Collateral, Funding

and Discounting

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1 Risk free rate

- Traditional derivatives valuation framework starts by assuming an existence of a (credit) risk-free rate
  - Little "r" in Black-Scholes
- Everything is discounted off risk-free rate
- Where is the risk-free rate now?
  - Give cash to another bank?
  - Give cash to a government?
- Nothing in modern economy looks like a theoretically-classic money market account
- How a banks funds itself, i.e. what it does with spare cash (and where it gets cash when it needs to), is of critical importance
  - Before: Bank funding rate $\approx$ Libor $\approx$ FedFunds $\approx$ Government rate...
  - Not anymore! That is why we need to revisit the foundations

2 Credit risk mitigation in OTC trading

- Over-the-counter (bilateral) trading is governed by legal documents, primary of which is ISDA Master Agreement
- Part of it, Credit Support Annex (CSA) specifies credit risk mitigation in form of collateral posting
- In broad strokes, it specifies that if party A owes money to party B, it has to post collateral in that amount, and vice versa
- So if A defaults, B could take that collateral in lieu of the promise of A
- CSA specifies other important credit risk mitigants such as netting – if A owes B on one contract and B owes A on some other, they can be offset against each other in the case of default
- CSAs between each two parties are (somewhat) different. CSA specifies
  - Eligible collateral (cash in a number of currencies, bonds)
  - Rates paid on collateral (party holding collateral typically pays certain rate to the collateral "owner")
  - frequency of collateral posting (e.g. daily)
  - Thresholds, minimum transfer amounts, etc

