, $K(t)$, $I_K(t)$, $I_D(t)$, $B(t)$, $t\in[0,\infty)$, that maximize the functional (2.1) subject to the constraints (2.2)-(2.6). For clarity, we assume the same deterioration rate $\delta_K = \delta_K = \delta$ for both physical capital K and adaptation capital D in equations (2.4) and (2.5). The capital deterioration rate $\delta>0$ will play a central role in the analysis below.

The optimal control problem (2.1)-(2.6) includes seven unknowns. Let us choose I_K, I_D, C as the independent decision variables, then K, D, B, P are four state variables determined from four constraints-equalities (2.3)-(2.6). The optimality condition for this problem is obtained using the standard Hamiltonian-based technique.

4.1. Steady-State Analysis

We restrict ourselves to the steady-state analysis of the optimal control problem (2.1)-(2.6). The steady state solution of this problem

$$K(t)=\overline{K}, \quad C(t)=\overline{C}, \quad B(t)=\overline{B}, \quad P(t)=\overline{P}, \quad D(t)=\overline{D}, \; t\in[0,\infty), \tag{4.1}$$

(if it exists) describes the long-term sustainable dynamics of the economic-environmental system under study, which is useful for policy recommendations. The steady-state analysis is the only analytic option in dynamic economic models with several endogenous controls.

As in (Brechet et al. 2013), the *environmental vulnerability* is expressed by the following exponential function:

$$\eta(D) = \underline{\eta} + (\overline{\eta} - \underline{\eta})e^{-aD}, \quad \text{with } \overline{\eta} > \underline{\eta} > 0, \quad a > 0, \tag{4.2}$$

where the term $(\overline{\eta} - \underline{\eta})$ is the range of physical adaptation opportunities, *i.e.* the benefits in terms of vulnerability reduction associated with adaptation measures. The parameter a represents the efficiency of adaptation. The function (4.2) monotonically decreases in D from a maximal value $\eta(0)=\overline{\eta} > \underline{\eta}$, when there is no adaptation at all, to a minimal value $\eta(\infty)=\underline{\eta}>0$, when adaptation efforts approach infinity.

Following (Brechet et al. 2013), the substitution of (4.1) and (4.2) into optimality conditions for the optimization problem (2.1)-(2.6) leads after transformations to the following two nonlinear equations with respect to unknown scalars $\overline{K}$ and $\overline{D}$:

$$\frac{\overline{K}^{\alpha}(A-\overline{K}^{1-\alpha}(\delta+\rho)/\alpha)^{2+\mu}}{\overline{K}(\delta+\rho)/\alpha-\delta(\overline{K}+\overline{D})} = \frac{\gamma^{1+\mu}A^{1+\mu}}{(\delta_P+\rho)\delta_P{}^{\mu}}[\underline{\eta}+(\overline{\eta}-\underline{\eta})e^{-a\overline{D}}], \tag{4.3}$$

$$a(\overline{\eta}-\underline{\eta})e^{-a\overline{D}}\,\frac{\gamma^{\mu+1}A^{\mu+1}}{\delta_P{}^{\mu+1}(\mu+1)} - \frac{\rho(A-\overline{K}^{1-\alpha}(\delta+\rho)/\alpha)^{\mu+1}}{\overline{K}(\delta+\rho)/\alpha-\delta(\overline{K}+\overline{D})} = 0. \tag{4.4}$$

The nonlinear system (4.3)-(4.4) determines the sustainable dynamics of the problem (2.1)-(2.6). If these two equations have a solution $\overline{K}\geq 0$ and $\overline{D}\geq 0$, then the other steady state components are expressed in the terms of $\overline{K}$ and $\overline{D}$ as

$$\overline{B} = A\overline{K}^{\alpha} - \overline{K}(\delta + \rho)/\alpha, \tag{4.5}$$

$$\overline{C} = \overline{K}(\delta + \rho)/\alpha - \delta(\overline{K} + \overline{D}), \tag{4.6}$$

$$\overline{P} = \frac{\gamma A}{\delta_P(A - \overline{K}^{1-\alpha}(\delta + \rho)/\alpha)}, \tag{4.7}$$

$$\bar{I}_K = \delta\overline{K}, \qquad \bar{I}_D = \delta\overline{D}. \tag{4.8}$$

Proving the existence of a solution ($\overline{K}, \overline{D}$) to the nonlinear system (4.3)-(4.4) is challeging. To keep the analysis tractable, some simplifying assumptions must be made. In particular, the existence of a unique solution ($\overline{K}$, $\overline{C}$, $\overline{B}$, $\overline{P}$, $\overline{D}$) in the model (4.4)-(4.4) without capital depreciation , *i.e*., at $\delta = 0$, is proven in (Brechet 2013).

One of key outcomes of (Brechet 2013) is that adaptation is not optimal when the capital level $\overline{K}$ is low. Here, we extend this result and prove that it holds in the general case of the problem, i.e., with capital depreciation. For this purpose, we introduce the auxiliary nonlinear equation

$$\frac{[A - \hat{K}^{1-\alpha}(\delta + \rho)/\alpha]^{2+\mu}}{\hat{K}^{1-\alpha}} = \frac{\gamma^{1+\mu} A^{1+\mu}\overline{\eta}[(\delta + \rho)/\alpha - \delta]}{(\delta_P + \rho)\delta_P{}^{\mu}}. \tag{4.9}$$

in the unknown $\hat{K}$, which is obtained from the equation $f_1(\overline{K}, \overline{D}) = 0$ at $\overline{D} = 0$. The nonlinear equation (4.9) always possesses a unique solution $\hat{K}$, $0 < \hat{K} < \left(\alpha A/(\delta + \rho)\right)^{\frac{1}{1-\alpha}}$. Then, the following major result holds.

Proposition 1. *If the solution* $\hat{K}$ *of the equation (4.9) satisfies the inequality*

$$A\hat{K}^{\alpha} - \hat{K}\frac{\delta + \rho}{\alpha} \le \frac{(1+\mu)\overline{\eta}\rho\delta_P}{a(\overline{\eta} - \underline{\eta})(\delta_P + \rho)}, \tag{4.10}$$

then the optimal steady-state adaptation $\overline{D} = 0$ *and the optimal capital level* $\overline{K} = \hat{K}$ *is given by (4.9). If (4.10) does not hold, then the optimal* $\overline{D} > 0$ *and* $\overline{K} > \hat{K}$ *are found from by the system of two nonlinear equations (4.3) and (4.4).*

Therefore, if the optimal adaptation $\overline{D}$ is positive, then the corresponding capital level $\overline{K}$ is larger, the pollution level $\overline{P}$ is higher, and less effort is devoted to abatement (the abatement investment ratio $\overline{B}/\overline{K}$ is smaller) compared to the case of no adaptation.

To analyze the sensitivity of the optimal dynamics to given model parameters, including the capital deterioration factor, we rely on numerical simulations.

4.2 Numerical Simulation

To illustrate the outcomes of analytic modeling, we solve the system of two nonlinear equations (4.3)-(4.4) numerically for different ranges of the model parameters. One of our objectives is to estimate the sensitivity of the steady state ($\overline{K}$, $\overline{C}$, $\overline{B}$, $\overline{P}$, $\overline{D}$) to the capital deterioration factor δ in order to validate or to revise the policy recommendations given in (Brechet et al. 2013). A sensitivity analysis was carried out for all economically relevant parameter values. The simulation was done using Excel Solver. The results are provided and discussed in the next two subsections.

Simulation Case 1: Economy with essential pollution

We estimate the effects of positive capital depreciation compared to the non-depreciation case which is commonly considered in the theoretical literature. Initial data in the first simulation case are based on (Brechet et al. 2013). Specifically, we consider the following model parameters: $\rho = 0.01$, $\alpha = 0.66$, $\delta_P = 0.0002$, $\gamma = 0.16$, $\mu = 1$, $\underline{\eta} = 0.0015$, $\overline{\eta} = 0.003$, $a = 0.001$. This data set reflects a common viewpoint of the climate science and presumes that the global environmental self–cleaning capability δ_P is negligible compared to the emission factor γ and the environmental vulnerability η. Also, we choose the capital depreciation $\delta = 0.01$ for both adaptation and productive capitals. We intentionally assume rather small capital depreciation to highlight the great sensitivity of our model to it (a larger more realistic δ is used in Section 4.2). Recalling that A represents the economic efficiency, we calculate the sustainable levels of the optimal capital K and adaptation capital D in cases with and without deterioration for different values of A from 1 to 40 in order to determine the optimal policies for various levels of economic development (the US approximately corresponds to A = 25). The main insights are the following.

Capital depreciation greatly affects the state of economy. The simulation outcomes demonstrate a significant impact of the capital deterioration on sustainable development. The optimal steady-state levels of the capital K, production output Y, abatement investment B, and other economic characteristics appear to be only 10-40% of their levels in the case without deterioration. The difference in Y increases for more productive economies.

No adaptation in inefficient economies. The optimal steady-state adaptation stock D with depreciation remains below the one without depreciation for any productivity A. The absence of adaptation for low productivity levels predicted by Proposition 1 clearly appears in Figure 1, which uses the logarithmic scale and the smaller range of A from 0.5 to 5 to demonstrate this effect. One can see the clear (and quite sharp) change in the optimal adaptation level when the economic efficiency A reaches some threshold (A approaching 1) in the presence of capital depreciation.