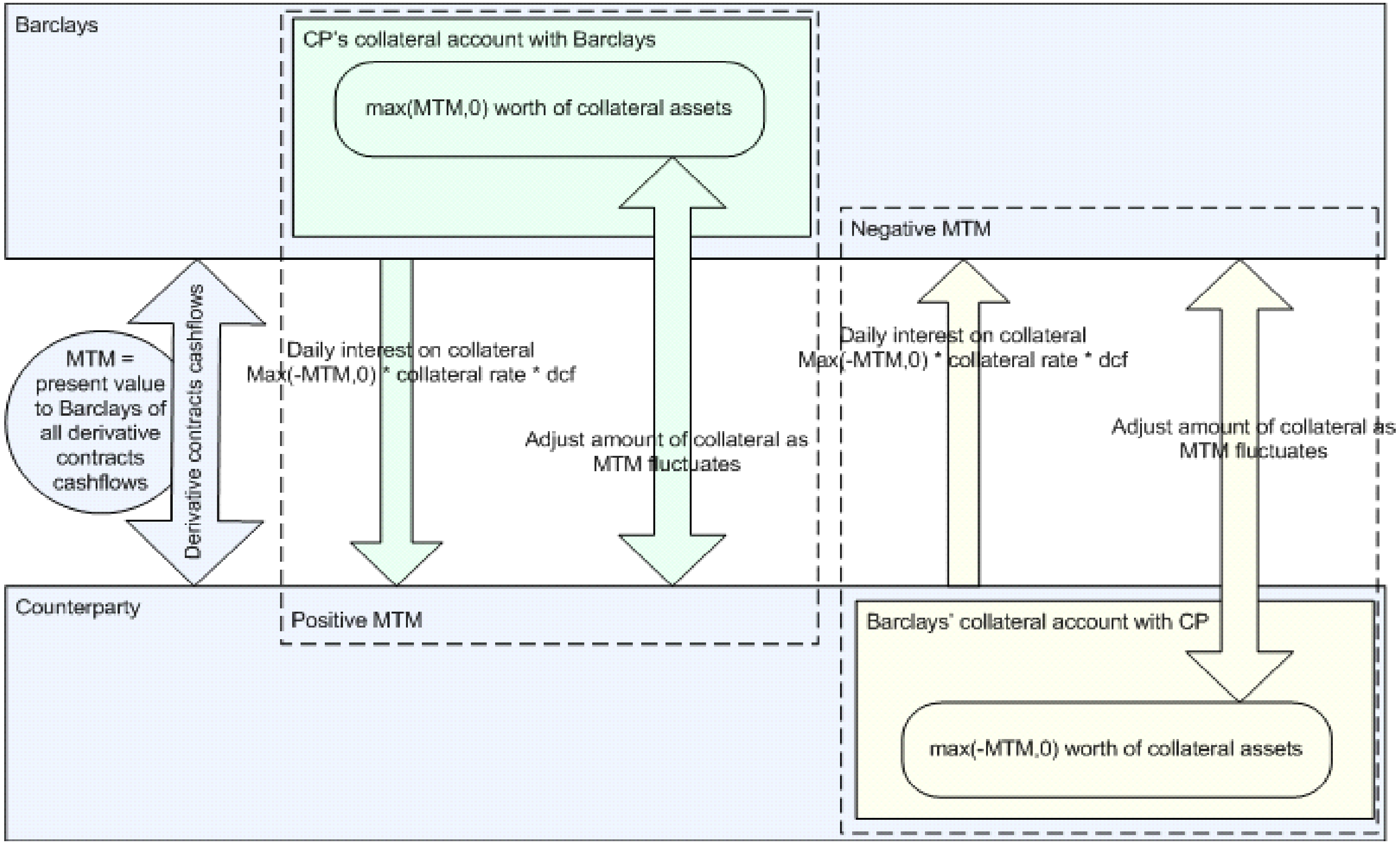
3 Collateralized Assets

- Let us look at the mechanics of collateralized trading
- Party A sells a call option to party B
- B pays $V(0)$ dollars to A
- A promises to pay the payoff of the option at expiry to B
- Any promise needs to be collateralized. A needs to post collateral. How much?
- Well, it is the value of the promise (option) so $V(0)$ dollars! They go right back to B
- During life, the value of the option fluctuates. Depending on the move A will post or claim back collateral
- B will pay an agreed-upon overnight rate on the outstanding collateral to A
- At any point in time the t total collateral posted by A will be $V(t)$ which is the value of the option on that day

4 Collateralized Assets

- No cash exchange at inception
- Pays daily (in math abstraction *continuous*) cashflows
- At any time the option contract could be dissolved and collateral kept – the collateral will exactly offset the market value of the option
- In particular, at option expiry B will just keep the collateral it has and A does not need to pay anything else
- Quite different from a classic picture of "buy and hold"
- Similar to a future. *Future = collateralized forward with collateral rate=0*

5 Collateralized Assets – Picture



6 Hedging instruments

- Trading in hedging instruments (stocks, bonds) fits the same pattern
- When we need to buy stock, where does the bank get money? (How does it fund the shares)
- By borrowing them, with the borrow secured by the shares just bough (repo)
- The rate for this loan is the repo rate
- Borrow the money, buy stock
- Deliver shares as collateral for the loan
- Get collateral back the next day
- Return the loan and the overnight interest (repo rate)
- Repeat for as many days as the shares are needed
- Paying repo rate more efficient than borrowing unsecured – lower rate due to absence of credit risk
- Same type of continuous cashflows as with collateralized derivatives

7 Discounting rate

- Consider fully collateralized counterparty
- Main question: what rate should we use to discount trades with this counterparty?
- In full generality a very difficult question to answer. We try in [Pit12]
- Notations
  - $V(t)$ is price of a collateralized asset between party A and B. If $V(t) > 0$ for A, party B will post $V(t)$ to A.
  - $c(t)$ is a contractually specified collateral rate $c(t)$ on $V(t)$. If $V(t) > 0$, A will pay this rate to B

- Assume A buys some collateralized asset from B ("buying" and "selling" in this context are somewhat meaningless but we keep the terminology for simplicity).

1. Purchase of the asset. The amount of $V(t)$ is paid by A to B
2. Collateral at t. Since A's mark-to-market is $V(t)$, the amount $V(t)$ of collateral is posted by B to A
3. Return of collateral. At time $t + dt$ A returns collateral $V(t)$ to B
4. Interest. At time $t + dt$, A also pays $V(t)c(t)\, dt$ interest to B
5. New collateral. The new mark-to-market is $V(t+dt)$. Party B pays $V(t+dt)$ in collateral to A.

Note that there is no actual cash exchange at time t. At time $t + dt$, net cash flow to A is given by

$$V(t + dt) - V(t)(1 + c(t))\, dt = dV(t) - c(t)V(t)\, dt.$$

As already noted, at time $t + dt$, the MTM+collateral for each party is 0, meaning they can terminate the contract (and keep the collateral) at no cost.