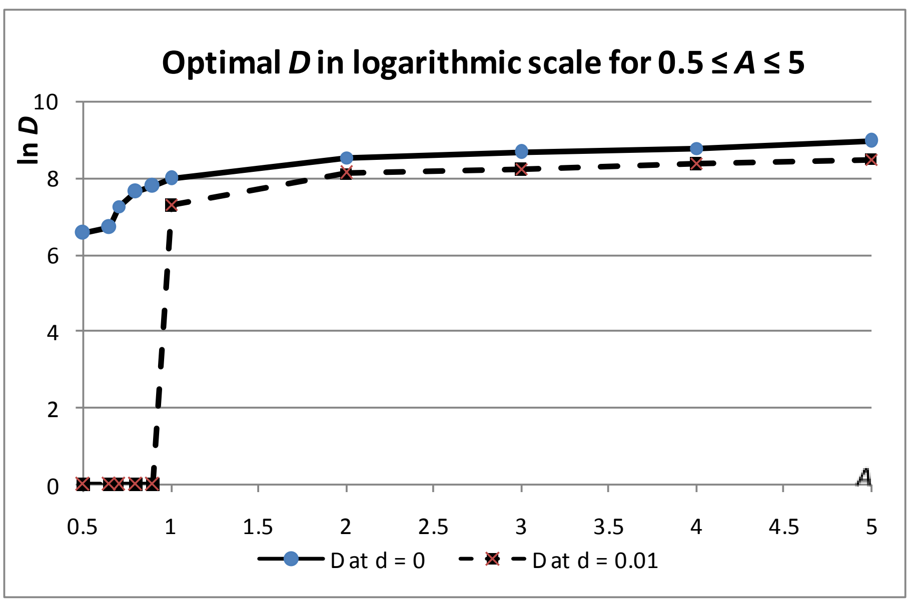

Figure 1. The optimal adaptation capital $\overline{D}$ (in logarithmic scale) for 0.5 < A < 5 without deterioration (solid line) and with deterioration at δ=0.01 (dashed line).

We cannot simulate this effect at $\delta = 0$ because then the numeric solution of system (4.3)-(4.4) at small values of A (shown with the solid curve in Fig.3) is economically impossible: the calculated optimal ratio D/K is 100-400 for $A < 1$. It happens because, in the absence of capital deterioration, the steady-state adaptation D appears to be a free force to compensate the economic inefficiency. Namely, the adaptation investment $\bar{I}_D = \delta\bar{D}$ is zero at $\delta = 0$ and does not affect the distribution equation (2.3). In other words, we do not need to invest to keep any desired steady-state adaptation level $\bar{D}$. Then, the unknown $\bar{D}$ appears in the equations (4.3)-(4.4) in the term $e^{-a\bar{D}}$ only, which leads to instability of their numeric solution. In summary, the presence of the adaptation investment $\delta\bar{D}$ in the distribution equation (2.3) at $\delta > 0$ is not only reasonable economically but it also regularizes numeric solution of the system (4.3)-(4.4).

Simulation Case 2: Less vulnerable economy

The second simulation case describes a more favorable environmental situation. Namely, we take the same values $\rho = 0.01$, $\alpha = 0.66$, $\delta_P = 0.0002$, $a = 0.001$, but with a much smaller polluting economy $\gamma = 0.0001$, less damaging pollution $\mu = 0.1$, and a wider range of environmental vulnerability $\underline{\eta} = 0.005$, $\overline{\eta} = 0.01$. For those parameters, we should also choose a more realistic capital depreciation rate rather than $\delta = 0.01$ as above. The reason is that the whole steady state of an economy is highly sensitive to the capital depreciation rate. If we keep the same $\delta = 0.01$, then the calculated steady-state adaptation more than 1000 times larger and steady-state capital would be 20-50 times larger compared to the case $\delta = 0.05$. So, we select the depreciation rate $\delta = 0.05$ that delivers more realistic simulation results. The chosen dataset reflects an optimistic case of an economy that can survive just with modest emission abatement because the environmental self–cleaning capability δ_P is larger than the pollution emission impact factor γ. As a result, simulation outcomes are less dramatic than in the previous case.

Capital depreciation. Our numerical experiments show that the optimal capital stock K at $\delta = 0.05$ is smaller than 1% of its level for $\delta = 0$. Meanwhile, the consumption C is ≈1.3%

and the output Y is ≈3% of their levels without depreciation. For comparison, we repeated the simulation in this case for δ = 0.01, then the optimal capital K level is 13-30%, the output Y is 25-60%, and the consumption C is 17-40% of their levels without deterioration, which is comparable to the Case 1. So, the impact of capital deterioration on sustainable development remains strong in the case of a less vulnerable economy.

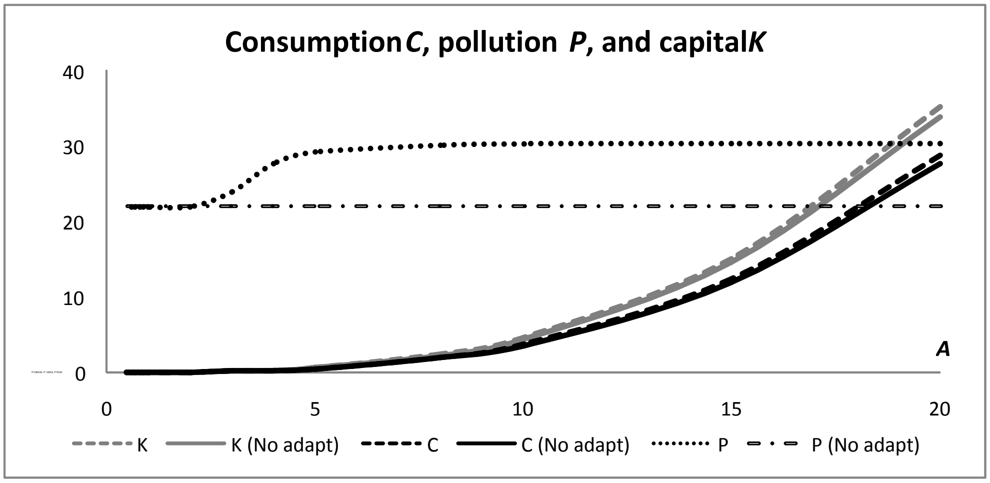

Figure 2. Consumption $\overline{C}$, capital $\overline{K}$, and pollution level $\overline{P}$ at δ=0.05 in two cases: (a) with adaptation and abatement, (b) abatement only.

Adaptation level versus Abatement? Figures 2 and 3 illustrate the impact of optimal adaptation and abatement investments on the economy with the capital deterioration factor δ= 5%. The simulation outcomes demonstrate that the adaptation capital D is in the range 0-2% of the physical capital K at different values of the economic efficiency A. The adaptation capital D is zero at A < 2 and monotonically increases for larger values of A. Figure 2 shows that an increase in the pollution level P due to the presence of adaptation is more essential (up to 40%) as compared to the increase of the economy size (both capital K and consumption C differ by less than 4% from their levels without adaptation).

Figure 3 shows that both adaptation D and abatement B increase in absolute units, but it also reveals that B increases faster. The abatement B is 30-35% larger without adaptation than in the case with adaptation. The most interesting effect is that the optimal ratio D/B

between adaptation and abatement has *an inverted U-shape* (the dotted curve in Fig. 5). The ratio D/B increases when the economic efficiency A increases from 2 to ≈4, and then decreases when A increases further. Specifically, the optimal ratio D/B changes from its maximal value of 6.6 at A = 4 to only 0.3 at A = 20.

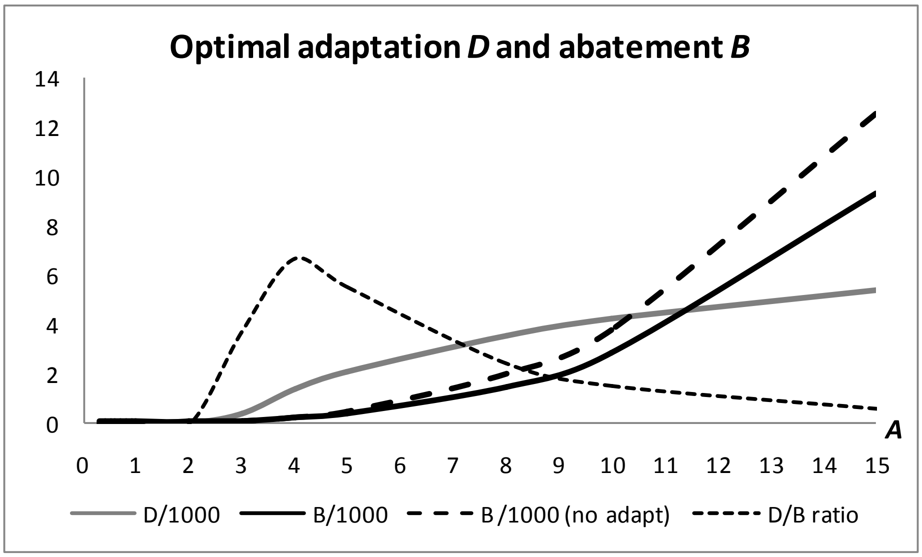

Figure 3. The optimal adaptation $\overline{D}$, abatement $\overline{B}$, and the adaptation/ abatement ratio $\overline{D}\,/\,\overline{B}$ for various values of the economic efficiency A.

The outcomes of this section significantly extend and clarify the result of (Brechet et al. 2013) that the optimal policy mix of adaptation and abatement depends on the country's economic potential in the case of positive capital depreciation. Specifically, investing into adaptation measures becomes profitable only starting with a certain threshold value of the economic efficiency. So, in the case of a poor country, the optimal environmental policy may be no adaptation to climate change at all. In the case of a developed country, the optimal adaptation/abatement ratio remains rather low. The maximum adaptation efforts (in terms of the adaptation/abatement ratio) should be done by countries at a certain intermediate stage of development.