9 Two Collateralized Assets

- Start by assuming two assets both collateralized with rate $c(t)$
- In real world measure the asset prices follow

$$dV_i(t) = \mu_i(t)V_i(t)\,dt + \sigma_i(t)V_i(t)\,dW(t), \quad i = 1, 2. \tag{1}$$

- Note the same Brownian motion. Case of a stock (i.e. a repo transaction with stock) and an option on that stock.
- At time t form a portfolio to hedge the effect of randomness of $dW(t)$ on the cash exchanged at time $t + dt$ (no cash exchange at t)
- Go long asset 1 notional $\sigma_2(t)V_2(t)$ and go short asset 2 notional $\sigma_1(t)V_1(t)$
- The cash exchange at time $t + dt$ is then equal to

$$\sigma_2(t)V_2(t)\,(dV_1(t) - c(t)V_1(t)\,dt) - \sigma_1(t)V_1(t)\,(dV_2(t) - c(t)V_2(t)\,dt)$$

  which, after some manipulation, gives us

$$\sigma_2(t)V_1(t)V_2(t)\,(\mu_1(t) - c(t))\;dt - \sigma_1(t)V_1(t)V_2(t)\,(\mu_2(t) - c(t))\;dt$$

- This amount is known at time t and the contract can be terminated at $t + dt$ at zero cost. Hence, the only way both parties agree to transact on this portfolio (no arbitrage), this cash flow must actually be zero

- Hence

$$\sigma_2(t)\left(\mu_1(t) - c(t)\right) = \sigma_1(t)\left(\mu_2(t) - c(t)\right).$$

- Using this we can rewrite (1) as

$$dV_i(t) = c(t)V_i(t)\,dt + \sigma_i(t)V_i(t)\,d\tilde{W}(t), \quad i = 1, 2, \tag{2}$$

  where

$$d\tilde{W}(t) = dW(t) + \frac{\mu_1(t) - c(t)}{\sigma_1(t)}dt = dW(t) + \frac{\mu_2(t) - c(t)}{\sigma_2(t)}dt.$$

- Now, looking at (2) we see that there exists a measure Q, equivalent to the real world one, in which both assets grow at rate $c(t)$. This is the analog to the traditional risk-neutral measure.

- In Q, the price process for each asset is given by

$$V_i(t) = \mathrm{E}_t^{\mathrm{Q}}\left(e^{-\int_t^T c(s)\,ds}V_i(T)\right), \quad i = 1, 2. \tag{3}$$

- Collateralized assets should be priced using collateral, usually OIS (Fed-Funds, Eonia) discounting curve

11 Different Collateral Rates

- Even if two assets can be collateralized at different rates, c_1 and c_2, and the same result would apply. In particular we would still have the condition

$$\sigma_2(t)\left(\mu_1(t) - c_1(t)\right) = \sigma_1(t)\left(\mu_2(t) - c_2(t)\right)$$

  from the cash flow analysis.

- Hence, the change of measure is still possible, and (3) still holds with $c(t)$ replaced by the appropriate c_1 or c_2:

$$V_i(t) = \mathrm{E}_t^{\mathrm{Q}}\left(e^{-\int_t^T c_i(s)\,ds} V_i(T)\right), \quad i = 1, 2. \tag{4}$$

- In the stock option example, the stock will grow at its repo rate and the option will grow at its collateral rate in the risk-neutral measure

- Consider more than two assets. Assume that $N+1$ collateralized (with the same collateral rate c) assets are traded, and their real-world dynamics are given by

$$dV = \mu V\,dt + \Sigma\,dW,$$

where dW is an N-dimensional Brownian motion and $dV = (dV_1, \ldots, dV_{N+1})^\top$

- By the cashflow/no arbitrage arguments similar to the above we can find a vector λ such that

$$\mu - c\mathbf{1} = \Sigma\lambda.$$

- Thus we can write

$$dV = cV\,dt + \Sigma\,(dW - \lambda\,dt)$$

and define the risk-neutral measure by the condition that $dW - \lambda\,dt$ is a driftless Brownian motion. In this measure all processes V have drift c.