The obtained analytic results and numeric simulation also demonstrate that the impact of capital depreciation on the optimal long-term sustainable growth is quite essential and cannot be ignored in designing optimal policies to combat climate change. We conclude that our modeling framework is applicable for making practical policy recommendations only with a proper estimation of the deterioration of physical and adaptation capital.

5. Optimal Adaptation and Mitigation in Case of Two Countries

This section considers an economic-environmental model of two countries engaged in both mitigation and adaption activities. Our specifications of production and pollution processes and social preferences are similar to the model (2.1)-(2.6). The objective is to investigate how the optimal policy of a given country depends on the country's stage of development and its position on the international area (in short, country's contribution to pollution). The development stage of a country can be characterized by its factor productivity and the rate of time preference. The other objective is to analyze the optimal environmental policy of a separate country in multi-countries world and how the international cooperation changes this policy.

Our first step is to analyze the problem (2.1)-(2.6) for one country in the case when the environmental pollution P has a component Z exogenous to the country. Then, the dynamics of the pollution stock is described by the modified equation (2.6):

$$P'(t) = -\delta_P P(t) + \gamma[Y(t)/B(t) + Z(t)], \qquad P(0) = P_0. \tag{5.1}$$

where $\gamma>0$ is the emission factor (the environmental dirtiness of the economy) and $\delta_P>0$ is a natural pollution decay rate (Brechet et al. 2013). The external pollution flow $Z(t)$ in (2.5) represents the rest of the world in our model. By choosing the size of Z, we will be able to show how the optimal policy of the country changes with respect to its contribution to climate change. It denotes the external exogenous economic activity of the rest of the world that leads to a pollution flow $\gamma Z(t)$.

Next, we apply the obtained results to investigate the case of two countries. First, we assume that the country makes its own environmental decisions and, then, that the decisions are made by an international governmental body on the behalf of both countries.

5.1. Optimization game of two countries: Nash equilibrium

Let us consider an economic-environmental system (the world) that consists of two countries that share the same pollution stock: Country 1 and Country 2. Then, the additional emission $\Delta E > 0$ in the pollution equation (5.1) of Country 1 is resulted from the economic activity of Country 2. Then

$$\Delta E = Y_2(t)/B_2(t) \tag{5.2}$$

in the pollution equation (5.1). Let us also assume that both countries follow the same management policy (to reach maximum utility (2.1)). Then, the corresponding optimization problem

$$\max_{I_{Ki}, I_{Di}, C_i,} \int_0^\infty e^{-\rho t} \left[\ln C_i(t) - \eta_i(D_i(t)) \frac{P(t)^{1+\mu}}{1+\mu} \right] dt, \tag{5.3}$$

$$I_{Ki}(t) \geq 0, \qquad I_{Di}(t) \geq 0, \qquad C_i(t) \geq 0, \qquad i = 1,2,$$

$$Y_i(t) = A_i K_i^{\alpha_i}(t) = I_{Ki}(t) + I_{Di}(t) + B_i(t) + C_i(t), \tag{5.4}$$

$$K_i'(t) = I_{Ki}(t) - \delta_{Ki} K_i(t), \qquad K_i(0) = K_{0i}, \tag{5.5}$$

$$D_i'(t) = I_{Di}(t) - \delta_{Di} D_i(t), \qquad D_i(0) = D_{0i}, \qquad i = 1,2, \tag{5.6}$$

$$P'(t) = -\delta_P P(t) + \gamma A_1 K_1^{\alpha_1}(t)/B_1(t) + \gamma A_2 K_2^{\alpha_2}(t)/B_2(t), \qquad P(0) = P_0, \tag{5.7}$$

is the dynamic continuous game of two players (Countries 1 and 2) with the pay-off functions (5.3), i=1,2. Each player solves the optimization problem (5.3)-(5.7) with its own endogenous variables: the consumption C_i, the investments I_{Ki}, I_{Di}, and the mitigation expense B_i, i = 1,2. Both countries share the same endogenous pollution P.

The solution concept of this game is the *Nash equilibrium* (Gibbons 1992). Indeed, each player knows the strategy of the other player and makes its best decision taking into account the other player's decision. If the continuous game (5.3)-(5.7) of two countries has an equilibrium state, it constitutes a Nash equilibrium.

As before, we restrict ourselves to the comparative static analysis of the problem (5.3)-(5.7) and assume $\delta_{Ki}=\delta_{Di}=0$. Then, the possible steady state $\overline{K}_i$, $\overline{B}_i$, $\overline{C}_i$, $\overline{D}_i$, $\overline{P}$ for each player i =1,2 should satisfy the formulas (14)-(18) at $\Delta E=\overline{Y}_{3-i}/\overline{B}_{3-i}$ or

$$\overline{B}_i = A_i \overline{K}_i^{\alpha_i} - \overline{K}_i \rho / \alpha_i, \tag{5.8}$$

$$\overline{C}_i = \overline{K}_i \rho / \alpha_i, \tag{5.9}$$

$$\overline{D}_i = \max\left\{0, \frac{1}{a_i} \ln \frac{b_i \overline{K}_i \overline{P}^{\mu+1}}{\alpha_i (\mu+1)}\right\}, \qquad i=1,2, \tag{5.10}$$

$$\overline{P} = \frac{\gamma}{\delta_P}\left(\frac{A_1}{A_1 - \overline{K}_1^{1-\alpha_1}\rho/\alpha_1} + \frac{A_2}{A_2 - \overline{K}_2^{1-\alpha_2}\rho/\alpha_2}\right), \tag{5.11}$$

$$\underline{\eta}_i \gamma^{1+\mu} \frac{\rho \overline{K}_i \overline{P}^{\mu+1}}{\alpha_i} - \frac{\rho(1+\mu)}{a_i} = \frac{(\delta_P+\rho)\overline{B}_i^2 \overline{P}}{\gamma A \overline{K}_i^{\alpha_i}}, \qquad i=1,2, \tag{5.12}$$

with respect to $\overline{K}_1$, $\overline{K}_2$, $\overline{B}_1$, $\overline{B}_2$, $\overline{C}_1$, $\overline{C}_2$, $\overline{D}_1$, $\overline{D}_2$, and $\overline{P}$.

The existence of the Nash equilibrium for continuous games is not guaranteed. In the particular game (5.3)-(5.7), it depends on the solvability of the nonlinear system (5.8)-(5.12). Here, we identify the special case of the game when the Nash equilibrium exists and has an important interpretation.

Case of two identical countries: Let

$$A_1=A_2, \quad \alpha_1=\alpha_2, \quad \underline{\eta}_1=\underline{\eta}_2, \quad a_1=a_2, \quad b_1=b_2. \tag{5.13}$$

Then, if the equilibrium steady state exists, it should satisfy $\overline{K}_1=\overline{K}_2=\overline{K}_N$, $\overline{B}_1=\overline{B}_2=\overline{B}_N$, $\overline{C}_1=\overline{C}_2=\overline{C}_N$, $\overline{D}_1=\overline{D}_2=\overline{D}_N$ (the subscript N stands for "Nash"). Then the equations (5.8)-(5.12) for the unknown $\overline{K}_N$, $\overline{B}_N$, $\overline{C}_N$, $\overline{D}_N$, and $\overline{P}_N$ lead to

$$\overline{B}_N = A\overline{K}_N{}^{\alpha} - \overline{K}_N \rho / \alpha, \tag{5.14}$$

$$\overline{C}_N = \overline{K}_N \rho / \alpha, \tag{5.15}$$

$$\overline{P}_N = \frac{2\gamma}{\delta_P}\left(\frac{A}{A - \overline{K}_N{}^{1-\alpha}\rho/\alpha}\right), \tag{5.16}$$

$$\overline{D}_N = \max\left\{0, \frac{1}{a}\ln\frac{b\overline{K}_N \overline{P}_N{}^{\mu+1}}{\alpha(\mu+1)}\right\}. \tag{5.17}$$

where $\overline{K}_N > 0$ is the unique solution of the nonlinear equation

$$\begin{aligned} \frac{A\alpha}{\overline{K}^{1-\alpha}\rho}\left(1 - \frac{\overline{K}^{1-\alpha}\rho}{A\alpha}\right)^{\mu+2} &= \frac{\eta\gamma^{1+\mu}}{(\delta_P + \rho)\delta_P{}^{\mu}}\left[1 + Z\left(1 - \frac{\overline{K}^{1-\alpha}\rho}{A\alpha}\right)\right]^{\mu} \\ &+ \frac{\alpha(\mu+1)}{a\overline{K}(1 + \rho/\delta_P)}\left(1 - \frac{\overline{K}^{1-\alpha}\rho}{A\alpha}\right)^{\mu+1}\left[1 + Z\left(1 - \frac{\overline{K}^{1-\alpha}\rho}{A\alpha}\right)\right]^{-1} \end{aligned} \tag{5.18}$$

obtained from the auxiliary problem (2.1)-(2.5),(5.1) at $Z = A\overline{K}_N{}^{\alpha} / \overline{B}_N$.

It is well known that the Nash equilibrium does not necessarily mean the best cumulative payoff for all players; in many cases all the players might improve their payoffs if they follow a kind of cooperative strategy. Such a strategy is considered in the next section.