- If we now consider the assets to be (collateralized) ZCBs, we obtain a model of interest rates that looks exactly like the standard HJM model
- Each (collateralized) zero coupon bond grows at the collateral rate $c(t)$ (or its own collateral rate if different). A zero-coupon bond

$$P(t,T) = \mathrm{E}_t^{\mathrm{Q}}\left(e^{-\int_t^T c(s)\,ds}\right)$$

13 Domestic and Foreign Collateral

- Important example of different collateral rates: cross-currency markets
  - On LCH, single-currency swaps are collateralized in the currency of the trade
  - Cross-currency swaps are collateralized in US dollars.
- Also major dealers normally have a choice of the currency to post as collateral
- We need to consider zero coupon bonds (ZCBs) collateralized in the domestic, and as well as some other (call it foreign) currency
- Economy with domestic and foreign assets and an (cash, T0) FX rate $X(t)$ expressed as a number of domestic ($\mathcal{D}$) units per one foreign ($\mathcal{F}$)
- The domestic collateral rate is $c_d(t)$ and the foreign rate is $c_f(t)$
- Domestic ZCB collateralized in domestic currency by $P_{d,d}(t,T)$. This bond generates the following cashflow at time $t + dt$,

$$dP_{d,d}(t,T) - c_d(t)P_{d,d}(t,T)\,dt. \tag{5}$$

- Now consider a foreign ZCB collateralized with the domestic rate. Let its price, in foreign currency, be $P_{f,d}(t,T)$. Cashflows:

1. Purchase of the asset. The amount of $P_{f,d}(t,T)$ is paid (in foreign currency $\mathcal{F}$) by party A to B.
2. Collateral at t. Since A's MTM is $P_{f,d}(t,T)$ in foreign currency, the amount $P_{f,d}(t,T)X(t)$ of collateral is posted in domestic currency $\mathcal{D}$ by B to A
3. Return of collateral. At time $t+dt$ A returns collateral $P_{f,d}(t,T)X(t)\mathcal{D}$ to B
4. Interest. At time $t+dt$, A also pays $c_d(t)P_{f,d}(t,T)X(t)\,dt$ interest to B in $\mathcal{D}$
5. New collateral. The new MTM is $P_{f,d}(t+dt,T)$. Party B pays $P_{f,d}(t+dt,T)X(t+dt)$ collateral to A in $\mathcal{D}$

The cash flow, in $\mathcal{D}$, at $t+dt$ is

$$d\left(P_{f,d}(t,T)X(t)\right) - c_d(t)P_{f,d}(t,T)X(t)\,dt. \tag{6}$$

15 Drift of FX Rate

- The equations (5), (6) are insufficient to determine the drift of X
- From (6) we can only deduce the drift of the combined quantity $XP_{f,d}$ and the drift of $P_{f,d}$ is in general *not* c_f (nor it is c_d, for that matter)
- To understand the drift of $X(\cdot)$, we need to understand what kind of (domestic) cash flow we can generate from holding a unit of foreign currency
- Suppose we have $1\mathcal{F}$. If it was a unit of stock, we could repo it out (i.e. borrow money secured by the stock) and pay a repo rate on the stock
- In FX, having $1\mathcal{F}$, we can give it to another dealer and receive its price in domestic currency, $X(t)\mathcal{D}$. The next instant $t + dt$ we would get back $1\mathcal{F}$, and pay back $X(t) + r_{d,f}(t)X(t)dt$, where $r_{d,f}(t)$ is a *rate agreed on this domestic loan collateralized by* $\mathcal{F}$. As we can sell $1\mathcal{F}$ for $X(t + dt)\mathcal{D}$ at time $t + dt$ the cash flow at $t + dt$ would be

$$dX(t) - r_{d,f}(t)X(t)\,dt$$

- This is an "instantaneous" FX swap, with a real-life equivalent an overnight (aka tom/next) FX swap
- Importantly, the rate $r_{d,f}(t)$ has no relationship to collateralization rates in two different currencies

1. Market in instantaneous FX swaps allows us to generate cash flow $dX(t) - r_{d,f}(t)X(t)\,dt$
2. Market in $P_{d,d}$ generates cash flow $dP_{d,d}(t,T) - c_d(t)P_{d,d}(t,T)\,dt$
3. Market in $P_{f,d}$ generates cash flow $d\left(P_{f,d}(t,T)X(t)\right) - c_d(t)P_{f,d}(t,T)X(t)\,dt$

- Assume real world measure dynamics (μ, dW are vectors and Σ is a matrix)

$$\begin{pmatrix} dX/X \\ dP_{d,d}/P_{d,d} \\ d\left(P_{f,d}X\right)/\left(P_{f,d}X\right) \end{pmatrix} = \mu\,dt + \Sigma\,dW,$$