5.2. Cooperative optimization

This section considers the international cooperation in our "two-countries" world. Let us assume a benevolent social planner represents a multinational governmental body with the complete control over the environmental policies of Countries 1 and 2. Both countries share the same pollution stock P. The corresponding optimization problem

$$\max_{I_{K1}, I_{D1}, C_1, I_{K2}, I_{D2}, C_2} \int_0^{\infty} e^{-\rho t}\left[\ln C_1(t) + \ln C_2(t) - \eta(D_1(t))\frac{P_1(t)^{1+\mu}}{1+\mu} - \eta(D_2(t))\frac{P_2(t)^{1+\mu}}{1+\mu}\right]dt \tag{5.19}$$

$$I_{K_i}(t) \geq 0, \qquad I_{D_i}(t) \geq 0, \qquad C_i(t) \geq 0,$$

$$A_i K_i^{\alpha_i}(t) = I_{K_i}(t) + I_{D_i}(t) + B_i(t) + C_i(t), \tag{5.20}$$

$$K_i'(t) = I_{K_i}(t) - \delta_{K_i} K_i(t), \qquad K_i(0)=K_{i_0}, \tag{5.21}$$

$$D_i'(t) = I_{D_i}(t) - \delta_{D_i} D_i(t), \qquad D_i(0)=D_{i_0}, \qquad i = 1,2, \tag{5.22}$$

$$P'(t) = -\delta_P P(t) + \gamma A_1 K_1^{\alpha_1}(t)/B_1(t) + \gamma A_2 K_2^{\alpha_2}(t)/B_2(t), \qquad P(0)=P_0, \tag{5.23}$$

is the extension of the optimization problem (2.1)-(2.6) with the double set of endogenous variables: the consumptions C_1 and C_2, the investments I_{K_1}, I_{K_2}, I_{D_1}, I_{D_2}, and the mitigation expenses B_1 and B_2 of Countries 1 and 2 correspondingly. The objective function describes the best cumulative payoff for both countries.

The optimization problem includes six decision variables I_{K_1}, I_{K_2}, I_{D_1}, I_{D_2}, C_1 and C_2, seven state variables K_1, K_2, D_1, D_2, B_1, B_2, P, and nine constraints-equalities (5.20)-(5.23). Despite its double size, it can be treated analogously to the problem (2.1)-(2.6). Assuming $\delta_{K_i} = \delta_{D_i} = \delta$, introducing and excluding dual variables for nine constraints-equalities (5.20)-(5.23) and providing the comparative static analysis, we obtain the following interior first order conditions for nine steady-state variables $\overline{K}_i$, $\overline{B}_i$, $\overline{C}_i$, $\overline{D}_i$, i =1,2, and $\overline{P}$:

$$\alpha_i A_i \overline{K}_i^{\alpha_i} - \alpha_i \overline{B}_i = \overline{K}_i(\delta + \rho), \tag{5.24}$$

$$A_i \overline{K}_i^{\alpha_i} = \delta \overline{K}_i + \delta \overline{D}_i + \overline{B}_i + \overline{C}_i, \tag{5.25}$$

$$-\eta_i'(\overline{D}_i)\frac{\overline{P}^{\mu+1}}{(1+\mu)} = \frac{(\delta+\rho)}{\overline{C}_i}, \quad i = 1,2, \tag{5.26}$$

$$\overline{P}^{\mu}[\eta_1(\overline{D}_1) + \eta_2(\overline{D}_2)] = \frac{(\delta_P + \rho)\overline{B}_1^{\,2}}{\gamma A_1 \overline{K}_1^{\alpha_1} \overline{C}_1} = \frac{(\delta_P + \rho)\overline{B}_2^{\,2}}{\gamma A_2 \overline{K}_2^{\alpha_2} \overline{C}_2}, \tag{5.27}$$

$$\delta_P \bar{P} = \gamma\left(\frac{A_1 \bar{K}_1^{\alpha_1}}{\bar{B}_1} + \frac{A_2 \bar{K}_2^{\alpha_2}}{\bar{B}_2}\right), \tag{5.28}$$

An analysis of the system (5.25)-(5.28) indicates a way of its sequential solution, which is the extension of the technique of Section 2. Namely, assuming $\delta = 0$, the steady state $\bar{K}_i$, $\bar{B}_i$, $\bar{C}_i$, $\bar{D}_i$, $i = 1,2$, $\bar{P}$ satisfies

$$\bar{B}_i = A_i \bar{K}_i^{\alpha_i} - \bar{K}_i \rho / \alpha_i, \tag{5.29}$$

$$\bar{C}_i = \bar{K}_i \rho / \alpha_i, \tag{5.30}$$

$$\bar{D}_i = \max\left\{0, \frac{1}{a_i} \ln \frac{b_i \bar{K}_i \bar{P}^{\mu+1}}{\alpha_i(\mu+1)}\right\}, \quad i = 1,2, \tag{5.31}$$