- By the same cashflow arguments as before, we can find a measure ("domestic risk-neutral") $\mathbb{Q}^d$ under which the dynamics are

$$\begin{pmatrix} dX/X \\ dP_{d,d}/P_{d,d} \\ d\left(P_{f,d}X\right)/\left(P_{f,d}X\right) \end{pmatrix} = \begin{pmatrix} r_{d,f} \\ c_d \\ c_d \end{pmatrix} dt + \Sigma\,dW^d \tag{7}$$

17 Cross-Currency Model under Domestic Collateral

With

$$\begin{pmatrix} dX/X \\ dP_{d,d}/P_{d,d} \\ d\left(P_{f,d}X\right)/\left(P_{f,d}X\right) \end{pmatrix} = \begin{pmatrix} r_{d,f} \\ c_d \\ c_d \end{pmatrix} dt + \Sigma\, dW^d,$$

we have

$$\begin{aligned} X(t) &= \mathrm{E}_t^d\left(e^{-\int_t^T r_{d,f}(s)\,ds} X(T)\right), \qquad (8) \\ P_{d,d}(t,T) &= \mathrm{E}_t^d\left(e^{-\int_t^T c_d(s)\,ds}\right), \\ P_{f,d}(t,T) &= \frac{1}{X(t)}\mathrm{E}_t^d\left(e^{-\int_t^T c_d(s)\,ds} X(T)\right). \end{aligned}$$

- Same model under foreign collateralization.
- Foreign bonds $P_{f,f}$ and domestic bonds collateralized in foreign currency $P_{d,f}$.
- By repeating the arguments above we can find a measure $\mathbb{Q}^f$ under which

$$\begin{pmatrix} d(1/X)/(1/X) \\ dP_{f,f}/P_{f,.f} \\ d\left(P_{d,f}/X\right)/\left(P_{d,f}/X\right) \end{pmatrix} = \begin{pmatrix} -r_{d,f} \\ c_f \\ c_f \end{pmatrix} dt + \tilde{\Sigma}\, dW^f \tag{9}$$

- In particular

$$P_{d,f}(t,T) = X(t)\mathrm{E}_t^f\left(e^{-\int_t^T c_f(s)\,ds}\frac{1}{X(T)}\right). \tag{10}$$

- Not all processes in (7) and (9) can be specified independently. In fact, with the addition of the dynamics of $P_{f,f}$ to (7), the model is fully specified, as the dynamics of $P_{d,f}$ can then be derived.
- Risk-free rates [FT11] have no economic meaning, their differences (for different currencies) do – they are rates quoted for instantaneous FX swaps and define the rate of growth of the FX rate

19 Forward FX

- A forward FX contract pays $X(T) - K$ at T (in $\mathcal{D}$). The price process of the *domestic-currency*-collateralized forward contract is

$$\mathrm{E}_t^d \left(e^{-\int_t^T c_d(s)\, ds} (X(T) - K) \right) = X(t) P_{f,d}(t,T) - K P_{d,d}(t,T)$$

- The *forward FX rate*, i.e. K that makes the price process have value zero is given by $X_d(t,T) = \frac{X(t) P_{f,d}(t,T)}{P_{d,d}(t,T)}$.
- We can also view a forward FX contract as paying $1 - K/X(T)$ in $\mathcal{F}$
- Then, with *foreign collateralization*, the value would be

$$\mathrm{E}_t^f \left(e^{-\int_t^T c_f(s)\, ds} (1 - K/X(T)) \right) = P_{f,f}(t,T) - K P_{d,f}(t,T)/X(t)$$

  and the forward FX rate collateralized in c_f is given by $X_f(t,T) = \frac{X(t) P_{f,f}(t,T)}{P_{d,f}(t,T)}$
- In the general model, there is no reason why $X_f(t,T)$ would be equal to $X_d(t,T)$, and the forward FX rate would depend on the collateral used. It appears, however, that in current market practice FX forwards are quoted without regard for the collateral arrangements

- Consider a domestic asset, with price process $V(t)$, that can be collateralized either in the domestic (rate c_d) or the foreign (rate c_f) currency.
- Common case for CSA agreements between dealers
- From previous analysis it follows that the foreign-collateralized domestic ZCB grows (in the domestic currency) at the rate $c_f + r_{d,f}$
- It can be shown rigorously that the same is true for any domestic asset
- When one can choose the collateral, one would maximize the rate received on it, so the choice collateral rate is equal to

$$\max(c_d(t), c_f(t) + r_{d,f}(t)) = c_d(t) + \max(c_f(t) + r_{d,f}(t) - c_d(t), 0)$$

- The simplest extension of the traditional cross-currency model that accounts for different collateralization would keep the spread

$$q_{d,f}(t) \triangleq c_f(t) + r_{d,f}(t) - c_d(t)$$

deterministic (intrinsic)

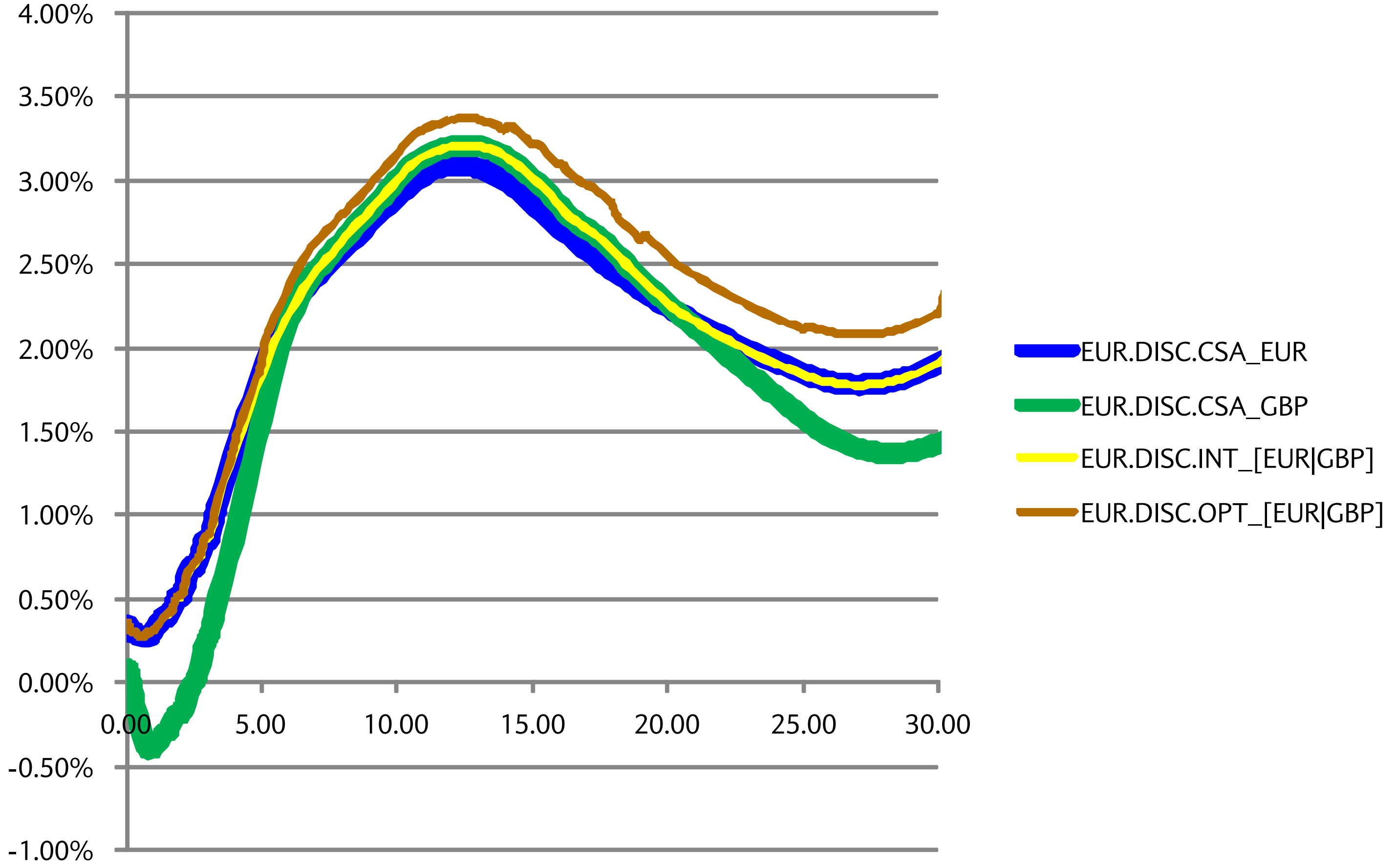
21 Choice Collateral

- In this case the collateral choice will not generate any optionality although the discounting curve for the choice collateral rate will be modified
- Anecdotal evidence suggests that at least some dealers do assign some value to the option to switch collateral in the future
- Full collateral choice model:

$$V(t) = \mathrm{E}_t^d \left(e^{-\int_t^T c_d(s)\,ds} e^{-\int_t^T \max(q_{d,f}(s),0)\,ds} V(T) \right)$$