$$\bar{P} = \frac{\gamma}{\delta_P}\left(\frac{A_1}{A_1 - \bar{K}_1^{1-\alpha_1}\rho/\alpha_1} + \frac{A_2}{A_2 - \bar{K}_2^{1-\alpha_2}\rho/\alpha_2}\right), \tag{5.32}$$

$$\frac{\gamma \bar{P}^{\mu}}{(\delta_P + \rho)}[\eta_1(\bar{D}_1) + \eta_1(\bar{D}_1)] = \frac{[\alpha_1 A_1 - \bar{K}_1^{1-\alpha_1}\rho]^2}{\rho \alpha_1 A_1 \bar{K}_1^{1+\alpha_1}} = \frac{[\alpha_2 A_2 - \bar{K}_2^{1-\alpha_2}\rho]^2}{\rho \alpha_2 A_2 \bar{K}_2^{1+\alpha_2}}, \tag{5.33}$$

that differs from the corresponding Nash equilibrium system (5.8)-(5.12) only by the last “double” equation (5.33). Substituting the expressions (5.31) and (5.32) of $\bar{D}_1$, $\bar{D}_2$ and $\bar{P}$ via $\bar{K}_1$ and $\bar{K}_2$ into (5.33), we obtain two nonlinear equations with respect to $\bar{K}_1$ and $\bar{K}_2$. This system of two equations will be an analogue of the equation (4.8) for $\bar{K}$ in the one country case.

Case of identical countries. The solution is essentially simple in the case (5.13) of identical countries. Then, by symmetry considerations, the steady state satisfies conditions $\bar{K}_1 = \bar{K}_2 = \bar{K}_{CO}$, $\bar{B}_1 = \bar{B}_2 = \bar{B}_{CO}$, $\bar{C}_1 = \bar{C}_2 = \bar{C}_{CO}$, $\bar{D}_1 = \bar{D}_2 = \bar{D}_{CO}$ (where the subscript CO stands for “cooperativeness”). Substituting the last expressions into the system (5.29)-(5.32), we obtain exactly the same system (5.8)–(5.12) for the unknown $\bar{K}_{CO}$, $\bar{B}_{CO}$, $\bar{C}_{CO}$, $\bar{D}_{CO}$, $\bar{P}_{CO}$, in which the coefficient γ is replaced with 2γ.

The obtained analytic expressions (5.14)-(5.18) and (5.29)-(5.33) have an imperative potential for analyzing the optimal adaptation-mitigation strategy for various combinations of different countries. Their comparison even in such simple case as two identical countries leads some expected and unexpected conclusions.

As it is natural to expect, if the environmental self–cleaning capability δ_P is essential and the emission impact factor γ and the environmental vulnerability η are negligible, then environmental considerations do not impact the optimal policies. Indeed, in the case $\eta\gamma^{\mu+1} << (\rho+\delta_P)\delta_P{}^{\mu}$, both Nash equilibrium (*NE*) and cooperative optimal strategies are similar and do not essentially depend on the environmental parameters.

An approximate analysis shows that, in the case of large emission impact factors γ and the environmental vulnerability η, large economic and adaptation efficiencies *A* and *a*, and negligible environmental self–cleaning δ_P, the NE value of capital is $2^{1/(1-\alpha)}$ times larger than the optimal capital amount in the cooperative optimization problem. The comparison of the optimal objective functional values demonstrates that the NE strategy delivers larger values of consumption and larger adaptation effort but the corresponding larger pollution amount diminishes the first two components (and makes the NE strategy non-optimal compared to the cooperative case). Moreover, if we consider *n* identical countries, *n* >2, then the similar conclusion holds with the replacing 2 with *n*.

In general, the NE strategy in the multi-country world under study is not optimal and involves overproduction, which leads to overconsumption, over-contamination and over-adaptation. The primary problem is the overproduction that appears because the country controls only its own pollution emission (so, a decrease in production causes twice smaller decrease in pollution emissions that the optimal one). The corresponding adaptation investment depends on both production and pollution so the over-adaption is the result of overproduction. The range $[\mathrm{M}_{\eta cr}, \infty)$ of the adaptation potential for which the countries engage in adaptation is larger when the regions do not cooperate.

Finally, let us stop on the mitigation. In both NE game and the cooperative optimization problem, the optimal mitigation is determined by the same formulas (5.17), (5.31). In both cases, the decision with regard to mitigation spending depends on the production level only and can be analyzed separately from the adaption decision. Formally, the NE mitigation $\overline{B}_N$ is larger than the optimal cooperative mitigation $\overline{B}_{CO}$. However, there is no mitigation overspending in the NE case, the mitigation is just right to keep the (larger) pollution under control. Because of the accepted hypotheses about the pollution motion law (2.6), we cannot decrease the economic pollution flow *Y/B* to zero, just to a constant value.

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