- At least 4 factors: one for each of c_d, c_f, X, $q_{d,f}$. "Standard" XC model recovered with $q_{d,f} \equiv 0$.

Example. For Intrinsic take FX-adjusted overnight forward curves and form a maximum.



23 Issues with Full Collateral Choice Model

- Large number of unobserved parameters (volatilities, correlations of $q_{d,f}$)
- Uncertain horizon – collateral choice may go away with developments in the industry (more clearing, standard CSA)
- Assumes that instantaneous replacement of collateral from one currency to another is possible
- More realistic assumptions (?)
  - Only *change* in collateral balance can be posted in a choice currency
  - Only currency previously posted can be recalled, not exceeding the total amount posted (and change in MTM)
- This results in a path-dependent, non-linear dynamic optimization problem
- All in all, swaps pricing is getting quite complicated!

- Consider an asset that we sold, with a positive MTM throughout the life, like a ZCB
- Price process is given by V_n (value to us is $-V_n$), $n = 0, \ldots, N+1$.
- We post collateral. Suppose we can post two types, with rates r_n and p_n. At time t_n we post A_n of type 1 and B_n of type 2.
- Basic restrictions are

$$A_n + B_n = V_n \tag{11}$$
$$A_n, B_n \geq 0. \tag{12}$$

- Path dependent restrictions. Having fixed $A_0, \ldots, A_{n-1}$, we have the following on A_n
  - If $V_n > V_{n-1}$ then the difference $\Delta V_n \triangleq V_n - V_{n-1}$ can be added to A or to B. So we must have

$$0 \leq A_n - A_{n-1} \leq \Delta V_n$$

  - If $V_n < V_{n-1}$ we can subtract up to $V_{n-1} - V_n$ from either A or B. But we cannot violate the restriction (12). After some manipulations

$$\max(-A_{n-1}, \Delta V_n) \leq A_n - A_{n-1} \leq \min(V_n - A_{n-1}, 0)$$

- Total interest on collateral account
$$\sum_{n=0}^{N} (r_n A_n + p_n B_n) = \sum_{n=0}^{N} (r_n A_n + p_n (V_n - A_n))$$
- Let $q_n = r_n - p_n$ then the total accrual is equal to
$$\sum_{n=0}^{N} p_n V_n + \sum_{n=0}^{N} q_n A_n$$
and the first term is independent of the strategy. Let us focus on the second one.
- Let $J_k(a_k)$ be the time-t_k the expected value of the optimal strategy for terms from k onwards, assuming $A_k = a_k$. Let $(A)_k$ be a particular strategy from k onwards, i.e. $(A)_k = \{A_k, \ldots, A_N\}$. Denote the allowed set of values for A_{k+1} given $A_k = a$ by $I_k(a)$, and the set of allowed strategies $(A)_k$ assuming $A_k = a$ by $\theta_k(a)$

- From the optimal control condition

$$
\begin{aligned}
J_k(a) &= \mathrm{E}_k\left(\max_{(A)_k\in\theta_k(a)}\left(\sum_{n=k}^{N} q_n A_n\right)\right) \\
&= q_k a + \mathrm{E}_k\left(\max_{A_{k+1}\in I_k(a)}\left(\max_{(A)_{k+1}\in\theta_{k+1}(A_{k+1})}\sum_{n=k+1}^{N} q_n A_n\right)\right) \\
&= q_k a + \mathrm{E}_k\left(\max_{\tilde{a}\in I_k(a)} J_{k+1}(\tilde{a})\right)
\end{aligned}
$$

- Only numerical solution, done at a portfolio level (not easy)
- Solution can be below intrinsic collateral choice (when $q_{d,f}$ deterministic)
- Strong dependence of solution on average daily (absolute) move in portfolio value
- No non-trivial $\Delta t \to 0$ limit as BM has infinite absolute variation (so full collateral choice formula is recovered)

27 Non-linear features

- Some CSA features prevent even an approximate “curve” valuation
  - One-way CSA
  - Material thresholds and minimum transfer amounts
  - Ratings triggers
  - Complicated netting rules not available in FO systems
- All in all, swaps pricing is getting quite complicated!

- Derivatives trading desk:
  - Trades derivatives
  - Hedges with underlying assets
  - Lends/borrows money
- Money comes from different sources
  - Unsecured
  - Secured by assets
  - Collateral (daily MTM paid/received in cash or high quality bonds)
- Historically do not differentiate between different rates
- But now large differences and volatility in *funding spread*
- How does it affect pricing (from funding prospective)?
- For inclusion of credit risk see [BK11]

29 Uncollateralized Derivatives

- Assume imperfect (allowing for none) collateralization
- Money generated/required by derivatives trading and not posted/received as collateral should come from somewhere
- For a trading desk, it comes from the funding desk
- Funding desk typically funds cash balances at the overnight funding rate determined by the cost of funds the banks can get externally
- Overnight funding can be included into our framework by considering it as another type of "collateral", i.e. cashflows that generate funding rate
- Have a model as before except "collateral account" pays a mixture of the external collateral and internal funding rate

- Asset $S(t)$
- Various rates
  - Risk-free overnight rate $r_C(t)$, paid on (true) collateral accounts (under CSA)
  - Asset dividend rate $r_D(t)$
  - Rate on borrowing secured by asset $r_R(t)$
  - Unsecured bank funding rate $r_F(t)$
  - funding spread $s_F(t) \triangleq r_F(t) - r_C(t)$
- Let $C(t)$ be the collateral held at time t against this derivative (allow to be different from $V(t)$). Pays rate $r_C(t)$
- The balance $V(t) - C(t)$ pays rate $r_F(t)$

Valuation with Partial Collateral

- Considering cashflows as before, we obtain that there exists a (risk-neutral) measure in which the price process grows as

  $$r_C(t)C(t)\,dt + r_F(t)\,(V(t) - C(t))\,dt = r_F(t)V(t)\,dt - (r_F(t) - r_C(t))\,C(t)\,dt$$

  while the stock grows at rate $r_R(t) - r_D(t)$
- The solution is given by

  $$V(t) = \mathrm{E}_t\left(e^{-\int_t^T r_F(u)du}V(T) + \int_t^T e^{-\int_t^u r_F(v)dv}\,(r_F(u) - r_C(u))\,C(u)\,du\right) \tag{13}$$

  in measure where

  $$dS(t)/S(t) = (r_R(t) - r_D(t))\;dt + \sigma_S(t)\,dW_S(t) \tag{14}$$

- Another useful formula

  $$V(t) = \mathrm{E}_t\left(e^{-\int_t^T r_C(u)\,du}V(T)\right) - \mathrm{E}_t\left(\int_t^T e^{-\int_t^u r_C(u)du}\,(r_F(u) - r_C(u))\,(V(u) - C(u))\;du\right) \tag{15}$$

- Same results obtained from the Black-Scholes extension in [Pit10]

32 Conclusions

- Careful considerations of the details of bank funding (through CSAs and externally) are critical to derivatives valuation
- Resulting framework is rather complex
  - Many unobservable parameters
  - Significant uncertainty in evolution of market structure
  - "Small" details of legal agreements (e.g. if collateral can be substituted) lead to significant differences in prices
- Full realism is probably unattainable. Need simplifications that still capture main effects
- Not only discounting is affected – forward curves are counterparty and CSA dependent
- A lot more work remains

References

[BK11] Christoph Burgard and Mats Kjaer. Partial differential equation representations of derivatives with counterparty risk and funding costs. *The Journal of Credit Risk*, 7(3):1–19, 2011.

[FT11] Masaaki Fujii and Akihiko Takahashi. Choice of collateral currency. *Risk*, 1:120–125, 2011.

[Pit10] Vladimir V. Piterbarg. Funding beyond discounting: Collateral agreements and derivatives pricing. *Risk*, 2:97–102, 2010.

[Pit12] Vladimir V. Piterbarg. Cooking with collateral. *Risk*, 8:58–63, 2012.